Novel properties of neighbourly edge irregular interval-valued neutrosophic graphs

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Abstract. In this paper, some types of edge irregular interval-valued neutrosophic graphs such as neighbourly edge irregular interval-valued neutrosophic graphs and neighbourly edge totally irregular interval-valued neutrosophic graphs are introduced. A comparative study between neighbourly edge irregular interval-valued neutrosophic graphs and neighbourly edge totally irregular interval-valued neutrosophic graphs is done. Likewise some properties of them are studied.

Keywords: edge degree in IVNG, edge totall degree in IVNG, edge irregular IVNG, neighbourly edge irregular IVNG, neighbourly edge totally irregular IVNG.

1. Introduction

In 1736, Euler first introduced the concept of graph theory. In the history of mathematics, the solution given by Euler of the well known Konigsberg bridge problem is considered to be the first theorem of graph theory. This has now become a subject generally regarded as a branch of combinatorics. The theory of graph is an extremely useful tool for solving combinatorial problems in different areas such as logic, geometry, algebra, topology, analysis, number theory, information theory, artificial intelligence, operations research, optimization, neural networks, planning, computer science and etc [9, 10, 11, 13].

Fuzzy set theory, introduced by Zadeh in 1965, is a mathematical tool for handling uncertainties like vagueness, ambiguity and imprecision in linguistic variables [31]. Research on theory of fuzzy sets has been witnessing an exponential growth; both within mathematics and in its application. Fuzzy set

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theory has emerged as a potential area of interdisciplinary research and fuzzy graph theory is of recent interest.

Atanassov [3, 4] proposed the extended form of fuzzy set theory by adding a new component, called, intuitionistic fuzzy sets. Smarandache [23, 24] introduced the concept of neutrosophic sets by combining the non-standard analysis. In neutrosophic set, the membership value is associated with three components: truth-membership (T), indeterminacy-membership (I) and falsity-membership (F), in which each membership value is a real standard or non-standard subset of the non-standard unit interval $]0^-, 1^+[$ and there is no restriction on their sum. Smarandache [25] and Wang et al. [29] presented the notion of single valued neutrosophic sets to apply neutrosophic sets in real life problems more conveniently. In single valued neutrosophic sets, three components are independent and their values are taken from the standard unit interval $[0, 1]$. Wang et al. [30] presented the concept of interval-valued neutrosophic sets, which is more precise and more flexible than the single valued neutrosophic set. An interval-valued neutrosophic set is a generalization of the concept of single valued neutrosophic set, in which three membership (T, I, F) functions are independent, and their values belong to the unit interval $[0, 1]$.

In 1975, Rosenfeld [19] introduced the concept of fuzzy graphs. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Later on, Bhattacharya gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Mordeson and Peng [12].

Later, Broumi et al. [5] presented the concept of single valued neutrosophic graphs by combining the single valued neutrosophic set theory and the graph theory, and defined different types of single valued neutrosophic graphs (SVNG). Recently, same authors [2, 6, 7, 8] introduced the concept of interval-valued neutrosophic graph as a generalization of fuzzy graph, intuitionistic fuzzy graph and single valued neutrosophic graph, and discussed some of their properties with examples. Moreover, Akram and Nasir [1] have introduced several concepts on interval-valued neutrosophic graphs.

A. Nagoorgani and K. Radha [15, 16] introduced the concept of regular fuzzy graphs and defined degree of a vertex in fuzzy graphs. A. Nagoorgani and S.R. Latha [14] introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2008. S.P.Nandhini and E.Nandhini introduced the concept of strongly irregular fuzzy graphs and discussed about its properties [17].

K. Radha and N. Kumaravel [18] introduced the concept of edge degree, total edge degree in fuzzy graph and edge regular fuzzy graphs and discussed about the degree of an edge in some fuzzy graphs. N.R. Santhi Maheswari and C. Sekar introduced the concept of edge irregular fuzzy graphs and edge totally irregular fuzzy graphs and discussed about its properties [20]. Also, N.R. Santhi Maheswari and C. Sekar introduced the concept of neighbourly edge irregular fuzzy graphs, neighbourly edge totally irregular fuzzy graphs, strongly edge irregular fuzzy graphs and strongly edge totally irregular fuzzy graphs and discussed about its properties [21, 22]. Then we introduced this concepts on intuitionistic fuzzy graphs, single valued neutrosophic graphs and interval-valued neutrosophic graphs and discussed about their properties [26, 27, 28].

This is the background to introduce neighbourly edge irregular interval-valued neutrosophic graphs, neighbourly edge totally irregular interval-valued neutrosophic graphs and discussed some of their properties. Also neighbourly edge irregularity and strongly edge irregularity on some interval-valued neutrosophic graphs whose underlying crisp graphs are a path, a cycle and a star are studied.

2. Preliminaries

We present some known definitions and results for ready reference to go through the work presented in this paper.

Definition 2.1. *A graph is an ordered pair $G^* = (V, E)$, where V is the set of vertices of G^* and E is the set of edges of G^*. A graph G^* is finite if its vertex set and edge set are finite.*

Definition 2.2. *The degree $d_{G^*}(v)$ of a vertex v in G^* or simply $d(v)$ is the number of edges of G^* incident with vertex v.*

Definition 2.3. *A Fuzzy graph denoted by $G : (\sigma, \mu)$ on the graph $G^* : (V, E)$ is a pair of functions (σ, μ) where $\sigma : V \to [0, 1]$ is a fuzzy subset of a non empty set V and $\mu : E \to [0, 1]$ is a symmetric fuzzy relation on σ such that for all u and v in V the relation $\mu(u, v) = \mu(uv) \leq min(\sigma(u), \sigma(v))$ is satisfied.*

Definition 2.4. *A single valued neutrosophic graph (SVNG) is of the form $G : (A, B)$ where $A = (T_A, I_A, F_A)$ and $B = (T_B, I_B, F_B)$ such that:*

(i) The functions $T_A : V \to [0, 1]$, $I_A : V \to [0, 1]$ and $F_A : V \to [0, 1]$ denote the degree of truth-membership, the degree of indeterminacy-membership and the degree of falsity-membership of the element $u \in V$, respectively, and $0 \leq T_A(u) + I_A(u) + F_A(u) \leq 3$ for every $u \in V$;

(ii) The functions $T_B : V \times V \to [0, 1], I_B : V \times V \to [0, 1]$ and $F_B : V \times V \to [0, 1]$ are the degree of truth-membership, the degree of indeterminacy-membership and the degree of falsity-membership of the edge $uv \in E$, respectively, such that $T_B(uv) \leq \min[T_A(u), T_A(v)]$, $I_B(uv) \geq \max[I_A(u), I_A(v)]$ and $F_B(uv) \geq \max[F_A(u), F_A(v)]$ and $0 \leq T_B(uv) + I_B(uv) + F_B(uv) \leq 3$ for every uv in E.

Definition 2.5. *Let $G : (A, B)$ be a SVNG on $G^* : (V, E)$. Then the degree of a vertex u is defined as $d_G(u) = (d_{T_A}(u), d_{I_A}(u), d_{F_A}(u))$ where $d_{T_A}(u) = \sum_{v \neq u} T_B(uv)$, $d_{I_A}(u) = \sum_{v \neq u} I_B(uv)$ and $d_{F_A}(u) = \sum_{v \neq u} F_B(uv)$.*

Definition 2.6. *Let $G : (A, B)$ be a SVNG on $G^* : (V, E)$. Then the total degree of a vertex u is defined by $td_G(u) = (td_{T_A}(u), td_{I_A}(u), td_{F_A}(u))$ where*

$td_{T_A}(u) = \sum_{v \neq u} T_B(uv) + T_A(u)$, $td_{I_A}(u) = \sum_{v \neq u} I_B(uv) + I_A(u)$ *and* $td_{F_A}(u) = \sum_{v \neq u} F_B(uv) + F_A(u)$.

Definition 2.7. *An interval-valued fuzzy graph (IVFG) is of the form* $G : (\sigma, \mu)$ *where* $\sigma = [\sigma^-, \sigma^+]$ *is an interval-valued fuzzy set in* V *and* $\mu = (\mu^-, \mu^+)$ *is an interval-valued fuzzy set in* $E \subseteq V \times V$ *such that* $\mu^-(uv) \leq min(\sigma^-(u), \sigma^-(v))$ *and* $\mu^+(uv) \leq min(\sigma^+(u), \sigma^+(v))$ *for every* uv *in* E.

