ON PRIMALITY OF THE SMARANDACHE SYMMETRIC SEQUENCES

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The study of primality for the Smarandache sequences represents a recent research direction on the Smarandache type notions. A few articles that were published recently deal with the primality of the direct and reverse Smarandache sequences. The primality of Smarandache symmetric sequences has not been studied yet. This article proposes some results concerning the non-primality of these symmetric sequences and presents some interesting conclusions on a large computational test on these.

Key Words: Smarandache Symmetric Sequences, Prime Numbers, Testing Primality

Smarandache type notions represents one of the important and recent directions on which the research on Elementary Number Theory has been carried out on the last years. Many theoretical and practical studies concerning the Smarandache functions, numbers or sequences have been developed so far. The practical studies have proved that many conjectures and open problems on this kind of notions are true. This article follows this line by developing a large computation on the Smarandache symmetric sequence and after that by proving some non-primality results. But, the most important fact is the article proposes two special numbers of the sequences that are primes.

1. INTRODUCTION

In the following, the main notions that are used in this article are summarized and some recent results concerning them are reviewed. All of them concern the Smarandache sequences. There are three types of the Smarandache sequences presented below [4]:
- The direct Smarandache sequence: 1, 12, 123, 1234, ...
- The reverse Smarandache sequence: 1, 21, 321, 4321, ...
- The symmetric Smarandache sequence: 1,121, 1221, 12321, 123321, 1234321, 12344321,...

Let these sequences be denoted by:

$$Sd(n) = 123..n \ (\forall n > 0) \quad (1.a.)$$

$$Sr(n) = n...321 \ (\forall n > 0) \quad (1.b.)$$

In order to simplify the study of the symmetric Smarandache sequence, we note

$$S_1(n) = 123...(n-1)n(n-1)...321 \ (\forall n > 0) \quad (2.a.)$$

$$S_2(n) = 123...(n-1)nn(n-1)...321 \ (\forall n > 0) \quad (2.b.)$$

and called them the symmetric Smarandache sequences of the first and second order, respectively. These sequences have been intensely studied and some interesting results have been proposed so far.

The direct and reverse Smarandache were the subject to an intense computational study. Stephan [5] developed the first large computational study on these sequences. He analyzed the factorization of the first one hundred terms of these sequences finding no prime numbers within. In order to find prime numbers in these sequences, Fleuren [3] extended the study up to two hundred finding no prime numbers too. In [3], a list of people who study these computationally these sequences was presented. No prime numbers in these sequences have been found so far. Unfortunately, a computational study has not been done for the symmetric sequences yet.

The only result concerning the symmetric sequences was proposed by Le [2]. This result states that the terms $S_2(n) = 123...(n-1)nn(n-1)...321$ of the second Smarandache symmetric sequence are not prime if $\frac{n}{2} \neq 1 \pmod{3}$. No computational results were furnished in this article for sustaining the theorem. Smarandache [4] proposed several proprieties on these three sequences, majority of them being open problems.

2. COMPUTATIONAL RESULTS

Testing primality has always represented a difficult problem. For deciding the primality of large numbers, special and complicated methods have been developed. The last generation ones use elliptic curve and are very efficient in finding prime factors large. For example, Fleuren used Elliptic Curve Primality Proving or Adleman-Promerance-Rumely tests obtaining all prime factors up to 20 digits. More information about these special tests could be found in [1].

The computation that we have done uses MAPLE 5, which is software oriented to mathematical computations. This software contains several functions for dealing with primes and factorization such us *isprime*, *ifactor*, *ifactors*, *ithfactor*, *etc*. The function *ifactor* that is based on the elliptic curves can find prime factors depending on the method used. The easy version discovers prime factor up to 10 digits. The "Lenstra" method can find prime factors up to 20 digits. We used the simple version of *ifactor* for testing the terms of the Smarandache symmetric sequences. The computation was done for all the numbers between 2 and 100. The results are presented in Tables 1,2 of Appendix. Table 1 gives the factorization of the terms of the first Smarandache symmetric sequence. Table 2 provides the factorization of the second Smarandache symmetric sequence.

Several simple observations can be made by analyzing Tables 1, 2. The most important of them is that two prime numbers are found within. The term $S_1(10) = 12345678910987654321$ of the first Smarandache symmetric sequence is a prime number with 20 digits. Similarly, the term $S_2(10) = 1234567891010987654321$ of the second Smarandache symmetric sequence is a prime number with 22 digits. No other prime numbers can be found in these two tables. The second remark is that the terms from the tables present similarities. For example, all the terms $S_1(3\cdot k), k>1$ and $S_2(3\cdot k-1), S_2(3\cdot k), k>1$ are divisible by 3. This will follow us to some theoretical results. The third remark is that some prime factors satisfy a very strange periodicity. The factor (333667) appear 29 times in the factorization of the second Smarandache symmetric sequence. A supposition that can be made is the following *there are no prime numbers in the Smarandache symmetric sequences others that* $S_1(10), S_2(10)$.