Definition 2.8. *Let* $G : (\sigma, \mu)$ *be an IVFG on* $G^* : (V, E)$. *Then the degree of a vertex* u *is defined as* $d_G(u) = (d_{\sigma^-}(u), d_{\sigma^+}(u))$ *where* $d_{\sigma^-}(u) = \sum_{v \neq u} \mu^-(u, v)$ *and* $d_{\sigma^+}(u) = \sum_{v \neq u} \mu^+(u, v)$.

Definition 2.9. *Let* $G : (\sigma, \mu)$ *be an IVFG on* $G^* : (V, E)$. *Then the total degree of a vertex* u *is defined by* $td_G(u) = (td_{\sigma^-}(u), td_{\sigma^+}(u))$ *where* $td_{\sigma^-}(u) = \sum_{v \neq u} \mu^-(u, v) + \sigma^-(u)$ *and* $td_{\sigma^+}(u) = \sum_{v \neq u} \mu^+(u, v) + \sigma^+(u)$.

3. Interval-valued neutrosophic graphs (IVNGs)

Throughout this paper, we denote $G^* : (V, E)$ a crisp graph, and $G : (A, B)$ an interval-valued neutrosophic graph.

Definition 3.1. *By an interval-valued neutrosophic graph(IVNG) of a graph* $G^* : (V, E)$ *we mean a pair* $G : (A, B)$, *where* $A : (T_A, I_A, F_A) = ((T_A^-, T_A^+), (I_A^-, I_A^+), (F_A^-, F_A^+))$ *is an interval-valued neutrosophic set on* V, *and* $B : (T_B, I_B, F_B) = ((T_B^-, T_B^+), (I_B^-, I_B^+), (F_B^-, F_B^+))$ *is an interval-valued neutrosophic relation on* E *satisfying the following condition:*

(i) $V = v_1, v_2, ..., v_n$ *such that* $T_A^- : V \to [0, 1]$, $T_A^+ : V \to [0, 1]$, $I_A^- : V \to [0, 1]$, $I_A^+ : V \to [0, 1]$, $F_A^- : V \to [0, 1]$ *and* $F_A^+ : V \to [0, 1]$ *denote the degree of truth-membership, the degree of indeterminacy-membership and falsity-membership of the element* $v_i \in V$, *respectively, and* $0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$ *for all* $v_i \in V, (i = 1, 2, ..., n)$.

(ii) The functions $T_B^- : V \times V \to [0, 1]$, $T_B^+ : V \times V \to [0, 1]$, $I_B^- : V \times V \to [0, 1]$, $I_B^+ : V \times V \to [0, 1]$, $F_B^- : V \times V \to [0, 1]$ *and* $F_B^+ : V \times V \to [0, 1]$ *are such that:*
$T_B^-(v_iv_j) \leq \min(T_A^-(v_i), T_A^-(v_j))$, $T_B^+(v_iv_j) \leq \min(T_A^+(v_i), T_A^+(v_j))$,
$I_B^-(v_iv_j) \geq \max(I_A^-(v_i), I_A^-(v_j))$, $I_B^+(v_iv_j) \geq \max(I_A^+(v_i), I_A^+(v_j))$,
$F_B^-(v_iv_j) \geq \max(F_A^-(v_i), F_A^-(v_j))$ *and* $F_B^+(v_iv_j) \geq \max(F_A^+(v_i), F_A^+(v_j))$
denotes the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge $v_iv_j \in E$ *respectively, where* $0 \leq T_B(v_iv_j) + I_B(v_iv_j) + F_B(v_iv_j) \leq 3$ *for all* $v_iv_j \in E, (i, j = 1, 2, ..., n)$.

Definition 3.2. *Let* $G : (A, B)$ *be an interval-valued neutrosophic graph on* $G^* : (V, E)$. *Then the degree of a vertex* v_i *is defined as*
$d_G(v_i) = ((d_{T_A^-}(v_i), d_{T_A^+}(v_i)), (d_{I_A^-}(v_i), d_{I_A^+}(v_i)), (d_{F_A^-}(v_i), d_{F_A^+}(v_i)))$ *where*
$d_{T_A^-}(v_i) = \sum_{v_i \neq v_j} T_B^-(v_iv_j)$ $d_{T_A^+}(v_i) = \sum_{v_i \neq v_j} T_B^+(v_iv_j)$,

$d_{I_A^-}(v_i) = \sum_{v_i \neq v_j} I_B^-(v_i v_j)$, $d_{I_A^+}(v_i) = \sum_{v_i \neq v_j} I_B^+(v_i v_j)$,
$d_{F_A^-}(v_i) = \sum_{v_i \neq v_j} F_B^-(v_i v_j)$ *and* $d_{F_A^+}(v_i) = \sum_{v_i \neq v_j} F_B^+(v_i, v_j)$.

Definition 3.3. *Let* $G : (A, B)$ *be an interval-valued neutrosophic graph on* $G^* : (V, E)$. *Then the total degree of a vertex* v_i *is defined as*
$td_G(v_i) = ((td_{T_A^-}(v_i), td_{T_A^+}(v_i)), (td_{I_A^-}(v_i), td_{I_A^+}(v_i)), (td_{F_A^-}(v_i), td_{F_A^+}(v_i)))$ *where*
$td_{T_A^-}(v_i) = \sum_{v_i \neq v_j} T_B^-(v_i v_j) + T_A^-(v_i)$, $td_{T_A^+}(v_i) = \sum_{v_i \neq v_j} T_B^+(v_i v_j) + T_A^+(v_i)$,
$td_{I_A^-}(v_i) = \sum_{v_i \neq v_j} I_B^-(v_i v_j) + I_A^-(v_i)$, $td_{I_A^+}(v_i) = \sum_{v_i \neq v_j} I_B^+(v_i v_j) + I_A^+(v_i)$,
$td_{F_A^-}(v_i) = \sum_{v_i \neq v_j} F_B^-(v_i v_j) + F_A^-(v_i)$ *and* $td_{F_A^+}(v_i) = \sum_{v_i \neq v_j} F_B^+(v_i, v_j) + F_A^+(v_i)$.

Definition 3.4. *Let* $G : (A, B)$ *be an interval-valued neutrosophic graph on* $G^* : (V, E)$. *Then:*

(i) G is irregular, if there is a vertex which is adjacent to vertices with distinct degrees.

(ii) G is totally irregular, if there is a vertex which is adjacent to vertices with distinct total degrees.

Definition 3.5. *Let* $G : (A, B)$ *be a connected interval-valued neutrosophic graph on* $G^* : (V, E)$. *Then:*

(i) G is said to be a neighbourly irregular IVNG if every pair of adjacent vertices have distinct degrees.

(ii) G is said to be a neighbourly totally IVNG if every pair of adjacent vertices have distinct total degrees.

(iii) G is said to be a strongly irregular IVNG if every pair of vertices have distinct degrees.

(iv) G is said to be a strongly totally irregular IVNG if every pair of vertices have distinct total degrees.

(v) G is said to be a highly irregular IVNG if every vertex in G is adjacent to the vertices having distinct degrees.

(vi) G is said to be a highly totally irregular IVNG if every vertex in G is adjacent to the vertices having distinct total degrees.

Definition 3.6. *Let* $G : (A, B)$ *be an interval-valued neutrosophic graph on* $G^* : (V, E)$. *The degree of an edge* $v_i v_j$ *is defined as*
$d_G(v_i v_j) = ((d_{T_B^-}(v_i v_j), d_{T_B^+}(v_i v_j)), (d_{I_B^-}(v_i v_j), d_{I_B^+}(v_i v_j)), (d_{F_B^-}(v_i v_j), d_{F_B^+}(v_i v_j)))$
where
$d_{T_B^-}(v_i v_j) = d_{T_A^-}(v_i) + d_{T_A^-}(v_j) - 2T_B^-(v_i v_j)$,
$d_{T_B^+}(v_i v_j) = d_{T_A^+}(v_i) + d_{T_A^+}(v_j) - 2T_B^+(v_i v_j)$,
$d_{I_B^-}(v_i v_j) = d_{I_A^-}(v_i) + d_{I_A^-}(v_j) - 2I_B^-(v_i v_j)$,
$d_{I_B^+}(v_i v_j) = d_{I_A^+}(v_i) + d_{I_A^+}(v_j) - 2I_B^+(v_i v_j)$,
$d_{F_B^-}(v_i v_j) = d_{F_A^-}(v_i) + d_{F_A^-}(v_j) - 2F_B^-(v_i v_j)$ *and*
$d_{F_B^+}(v_i v_j) = d_{F_A^+}(v_i) + d_{F_A^+}(v_j) - 2F_B^+(v_i v_j)$.

Definition 3.7. *Let $G:(A,B)$ be an interval-valued neutrosophic graph on $G^*:(V,E)$. The total degree of an edge v_iv_j is defined as*
$td_G(v_iv_j)=((td_{T_B^-}(v_iv_j),td_{T_B^+}(v_iv_j)),(td_{I_B^-}(v_iv_j),td_{I_B^+}(v_iv_j)),(td_{F_B^-}(v_iv_j),$
$td_{F_B^+}(v_iv_j)))$ *where*
$td_{T_B^-}(v_iv_j)=d_{T_A^-}(v_i)+d_{T_A^-}(v_j)-T_B^-(v_iv_j)=d_{T_B^-}(v_iv_j)+T_B^-(v_iv_j)$,
$td_{T_B^+}(v_iv_j)=d_{T_A^+}(v_i)+d_{T_A^+}(v_j)-T_B^+(v_iv_j)=d_{T_B^+}(v_iv_j)+T_B^+(v_iv_j)$,
$td_{I_B^-}(v_iv_j)=d_{I_A^-}(v_i)+d_{I_A^-}(v_j)-I_B^-(v_iv_j)=d_{I_B^-}(v_iv_j)+I_B^-(v_iv_j)$,
$td_{I_B^+}(v_iv_j)=d_{I_A^+}(v_i)+d_{I_A^+}(v_j)-I_B^+(v_iv_j)=d_{I_B^+}(v_iv_j)+I_B^+(v_iv_j)$,
$td_{F_B^-}(v_iv_j)=d_{F_A^-}(v_i)+d_{F_A^-}(v_j)-F_B^-(v_iv_j)=d_{F_B^-}(v_iv_j)+F_B^-(v_iv_j)$ *and*
$td_{F_B^+}(v_iv_j)=d_{F_A^+}(v_i)+d_{F_A^+}(v_j)-F_B^+(v_iv_j)=d_{F_B^+}(v_iv_j)+F_B^+(v_iv_j)$.

4. Neighbourly edge irregular interval-valued neutrosophic graphs and neighbourly edge totally irregular interval-valued neutrosophic graphs

In this section, neighbourly edge irregular interval-valued neutrosophic graphs and neighbourly edge totally irregular interval-valued neutrosophic graphs are introduced.

Definition 4.1. *Let $G:(A,B)$ be a connected interval-valued neutrosophic graph on $G^*:(V,E)$. Then G is said to be:*

(i) A neighbourly edge irregular interval-valued neutrosophic graph if every pair of adjacent edges have distinct degrees.

(ii) A neighbourly edge totally irregular interval-valued neutrosophic graph if every pair of adjacent edges have distinct total degrees.

Example 4.1. Graph which is both neighbourly edge irregular interval-valued neutrosophic graph and neighbourly edge totally irregular interval-valued neutrosophic graph.

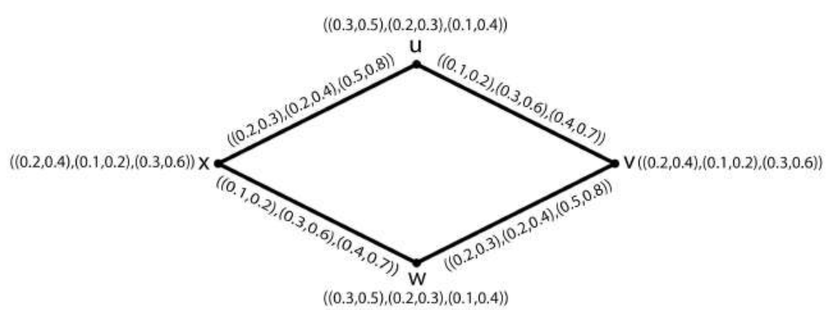

Figure 1: Both neighbourly edge irregular IVNG and neighbourly edge totally irregular IVNG

Consider $G^* : (V, E)$ where $V = \{u, v, w, x\}$ and $E = \{uv, vw, wx, xu\}$. From Figure 1, $d_G(u) = d_G(v) = d_G(w) = d_G(x) = ((0.3, 0.5), (0.5, 1.0), (0.9, 1.5))$.

Degrees of the edges are calculated as follows $d_G(uv) = d_G(wx) = ((0.4, 0.6), (0.4, 0.8), (1.0, 1.6)), d_G(vw) = d_G(xu) = ((0.2, 0.4), (0.6, 1.2), (0.8, 1.4))$.

It is noted that every pair of adjacent edges have distinct degrees. Hence, G is a neighbourly edge irregular interval-valued neutrosophic graph.

Total degrees of the edges are calculated as follows $td_G(uv) = td_G(wx) = ((0.5, 0.8), (0.7, 1.4), (1.4, 2.3)), td_G(vw) = td_G(xu) = ((0.4, 0.7), (0.8, 1.6), (1.3, 2.2))$.

It is observed that every pair of adjacent edges having distinct total degrees. So, G is a neighbourly edge totally irregular interval-valued neutrosophic graph.

Hence G is both neighbourly edge irregular interval-valued neutrosophic graph and neighbourly edge totally irregular interval-valued neutrosophic graph.

Example 4.2. Neighbourly edge irregular interval-valued neutrosophic graph don't need to be neighbourly edge totally irregular interval-valued neutrosophic graph.

Consider $G : (A, B)$ be an interval-valued neutrosophic graph such that $G^* : (V, E)$ is a star on four vertices.

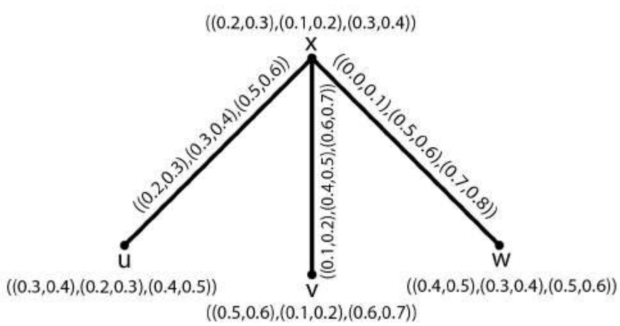

Figure 2: Neighbourly edge irregular IVNG, not neighbourly edge totally irregular IVNG

From Figure 2, $d_G(u) = ((0.2, 0.3), (0.3, 0.4), (0.5, 0.6)), d_G(v) = ((0.1, 0.2), (0.4, 0.5), (0.6, 0.7)), d_G(w) = ((0.0, 0.1), (0.5, 0.6), (0.7, 0.8)), d_G(x) = ((0.3, 0.6), (1.2, 1.5), (1.8, 2.1))$; $d_G(ux)=((0.1, 0.3), (0.9, 1.1), (1.3, 1.5)), d_G(vx)=((0.2, 0.4), (0.8, 1.0), (1.2, 1.4)), d_G(wx)=((0.3, 0.5), (0.7, 0.9), (1.1, 1.3))$; $td_G(ux)=td_G(vx) = td_G(wx) = ((0.3, 0.6), (1.2, 1.5), (1.8, 2.1))$.

Here, $d_G(ux) \neq d_G(vx) \neq d_G(wx)$. Hence G is a neighbourly edge irregular interval-valued neutrosophic graph. But G is not a neighbourly edge totally irregular interval-valued neutrosophic graph, since all edges have same total degrees.

Example 4.3. Neighbourly edge totally irregular interval-valued neutrosophic graphs don't need to be neighbourly edge irregular interval-valued neutrosophic graphs. Following shows this subject:

Consider $G : (A, B)$ be an interval-valued neutrosophic graph such that $G^* : (V, E)$ is a path on four vertices.