In the following, the prime numbers $S_1(10) = 12345678910987654321$ and $S_2(10) = 1234567891010987654321$ are named the Smarandache gold numbers. Perhaps, they are the largest and simplest prime numbers known so far.

3. PRIMALITY OF THE SMARANDACHE SYMMETRIC SEQUENCES

In this section, some theoretical results concerning the primality of the Smarandache symmetric sequences are presented. The remarks drawn from Tables 1,2 are proved to be true in general.

Let $ds(n), n \in N^*$ be the digits sum of number n. It is know that a natural number n is divisible by 3 *if and only if* $3 \mid ds(n)$. A few simple results on this function are given in the following.

Proposition 1. (∀n>1) *ds*(3n) is M3.

Proof

The proof is obvious by using the simple remarks 3n is M3. Thus, *ds(3n)* is M3. ♣

Proposition 2. (∀n>1) *ds*(3*n*-1) + *ds*(3*n*-2) is M3.

Proof

Let us suppose that the forms of the numbers 3*n*-1,3*n*-2 are

$$3\cdot n-2 = \overline{a_1a_2a_3...a_p} \tag{3.}$$

$$3\cdot n-1 = \overline{b_1b_2b_3...b_p}. \tag{4.}$$

Both of them have the same number of digits because 3*n*-2 cannot be 999...9. The equation

$$ds(3\cdot n-2)+ds(3\cdot n-1) = a_1+a_2+...+a_p+b_1+b_2+...+b_p = ds(a_1a_2...a_pb_1b_2...b_p)$$

gives $ds(3\cdot n-2)+ds(3\cdot n-1) = ds\left((3\cdot n-2)\cdot 10^p + 3\cdot n-1\right)$.

The number $(3\cdot n-2)\cdot 10^p + 3\cdot n-1$ is divisible by 3 as follows

$$(3\cdot n-2)\cdot 10^p + 3\cdot n-1 = 10^p - 1 = (9+1)^p - 1 = 0 \pmod 3.$$

Thus, *ds*(3n-2) + *ds*(3n-1) = *ds*((3n-2)10^p + 3n-1) is M3. ♣

In order to prove that the number $S_1(3\cdot k), k>1$ and $S_2(3\cdot k-1), S_2(3\cdot k), k>1$ are divisible by 3, Equations (5-6) are used.

$$ds(S_1(3\cdot k)) = ds(S_1(3\cdot k-3)) + 2\cdot ds(3\cdot k-2) + 2\cdot ds(3\cdot k-1) + ds(3\cdot k) \tag{5.}$$

$$ds(S_2(3\cdot k)) = ds(S_2(3\cdot k-3)) + 2\cdot ds(3\cdot k-2) + 2\cdot ds(3\cdot k-1) + 2\cdot ds(3\cdot k) \tag{6.a.}$$

$$ds(S_2(3\cdot k-1)) = ds(S_2(3\cdot k-4)) + 2\cdot ds(3\cdot k-3) + 2\cdot ds(3\cdot k-2) + 2\cdot ds(3\cdot k-1) \tag{6.b.}$$

Based on Propositions 1,2, Equations (5-6) give

$$ds(S_1(3\cdot k)) = ds(S_1(3\cdot k-3)) \pmod 3 \tag{7.}$$

$$ds(S_2(3\cdot k)) = ds(S_2(3\cdot k-3))\,(\text{mod}3) \text{ and } ds(S_2(3\cdot k-1)) = ds(S_2(3\cdot k-4))\,(\text{mod}3). \qquad (8.)$$

•

The starting point is given by $S_1(3) = 3^2\cdot37^2$ is M3, $S_2(2) = 3\cdot11\cdot37$ is M3, and $S_2(3) = 3\cdot11\cdot37\cdot101$ is M3. All the above facts provide an induction mechanism that proves obviously the following theorem.

Theorem 3. The numbers $S_1(3\cdot k), k\geq1$ and $S_2(3\cdot k-1), S_2(3\cdot k), k\geq1$ are divisible by 3, thus are not prime.

Proof

This proof is given by the below implications.

$S_1(3) = 3^2\cdot37^2$ is M3, $ds(S_1(3\cdot k)) = ds(S_1(3\cdot k-3))\,(\text{mod}3) \Rightarrow S_1(3k)$ is M3, $k\geq1$.

$S_2(2) = 3\cdot11\cdot37$ is M3, $ds(S_1(3\cdot k)) = ds(S_1(3\cdot k-3))\,(\text{mod}3) \Rightarrow S_2(3k)$ is M3, $k\geq1$.

$S_2(2) = 3\cdot11\cdot37\cdot101$ is M3, $ds(S_2(3\cdot k-1)) = ds(S_2(3\cdot k-4))\,(\text{mod}3) \Rightarrow S_2(3k-1)$ is M3, $k\geq1$.