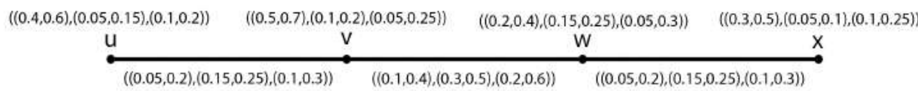

Figure 3: Neighbourly edge totally irregular IVNG, not neighbourly edge irregular IVNG

From Figure 3, $d_G(u) = d_G(x) = ((0.05, 0.20), (0.15, 0.25), (0.1, 0.3)), d_G(v) = d_G(w) = ((0.15, 0.60), (0.45, 0.75), (0.3, 0.9))$; $d_G(uv) = d_G(vw) = d_G(wx) = ((0.1, 0.4), (0.3, 0.5), (0.2, 0.6))$; $td_G(uv) = ((0.15, 0.60), (0.45, 0.75), (0.3, 0.9))$, $td_G(vw) = ((0.2, 0.8), (0.6, 1.0), (0.4, 1.2)), td_G(wx) = ((0.15, 0.60), (0.45, 0.75), (0.3, 0.9))$.

Here, $d_G(uv) = d_G(vw) = d_G(wx)$. Hence G is not a neighbourly edge irregular interval-valued neutrosophic graph. But G is a neighbourly edge totally irregular interval-valued neutrosophic graph, since $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$.

Theorem 4.1. *Let $G : (A, B)$ be a connected interval-valued neutrosophic graph on $G^* : (V, E)$ and $B : ((T_B^-, T_B^+), (I_B^-, I_B^+), (F_B^-, F_B^+))$ a constant function. Then G is a neighbourly edge irregular interval-valued neutrosophic graph, if and only if G is a neighbourly edge totally irregular interval-valued neutrosophic graph.*

Proof. Assume that $B : ((T_B^-, T_B^+), (I_B^-, I_B^+), (F_B^-, F_B^+))$ is a constant function, let $B(uv) = C$, for all uv in E, where $C = ((C_T^-, C_T^+), (C_I^-, C_I^+), (C_F^-, C_F^+))$ is constant.

Let uv and vw be pair of adjacent edges in E, then we have $d_G(uv) \neq d_G(vw) \Leftrightarrow d_G(uv)+C \neq d_G(vw)+C \Leftrightarrow ((d_{T_B^-}(uv), d_{T_B^+}(uv)), (d_{I_B^-}(uv), d_{I_B^+}(uv)), (d_{F_B^-}(uv), d_{F_B^+}(uv))) + ((C_T^-, C_T^+), (C_I^-, C_I^+), (C_F^-, C_F^+)) \neq ((d_{T_B^-}(vw), d_{T_B^+}(vw)), (d_{I_B^-}(vw), d_{I_B^+}(vw)), (d_{F_B^-}(vw), d_{F_B^+}(vw))) + ((C_T^-, C_T^+), (C_I^-, C_I^+), (C_F^-, C_F^+)) \Leftrightarrow ((d_{T_B^-}(uv) + C_T^-, d_{T_B^+}(uv) + C_T^+), (d_{I_B^-}(uv) + C_I^-, d_{I_B^+}(uv) + C_I^+), (d_{F_B^-}(uv) + C_F^-, d_{F_B^+}(uv) + C_F^+)) \neq ((d_{T_B^-}(vw) + C_T^-, d_{T_B^+}(vw) + C_T^+), (d_{I_B^-}(vw) + C_I^-, d_{I_B^+}(vw) + C_I^+), (d_{F_B^-}(vw)+C_F^-, d_{F_B^+}(vw)+C_F^+)) \Leftrightarrow ((d_{T_B^-}(uv)+T_B^-(uv), d_{T_B^+}(uv)+T_B^+(uv)), (d_{I_B^-}(uv)+I_B^-(uv), d_{I_B^+}(uv)+I_B^+(uv)), (d_{F_B^-}(uv)+F_B^-(uv), d_{F_B^+}(uv)+F_B^+(uv))) \neq ((d_{T_B^-}(vw)+T_B^-(vw), d_{T_B^+}(vw)+T_B^+(vw)), (d_{I_B^-}(vw)+I_B^-(vw), d_{I_B^+}(vw)+I_B^+(vw)),$

$(d_{F_B^-}(vw) + F_B^-(vw), d_{F_B^+}(vw) + F_B^+(vw))) \Leftrightarrow ((td_{T_B^-}(uv), td_{T_B^+}(uv)), (td_{I_B^-}(uv), td_{I_B^+}(uv)), (td_{F_B^-}(uv), td_{F_B^+}(uv))) \neq ((td_{T_B^-}(vw), td_{T_B^+}(vw)), (td_{I_B^-}(vw), td_{I_B^+}(vw)), (td_{F_B^-}(vw), td_{F_B^+}(vw))) \Leftrightarrow td_G(uv) \neq td_G(vw)$. Therefore, every pair of adjacent edges have distinct degrees if and only if have distinct total degrees. Hence G is a neighbourly edge irregular interval-valued neutrosophic graph if and only if G is a neighbourly edge totally irregular interval-valued neutrosophic graph. □

Remark 4.1. Let $G : (A, B)$ be a connected interval-valued neutrosophic graph on $G^* : (V, E)$. If G is both neighbourly edge irregular interval-valued neutrosophic graph and neighbourly edge totally irregular interval-valued neutrosophic graph, Then B don't need to be a constant function.

Example 4.4. Consider $G : (A, B)$ be an interval-valued neutrosophic graph such that $G^* : (V, E)$ is a path on four vertices.

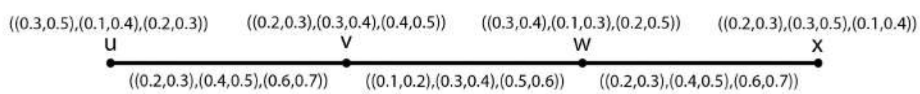

Figure 4: B is not a constant function.

From Figure 4,
$d_G(u) = d_G(x) = ((0.2, 0.3), (0.4, 0.5), (0.6, 0.7))$,
$d_G(v) = d_G(w) = ((0.3, 0.5), (0.7, 0.9), (1.1, 1.3))$;
$d_G(uv) = d_G(wx) = ((0.1, 0.2), (0.3, 0.4), (0.5, 0.6))$,
$d_G(vw) = ((0.4, 0.6), (0.8, 1.0), (1.2, 1.4))$;
$td_G(uv) = td_G(wx) = ((0.3, 0.5), (0.7, 0.9), (1.1, 1.3))$,
$td_G(vw) = ((0.5, 0.8), (1.1, 1.4), (1.7, 2.0))$.

Here, $d_G(uv) \neq d_G(vw)$ and $d_G(vw) \neq d_G(wx)$. Hence G is a neighbourly edge irregular interval-valued neutrosophic graph. Also, $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$. Hence G is a neighbourly edge totally irregular interval-valued neutrosophic graph. But B is not constant function.

Theorem 4.2. *Let $G : (A, B)$ be a connected interval-valued neutrosophic graph on $G^* : (V, E)$ and $B : ((T_B^-, T_B^+), (I_B^-, I_B^+), (F_B^-, F_B^+))$ a constant function. If G is a strongly irregular interval-valued neutrosophic graph, then G is a neighbourly edge irregular interval-valued neutrosophic graph.*

Proof. Let $G : (A, B)$ be a connected interval-valued neutrosophic graph on $G^* : (V, E)$. Assume that $B : ((T_B^-, T_B^+), (I_B^-, I_B^+), (F_B^-, F_B^+))$ is a constant function, let $B(uv) = C$, for all uv in E, where $C = ((C_T^-, C_T^+), (C_I^-, C_I^+), (C_F^-, C_F^+))$ is constant.

Let uv and vw be any two adjacent edges in G. Let us suppose that G is a strongly irregular interval-valued neutrosophic graph. Then, every pair of vertices in G having distinct degrees, and hence $d_G(u) \neq d_G(v) \neq d_G(w) \Rightarrow ((d_{T_A^-}(u), d_{T_A^+}(u)), (d_{I_A^-}(u), d_{I_A^+}(u)), (d_{F_A^-}(u), d_{F_A^+}(u))) \neq ((d_{T_A^-}(v), d_{T_A^+}(v)), (d_{I_A^-}(v), d_{I_A^+}(v)), (d_{F_A^-}(v), d_{F_A^+}(v))) \neq ((d_{T_A^-}(w), d_{T_A^+}(w)), (d_{I_A^-}(w), d_{I_A^+}(w)), (d_{F_A^-}(w), d_{F_A^+}(w))) \Rightarrow ((d_{T_A^-}(u), d_{T_A^+}(u)), (d_{I_A^-}(u), d_{I_A^+}(u)), (d_{F_A^-}(u), d_{F_A^+}(u))) + ((d_{T_A^-}(v), d_{T_A^+}(v)), (d_{I_A^-}(v), d_{I_A^+}(v)), (d_{F_A^-}(v), d_{F_A^+}(v))) - 2((C_T^-, C_T^+), (C_I^-, C_I^+), (C_F^-, C_F^+)) \neq ((d_{T_A^-}(v), d_{T_A^+}(v)), (d_{I_A^-}(v), d_{I_A^+}(v)), (d_{F_A^-}(v), d_{F_A^+}(v))) + ((d_{T_A^-}(w), d_{T_A^+}(w)), (d_{I_A^-}(w), d_{I_A^+}(w)), (d_{F_A^-}(w), d_{F_A^+}(w))) - 2((C_T^-, C_T^+), (C_I^-, C_I^+), (C_F^-, C_F^+)) \Rightarrow ((d_{T_A^-}(u)+d_{T_A^-}(v)-2C_T^-, d_{T_A^+}(u)+d_{T_A^+}(v)-2C_T^+), (d_{I_A^-}(u)+d_{I_A^-}(v)-2C_I^-, d_{I_A^+}(u)+d_{I_A^+}(v)-2C_I^+), (d_{F_A^-}(u)+d_{F_A^-}(v)-2C_F^-, d_{F_A^+}(u)+d_{F_A^+}(v)-2C_F^+)) \neq ((d_{T_A^-}(v)+d_{T_A^-}(w)-2C_T^-, d_{T_A^+}(v)+d_{T_A^+}(w)-2C_T^+), (d_{I_A^-}(v)+d_{I_A^-}(w)-2C_I^-, d_{I_A^+}(v)+d_{I_A^+}(w)-2C_I^+), (d_{F_A^-}(v)+d_{F_A^-}(w)-2C_F^-, d_{F_A^+}(v)+d_{F_A^+}(w)-2C_F^+)) \Rightarrow ((d_{T_A^-}(u)+d_{T_A^-}(v)-2T_B^-(uv), d_{T_A^+}(u)+d_{T_A^+}(v)-2T_B^-(uv)), (d_{I_A^-}(u)+d_{I_A^-}(v)-2I_B^-(uv), d_{I_A^+}(u)+d_{I_A^+}(v)-2I_B^-(uv)), (d_{F_A^-}(u)+d_{F_A^-}(v)-2F_B^-(uv), d_{F_A^+}(u)+d_{F_A^+}(v)-2F_B^-(uv))) \neq ((d_{T_A^-}(v)+d_{T_A^-}(w)-2T_B^-(vw), d_{T_A^+}(v)+d_{T_A^+}(w)-2T_B^-(vw)), (d_{I_A^-}(v)+d_{I_A^-}(w)-2I_B^-(vw), d_{I_A^+}(v)+d_{I_A^+}(w)-2I_B^-(vw)), (d_{F_A^-}(v)+d_{F_A^-}(w)-2F_B^-(vw), d_{F_A^+}(v)+d_{F_A^+}(w)-2F_B^-(vw))) \Rightarrow ((d_{T_B^-}(uv), d_{T_B^+}(uv)), (d_{I_B^-}(uv), d_{I_B^+}(uv)), (d_{F_B^-}(uv), d_{F_B^+}(uv))) \neq ((d_{T_B^-}(vw), d_{T_B^+}(vw)), (d_{I_B^-}(vw), d_{I_B^+}(vw)), (d_{F_B^-}(vw), d_{F_B^+}(vw))) \Rightarrow d_G(uv) \neq d_G(vw)$.

Therefore, every pair of adjacent edges have distinct degrees, hence G is a neighbourly edge irregular interval-valued neutrosophic graph. □

Similar to the above theorem can be considered the following theorem:

Theorem 4.3. *Let $G : (A, B)$ be a connected interval-valued neutrosophic graph on $G^* : (V, E)$ and $B : ((T_B^-, T_B^+), (I_B^-, I_B^+), (F_B^-, F_B^+))$ a constant function. If G is a strongly irregular interval-valued neutrosophic graph, then G is a neighbourly edge totally irregular interval-valued neutrosophic graph.*

Remark 4.2. Converse of the above theorems don't need to be true.

Example 4.5. Consider $G : (A, B)$ be an interval-valued neutrosophic graph such that $G^* : (V, E)$ is a path on four vertices.

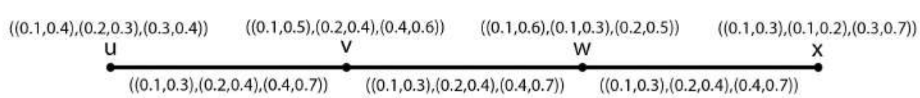

Figure 5: Both neighbourly edge irregular IVNG and neighbourly edge totally irregular IVNG, not strongly irregular IVNG

From Figure 5,
$d_G(u) = d_G(x) = ((0.1, 0.3), (0.2, 0.4), (0.4, 0.7))$,
$d_G(v) = d_G(w) = ((0.2, 0.6), (0.4, 0.8), (0.8, 1.4))$.

Here, G is not a strongly irregular interval-valued neutrosophic graph.

$d_G(uv) = d_G(wx) = ((0.1, 0.3), (0.2, 0.4), (0.4, 0.7))$,
$d_G(vw) = ((0.2, 0.6), (0.4, 0.8), (0.8, 1.4))$;

$td_G(uv) = td_G(wx) = ((0.2, 0.6), (0.4, 0.8), (0.8, 1.4))$,
$td_G(vw) = ((0.3, 0.9), (0.6, 1.2), (1.2, 2.1))$.

It is noted that $d_G(uv) \neq d_G(vw)$ and $d_G(vw) \neq d_G(wx)$. And also, $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$. Hence G is both neighbourly edge irregular interval-valued neutrosophic graph and neighbourly edge totally irregular interval-valued neutrosophic graph. But G is not a strongly irregular interval-valued neutrosophic graph.

Theorem 4.4. *Let $G : (A, B)$ be a connected interval-valued neutrosophic graph on $G^* : (V, E)$ and $B : ((T_B^-, T_B^+), (I_B^-, I_B^+), (F_B^-, F_B^+))$ a constant function. Then G is a highly irregular interval-valued neutrosophic graph if and only if G is a neighbourly edge irregular interval-valued neutrosophic graph.*

Proof. Let $G : (A, B)$ be a connected interval-valued neutrosophic graph on $G^* : (V, E)$. Assume that $B : ((T_B^-, T_B^+), (I_B^-, I_B^+), (F_B^-, F_B^+))$ is a constant function, let $B(uv) = C$, for all uv in E, where $C = ((C_T^-, C_T^+), (C_I^-, C_I^+), (C_F^-, C_F^+))$ is constant.

Let uv and vw be any two adjacent edges in G. Then, we have $d_G(u) \neq d_G(w) \Leftrightarrow ((d_{T_A^-}(u), d_{T_A^+}(u)), (d_{I_A^-}(u), d_{I_A^+}(u)), (d_{F_A^-}(u), d_{F_A^+}(u))) \neq ((d_{T_A^-}(w), d_{T_A^+}(w)), (d_{I_A^-}(w), d_{I_A^+}(w)), (d_{F_A^-}(w), d_{F_A^+}(w))) \Leftrightarrow ((d_{T_A^-}(u), d_{T_A^+}(u)), (d_{I_A^-}(u), d_{I_A^+}(u)), (d_{F_A^-}(u), d_{F_A^+}(u))) + ((d_{T_A^-}(v), d_{T_A^+}(v)), (d_{I_A^-}(v), d_{I_A^+}(v)), (d_{F_A^-}(v), d_{F_A^+}(v))) - 2((C_T^-, C_T^+), (C_I^-, C_I^+), (C_F^-, C_F^+)) \neq ((d_{T_A^-}(v), d_{T_A^+}(v)), (d_{I_A^-}(v), d_{I_A^+}(v)), (d_{F_A^-}(v), d_{F_A^+}(v))) + ((d_{T_A^-}(w), d_{T_A^+}(w)), (d_{I_A^-}(w), d_{I_A^+}(w)), (d_{F_A^-}(w), d_{F_A^+}(w))) - 2((C_T^-, C_T^+), (C_I^-, C_I^+), (C_F^-, C_F^+)) \Leftrightarrow ((d_{T_A^-}(u) + d_{T_A^-}(v) - 2C_T^-, d_{T_A^+}(u) + d_{T_A^+}(v) - 2C_T^+), (d_{I_A^-}(u) + d_{I_A^-}(v) - 2C_I^-, d_{I_A^+}(u) + d_{I_A^+}(v) - 2C_I^+), (d_{F_A^-}(u) + d_{F_A^-}(v) - 2C_F^-, d_{F_A^+}(u) + d_{F_A^+}(v) - 2C_F^+)) \neq ((d_{T_A^-}(v) + d_{T_A^-}(w) - 2C_T^-, d_{T_A^+}(v) + d_{T_A^+}(w) - 2C_T^+), (d_{I_A^-}(v) + d_{I_A^-}(w) - 2C_I^-, d_{I_A^+}(v) + d_{I_A^+}(w) - 2C_I^+), (d_{F_A^-}(v) + d_{F_A^-}(w) - 2C_F^-, d_{F_A^+}(v) + d_{F_A^+}(w) - 2C_F^+)) \Leftrightarrow ((d_{T_A^-}(u) + d_{T_A^-}(v) - 2T_B^-(uv), d_{T_A^+}(u) + d_{T_A^+}(v) - 2T_B^-(uv)), (d_{I_A^-}(u) + d_{I_A^-}(v) - 2I_B^-(uv), d_{I_A^+}(u) + d_{I_A^+}(v) - 2I_B^-(uv)), (d_{F_A^-}(u) + d_{F_A^-}(v) - 2F_B^-(uv), d_{F_A^+}(u) + d_{F_A^+}(v) - 2F_B^-(uv))) \neq ((d_{T_A^-}(v) + d_{T_A^-}(w) - 2T_B^-(vw), d_{T_A^+}(v) + d_{T_A^+}(w) - 2T_B^-(vw)), (d_{I_A^-}(v) + d_{I_A^-}(w) - 2I_B^-(vw), d_{I_A^+}(v) + d_{I_A^+}(w) - 2I_B^-(vw)), (d_{F_A^-}(v) + d_{F_A^-}(w) - 2F_B^-(vw), d_{F_A^+}(v) + d_{F_A^+}(w) - 2F_B^-(vw))) \Leftrightarrow$

$((d_{T_B^-}(uv), d_{T_B^+}(uv)), (d_{I_B^-}(uv), d_{I_B^+}(uv)), (d_{F_B^-}(uv), d_{F_B^+}(uv))) \neq ((d_{T_B^-}(vw), d_{T_B^+}(vw)), (d_{I_B^-}(vw), d_{I_B^+}(vw)), (d_{F_B^-}(vw), d_{F_B^+}(vw))) \Leftrightarrow d_G(uv) \neq d_G(vw)$.

Therefore, every pair of adjacent edges have distinct degrees, if and only if every vertex adjacent to the vertices having distinct degrees. Hence G is a highly irregular interval-valued neutrosophic graph, if and only if G is a neighbourly edge irregular interval-valued neutrosophic graph. □

Theorem 4.5. *Let $G : (A, B)$ be a connected interval-valued neutrosophic graph on $G^* : (V, E)$ and $B : ((T_B^-, T_B^+), (I_B^-, I_B^+), (F_B^-, F_B^+))$ a constant function. Then G is highly irregular interval-valued neutrosophic graph if and only if G is neighbourly edge totally irregular interval-valued neutrosophic graph.*

Proof. Proof is similar to the above Theorem 4.4. □

Theorem 4.6. *Let $G : (A, B)$ be an interval-valued neutrosophic graph on $G^* : (V, E)$, a star $K_{1,n}$. Then G is a totally edge regular interval-valued neutrosophic graph. Also, if the degrees of truth-membership, indeterminacy-membership and falsity-membership of no two edges are same, then G is a neighbourly edge irregular interval-valued neutrosophic graph.*

Proof. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices adjacent to the vertex x. Let $e_1, e_2, e_3, \ldots, e_n$ be the edges of a star G^* in that order having the degrees of truth-membership $p_1, p_2, p_3, \ldots, p_n$, the degrees of indeterminacy-membership $q_1, q_2, q_3, \ldots, q_n$ and the degrees of falsity-membership $r_1, r_2, r_3, \ldots, r_n$ where $p_i = (p_i^-, p_i^+)$, $q_i = (q_i^-, q_i^+)$ and $r_i = (r_i^-, r_i^+)$ for $i = 1, 2, ..., n$ such that $0 \leq p_i + q_i + r_i \leq 3$, for every $1 \leq i \leq n$. Then, $td_G(e_i) = ((td_{T_B^-}(e_i), td_{T_B^+}(e_i)), (td_{I_B^-}(e_i), td_{I_B^+}(e_i)), (td_{F_B^-}(e_i), td_{F_B^+}(e_i))) = ((d_{T_B^-}(e_i) + T_B^-(e_i), d_{T_B^+}(e_i) + T_B^+(e_i)), (d_{I_B^-}(e_i) + I_B^-(e_i), d_{I_B^+}(e_i) + I_B^+(e_i)), (d_{F_B^-}(e_i) + F_B^-(e_i), d_{F_B^+}(e_i) + F_B^+(e_i))) = ((\sum_{k=1}^n p_k^- - p_i^- + p_i^-, \sum_{k=1}^n p_k^+ - p_i^+ + p_i^+), (\sum_{k=1}^n q_k^- - q_i^- + q_i^-, \sum_{k=1}^n q_k^+ - q_i^+ + q_i^+), (\sum_{k=1}^n r_k^- - r_i^- + r_i^-, \sum_{k=1}^n r_k^+ - r_i^+ + r_i^+)) = ((\sum_{k=1}^n p_k^-, \sum_{k=1}^n p_k^+), (\sum_{k=1}^n q_k^-, \sum_{k=1}^n q_k^+), (\sum_{k=1}^n r_k^-, \sum_{k=1}^n r_k^+))$.

All edges e_i , $(1 \leq i \leq n)$, having same total degrees. Hence G is a totally edge regular interval-valued neutrosophic graph.

Now, if $p_i^- \neq p_j^-$, $p_i^+ \neq p_j^+$, $q_i^- \neq q_j^-$, $q_i^+ \neq q_j^+$, $r_i^- \neq r_j^-$ and $r_i^+ \neq r_j^+$, for every $1 \leq i, j \leq n$ then, we have $d_G(e_i) = ((d_{T_B^-}(e_i), d_{T_B^+}(e_i)), (d_{I_B^-}(e_i), d_{I_B^+}(e_i)), (d_{F_B^-}(e_i), d_{F_B^+}(e_i)))$=$((d_{T_A^-}(x) + d_{T_A^-}(v_i) - 2T_B^-(xv_i), d_{T_A^+}(x) + d_{T_A^+}(v_i) - 2T_B^+(xv_i)), (d_{I_A^-}(x) + d_{I_A^-}(v_i) - 2I_B^-(xv_i), d_{I_A^+}(x) + d_{I_A^+}(v_i) - 2I_B^+(xv_i)), (d_{F_A^-}(x) + d_{F_A^-}(v_i) - 2F_B^-(xv_i), d_{F_A^+}(x) + d_{F_A^+}(v_i) - 2F_B^+(xv_i))) = ((\sum_{k=1}^n p_k^- + p_i^- - 2p_i^-, \sum_{k=1}^n p_k^+ + p_i^+ - 2p_i^+), (\sum_{k=1}^n q_k^- + q_i^- - 2q_i^-, \sum_{k=1}^n q_k^+ + q_i^+ - 2q_i^+), (\sum_{k=1}^n r_k^- + r_i^- - 2r_i^-, \sum_{k=1}^n r_k^+ + r_i^+ - 2r_i^+))$=$((\sum_{k=1}^n p_k^- - p_i^-, \sum_{k=1}^n p_k^+ - p_i^+), (\sum_{k=1}^n q_k^- - q_i^-, \sum_{k=1}^n q_k^+ - q_i^+), (\sum_{k=1}^n r_k^- - r_i^-, \sum_{k=1}^n r_k^+ - r_i^+))$ for every $1 \leq i \leq n$.

Therefore, all edges e_i ,$(1 \leq i \leq n)$, having distinct degrees. Hence G is a neighbourly edge irregular interval-valued neutrosophic graph. □

Theorem 4.7. *Let $G:(A,B)$ be an interval-valued neutrosophic graph such that $G^*:(V,E)$ is a path on $2m(m>1)$ vertices. If the degrees of truth-membership, indeterminacy-membership and falsity-membership of the edges e_i, $i=1,3,5,...,2m-1$, are $p_1=(p_1^-,p_1^+)$, $q_1=(q_1^-,q_1^+)$ and $r_1=(r_1^-,r_1^+)$, respectively, and the degrees of truth-membership, indeterminacy-membership and falsity-membership of the edges e_i, $i=2,4,6,...,2m-2$, are $p_2=(p_2^-,p_2^+)$, $q_2=(q_2^-,q_2^+)$ and $r_2=(r_2^-,r_2^+)$, respectively, such that $p_1\neq p_2$ and $p_2\neq 2p_1$ and $q_1\neq q_2$ and $q_2\neq 2q_1$ and $r_1\neq r_2$ and $r_2\neq 2r_1$, then G is both neighbourly edge irregular interval-valued neutrosophic graph and neighbourly edge totally irregular interval-valued neutrosophic graph.*

Proof. Let $G:(A,B)$ be an interval-valued neutrosophic graph on $G^*:(V,E)$, a path on $2m(m>1)$ vertices.

Let $e_1,e_2,e_3,\ldots,e_{2m-1}$ be the edges of path G^*. If the alternate edges have the same degrees of truth-membership, indeterminacy-membership and falsity-membership, such that

$$\begin{aligned}B(e_i)&=(T_B(e_i),I_B(e_i),F_B(e_i))\\&=((T_B^-(e_i),T_B^+(e_i)),(I_B^-(e_i),I_B^+(e_i)),(F_B^-(e_i),F_B^+(e_i)))\\&=\begin{cases}(p_1,q_1,r_1), & \text{if } i \text{ is odd,}\\(p_2,q_2,r_2), & \text{if } i \text{ is even}\end{cases}=\begin{cases}((p_1^-,p_1^+),(q_1^-,q_1^+),(r_1^-,r_1^+)), & \text{if } i \text{ is odd,}\\((p_2^-,p_2^+),(q_2^-,q_2^+),(r_2^-,r_2^+)), & \text{if } i \text{ is even}\end{cases}\end{aligned}$$

where $0\leq p_i+q_i+r_i\leq 3$ for $i=1,2$ and $(p_1^-,p_1^+)\neq(p_2^-,p_2^+)$ and $(p_2^-,p_2^+)\neq 2(p_1^-,p_1^+)$ and $(q_1^-,q_1^+)\neq(q_2^-,q_2^+)$ and $(q_2^-,q_2^+)\neq 2(q_1^-,q_1^+)$ and $(r_1^-,r_1^+)\neq(r_2^-,r_2^+)$ and $(r_2^-,r_2^+)\neq 2(r_1^-,r_1^+)$, then $d_G(e_1)=(((p_1^-)+(p_1^-+p_2^-)-2p_1^-,(p_1^+)+(p_1^++p_2^+)-2p_1^+),((q_1^-)+(q_1^-+q_2^-)-2q_1^-,(q_1^+)+(q_1^++q_2^+)-2q_1^+),((r_1^-)+(r_1^-+r_2^-)-2r_1^-,(r_1^+)+(r_1^++r_2^+)-2r_1^+))=((p_2^-,p_2^+),(q_2^-,q_2^+),(r_2^-,r_2^+))=(p_2,q_2,r_2)$

for $i=3,5,7,\ldots,2m-3$; $d_G(e_i)=(((p_1^-+p_2^-)+(p_1^-+p_2^-)-2p_1^-,(p_1^++p_2^+)+(p_1^++p_2^+)-2p_1^+),((q_1^-+q_2^-)+(q_1^-+q_2^-)-2q_1^-,(q_1^++q_2^+)+(q_1^++q_2^+)-2q_1^+),((r_1^-+r_2^-)+(r_1^-+r_2^-)-2r_1^-,(r_1^++r_2^+)+(r_1^++r_2^+)-2r_1^+))=((2p_2^-,2p_2^+),(2q_2^-,2q_2^+),(2r_2^-,2r_2^+))=(2p_2,2q_2,2r_2)$

for $i=2,4,6,\ldots,2m-2$; $d_G(e_i)=(((p_1^-+p_2^-)+(p_1^-+p_2^-)-2p_2^-,(p_1^++p_2^+)+(p_1^++p_2^+)-2p_2^+),((q_1^-+q_2^-)+(q_1^-+q_2^-)-2q_2^-,(q_1^++q_2^+)+(q_1^++q_2^+)-2q_2^+),((r_1^-+r_2^-)+(r_1^-+r_2^-)-2r_2^-,(r_1^++r_2^+)+(r_1^++r_2^+)-2r_2^+))=((2p_1^-,2p_1^+),(2q_1^-,2q_1^+),(2r_1^-,2r_1^+))=(2p_1,2q_1,2r_1)$ $d_G(e_{2m-1})=(((p_1^-+p_2^-)+(p_1^-)-2p_1^-,(p_1^++p_2^+)+(p_1^+)-2p_1^+),((q_1^-+q_2^-)+(q_1^-)-2q_1^-,(q_1^++q_2^+)+(q_1^+)-2q_1^+),((r_1^-+r_2^-)+(r_1^-)-2r_1^-,(r_1^++r_2^+)+(r_1^+)-2r_1^+))=((p_2^-,p_2^+),(q_2^-,q_2^+),(r_2^-,r_2^+))$ $=(p_2,q_2,r_2)$.

We observe that the adjacent edges have distinct degrees. Hence G is a neighbourly edge irregular interval-valued neutrosophic graph. Also, we have $td_G(e_1)=(p_1+p_2,q_1+q_2,r_1+r_2)$ $td_G(e_i)=(2p_1+p_2,2q_1+q_2,2r_1+r_2)$ for $i=2,4,6,\ldots,2m-2$, $td_G(e_i)=(p_1+2p_2,q_1+2q_2,r_1+2r_2)$ for $i=3,5,7,\ldots,2m-3$ $td_G(e_{2m-1})=(p_1+p_2,q_1+q_2,r_1+r_2)$.

Therefore, the adjacent edges have distinct total degrees, hence G is a neighbourly edge totally irregular interval-valued neutrosophic graph. □

Theorem 4.8. *Let $G:(A,B)$ be an interval-valued neutrosophic graph such that $G^*:(V,E)$ is an even cycle of length $2m$. If the alternate edges have the same degrees of truth-membership, the same degrees of indeterminacy-membership and the same degrees of falsity-membership , then G is both neighbourly edge irregular interval-valued neutrosophic graph and neighbourly edge totally irregular interval-valued neutrosophic graph.*

Proof. Let $G:(A,B)$ be an interval-valued neutrosophic graph on $G^*:(V,E)$, an even cycle of length $2m$. Let $e_1,e_2,e_3,\dots,e_{2m}$ be the edges of cycle G^*. If the alternate edges have the same degrees of truth-membership, the same degrees of indeterminacy-membership and the same degrees of falsity-membership, such that

$$\begin{aligned} B(e_i) &= (T_B(e_i), I_B(e_i), F_B(e_i)) \\ &= ((T_B^-(e_i),T_B^+(e_i)),(I_B^-(e_i),I_B^+(e_i)),(F_B^-(e_i),F_B^+(e_i))) \\ &= \begin{cases} (p_1,q_1,r_1), & \text{if } i \text{ is odd,} \\ (p_2,q_2,r_2), & \text{if } i \text{ is even} \end{cases} = \begin{cases} ((p_1^-,p_1^+),(q_1^-,q_1^+),(r_1^-,r_1^+)), & \text{if } i \text{ is odd,} \\ ((p_2^-,p_2^+),(q_2^-,q_2^+),(r_2^-,r_2^+)), & \text{if } i \text{ is even} \end{cases} \end{aligned}$$

where $0 \le p_i+q_i+r_i \le 3$ for $i=1,2$ and $p_1 \neq p_2$ and $q_1 \neq q_2$ and $r_1 \neq r_2$, then for $i=1,3,5,7,\dots,2m-1$: $d_G(e_i) = (((p_1^-+p_2^-)+(p_1^-+p_2^-)-2p_1^-,(p_1^++p_2^+)+(p_1^++p_2^+)-2p_1^+),((q_1^-+q_2^-)+(q_1^-+q_2^-)-2q_1^-,(q_1^++q_2^+)+(q_1^++q_2^+)-2q_1^+),((r_1^-+r_2^-)+(r_1^-+r_2^-)-2r_1^-,(r_1^++r_2^+)+(r_1^++r_2^+)-2r_1^+)) = ((2p_2^-,2p_2^+),(2q_2^-,2q_2^+),(2r_2^-,2r_2^+)) = (2p_2,2q_2,2r_2)$,
for $i=2,4,6,\dots,2m$: $d_G(e_i) = (((p_1^-+p_2^-)+(p_1^-+p_2^-)-2p_2^-,(p_1^++p_2^+)+(p_1^++p_2^+)-2p_2^+),((q_1^-+q_2^-)+(q_1^-+q_2^-)-2q_2^-,(q_1^++q_2^+)+(q_1^++q_2^+)-2q_2^+),((r_1^-+r_2^-)+(r_1^-+r_2^-)-2r_2^-,(r_1^++r_2^+)+(r_1^++r_2^+)-2r_2^+)) = ((2p_1^-,2p_1^+),(2q_1^-,2q_1^+),(2r_1^-,2r_1^+)) = (2p_1,2q_1,2r_1)$.

We observe that the adjacent edges have distinct degrees. Hence G is a neighbourly edge irregular interval-valued neutrosophic graph. Also, we have $td_G(e_i) = (p_1+2p_2, q_1+2q_2, r_1+2r_2)$, for $i=1,3,5,7,\dots,2m-1$, $td_G(e_i) = (2p_1+p_2, 2q_1+q_2, 2r_1+r_2)$, for $i=2,4,6,\dots,2m$.

Therefore, the adjacent edges have distinct total degrees, hence G is a neighbourly edge totally irregular interval-valued neutrosophic graph. □

Theorem 4.9. *Let $G:(A,B)$ be an interval-valued neutrosophic graph on $G^*:(V,E)$, a cycle on $m (m \ge 4)$ vertices. If the degrees of truth-membership, indeterminacy-membership and falsity-membership of the edges $e_1,e_2,e_3,\dots,e_m$ are $p_1,p_2,p_3,\dots,p_m$ such that $p_1<p_2<p_3<\dots<p_m$ and $q_1,q_2,q_3,\dots,q_m$ such that $q_1>q_2>q_3>\dots>q_m$ and $r_1,r_2,r_3,\dots,r_m$ such that $r_1>r_2>r_3>\dots>r_m$, respectively, then G is both neighbourly edge irregular interval-valued neutrosophic graph and neighbourly edge totally irregular interval-valued neutrosophic graph.*

Proof. Let $G : (A, B)$ be an interval-valued neutrosophic graph on $G^* : (V, E)$, a cycle on $m(m \geq 4)$ vertices. Let $e_1, e_2, e_3, \ldots, e_m$ be the edges of cycle G^* in that order. Let degrees of truth-membership, indeterminacy-membership and falsity-membership of the edges $e_1, e_2, e_3, \ldots, e_m$ are $p_1, p_2, p_3, \ldots, p_m$ such that $p_1 < p_2 < p_3 < ... < p_m$ and $q_1, q_2, q_3, \ldots, q_m$ such that $q_1 > q_2 > q_3 > ... > q_m$ and $r_1, r_2, r_3, \ldots, r_m$ such that $r_1 > r_2 > r_3 > ... > r_m$, respectively, where $p_i = (p_i^-, p_i^+)$ and $q_i = (q_i^-, q_i^+)$ and $r_i = (r_i^-, r_i^+)$ for $i = 1, 2, ..., m$, then $d_G(v_1) = ((p_1^- + p_m^-, p_1^+ + p_m^+), (q_1^- + q_m^-, q_1^+ + q_m^+), (r_1^- + r_m^-, r_1^+ + r_m^+)) = (p_1 + p_m, q_1 + q_m, r_1 + r_m)$, for $i = 2, 3, 4, 5, \ldots, m$: $d_G(v_i) = ((p_{i-1}^- + p_i^-, p_{i-1}^+ + p_i^+), (q_{i-1}^- + q_i^-, q_{i-1}^+ + q_i^+), (r_{i-1}^- + r_i^-, r_{i-1}^+ + r_i^+)) = (p_{i-1} + p_i, q_{i-1} + q_i, r_{i-1} + r_i)$, $d_G(e_1) = ((p_2^- + p_m^-, p_2^+ + p_m^+), (q_2^- + q_m^-, q_2^+ + q_m^+), (r_2^- + r_m^-, r_2^+ + r_m^+)) = (p_2 + p_m, q_2 + q_m, r_2 + r_m)$, for $i = 2, 3, 4, 5, \ldots, m-1$: $d_G(e_i) = ((p_{i-1}^- + p_{i+1}^-, p_{i-1}^+ + p_{i+1}^+), (q_{i-1}^- + q_{i+1}^-, q_{i-1}^+ + q_{i+1}^+), (r_{i-1}^- + r_{i+1}^-, r_{i-1}^+ + r_{i+1}^+)) = (p_{i-1} + p_{i+1}, q_{i-1} + q_{i+1}, r_{i-1} + r_{i+1})$, $d_G(e_m) = ((p_1^- + p_{m-1}^-, p_1^+ + p_{m-1}^+), (q_1^- + q_{m-1}^-, q_1^+ + q_{m-1}^+), (r_1^- + r_{m-1}^-, r_1^+ + r_{m-1}^+)) = (p_1 + p_{m-1}, q_1 + q_{m-1}, r_1 + r_{m-1})$.

We observe that the adjacent edges have distinct degrees. Hence G is a neighbourly edge irregular interval-valued neutrosophic graph.

$td_G(e_1) = (p_1 + p_2 + p_m, q_1 + q_2 + q_m, r_1 + r_2 + r_m)$, for $i = 2, 3, 4, 5, \ldots, m-1$, $td_G(e_i) = (p_{i-1} + p_i + p_{i+1}, q_{i-1} + q_i + q_{i+1}, r_{i-1} + r_i + r_{i+1})$ for $i = 2, 3, 4, 5, \ldots, m-1$, $td_G(e_m) = (p_1 + p_{m-1} + p_m, q_1 + q_{m-1} + q_m, r_1 + r_{m-1} + r_m)$.

We note that the adjacent edges have distinct total degrees. Hence G is a neighbourly edge totally irregular interval-valued neutrosophic graph. □

5. Conclusion

It is well known that graphs are among the most ubiquitous models of both natural and human-made structures. They can be used to model many types of relations and process dynamics in computer science, physical, biological and social systems. In general graphs theory has a wide range of applications in diverse fields. IVNG is an extended structure of a fuzzy graph which gives more precision, flexibility, and compatibility to the system when compared with the classical, fuzzy and neutrosophic models.

In this paper, we defined degree of an edge and total degree of an edge. Also, we introduced some types of edge irregular interval-valued neutrosophic graphs and properties of them.

A comparative study between neighbourly edge irregular interval-valued neutrosophic graphs and neighbourly edge totally irregular interval-valued neutrosophic graphs did. Also some properties of neighbourly edge irregular interval-valued neutrosophic graphs and neighbourly edge totally irregular interval-valued neutrosophic graphs studied.

In our future work, we will introduce strongly edge irregular interval-valued neutrosophic graphs and highly edge irregular interval-valued neutrosophic graphs. Also, we will study some properties of them.

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