4. FINAL REMARKS

This article has provided both a theoretical and computational study on the Smarandache symmetric sequences. This present study can be further developed on two ways. Firstly, the factorization can be refined by using a more powerful primality testing technique. Certainly, the function *ifactor* used by Lenstra's method may give factors up to 20 digits. Secondly, the computation can be extended up to 150 in order to check divisibility property. Perhaps, the most interested fact to be followed is if the factor (333667) appears periodically in the factorization of the second Smarandache sequence.

The important remark that can be outlined is two important prime numbers were found. These are 12345678910987654321 and 1234567891010987654321. We have named them the Smarandache gold numbers and represent large numbers that can be memorized easier. Moreover, they seem to be the only prime numbers within the Smarandache symmetric sequences.

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APPENDIX - The results of the computation.

n	*Digits*	*Factorisation*
2	3	$(11)^2$
3	5	$(3)^2(37)^2$
4	7	$(11)^2(101)^2$
5	9	$(41)^2(271)^2$
6	11	$(3)^2(7)^2(11)^2(13)^2(37)^2$
7	13	$(239)^2(4649)^2$
8	15	$(11)^2(73)^2(101)^2(137)^2$
9	17	$(3)^4(37)^2(333667)^2$
10	**20**	**PRIME**
11	24	(7)(17636684157301569664903)
12	28	$(3)^2(7)^2$(27994736757621793899946 81)
13	32	(1109)(4729)(2354041513534224607850261)
14	36	(7)(571)(3167)(10723)(439781)(2068140300159522133)
15	40	$(3)^2$(7)(3167)(10723)(75401)(439781)(687437)_c27
16	44	(71)(18428)_c37
17	48	$(7)^2$(31)_c44
18	52	$(3)^5$(7)(8087)(897447779391490639058918259893784003373302 83)
19	56	(251)(281)(5519)(96601)_c42
20	60	(7)(1763668415730173305930881688457416881659305901730156966 4903)
21	64	$(3)^2$(7)_c62
22	68	(70607)_c63
23	72	(7)(15913)_c67
24	76	$(3)^2$(7)(659)(56383)_c66
25	80	NO Answer, Yet
26	84	(7)(3209)(17627)_c75
27	88	$(3)^4$(7)(223)(28807)_78
28	92	(149)(828569054369880076611823612026988069290286089245813774652497450951793030296182624898500 29)
29	96	(7)_c95
30	100	$(3)^2$(7)(167)(761)_93
31	104	(827)_c101
32	108	(7)(31)(42583813)_c98
33	112	$(3)^2(7)^2$(281)_c106
34	116	(197)(509)_c111
35	120	(7)(10243)_c115
36	124	$(3)^6$(7)(2399)_c117
37	128	NO Answer, Yet
38	132	$(7)^2$(313)_c127
39	136	$(3)^2$(7)(733)(2777)_c127
40	140	(17047)(28219)_131
41	144	(7)(5153)(7687)(79549)_c130
42	148	$(3)^2$(7)(9473)_c142
43	152	(191)(4567)_c?
44	156	(7)(223)(251)_c150
45	160	$(3)^6$(7)(643303)_c150
46	164	(967)(33289)_c156
47	168	(7)(31)(199)(281)_c161
48	172	$(3)^2$(7)(557)(38995472881)_c156
49	176	(139121)_c170
50	180	(7)(179)
51	184	$(3)^2$(7)(71)(55697)_c175
52	188	(109)(181)_c183

53	192	(7)(14771)_c187
54	196	$(3)^4(7)^3$(191)(3877)_c185
55	200	(5333)_c196
56	204	(7)(73589)_c198
57	208	$(3)^2$(7)(3389)(56591)_c198
58	212	NO Answer, Yet
59	216	$(7)^2$_c214
60	220	$(3)^2$(7)(14769967)_c211
61	224	(281)(286813)_c216
62	228	(7)(31)_c?
63	232	$(3)^2$(7)_c228
64	236	NO Answer, Yet
65	240	(7)_c239
66	244	$(3)^2$(7)_c242
67	248	NO Answer, Yet
68	252	(7)(1861)(12577)(19163)_c?
69	256	$(3)^2$(7)(251)(1861)_c248
70	260	NO Answer, Yet
71	264	(7)_c263
72	268	$(3)^5$(7)(563)(3323)_c258
73	272	(2477)(3323)(3943)_c265
74	276	(7)(47279)_c270
75	280	$(3)^2(7)^2$(281)(7681)_c271
76	284	NO Answer, Yet
77	288	(7)(31)_c285
78	292	$(3)^2$(7)_c290
79	296	(313)(6529)(63311)_c284
80	300	$(7)^3$(130241)_c292
81	304	$(3)^4$(7)_c301
82	308	NO Answer, Yet
83	312	(7)(197)_c308
84	316	$(3)^2$(7)(1931)(110323)_c305
85	320	(953)(1427)(103573)_c308
86	324	(7)(71)(181)_c319
87	328	$(3)^2$(7)(491)_c?
88	332	NO Answer, Yet
89	336	(7)(281)(50581)_c328
90	340	$(3)^5$(7)(67121)_c332
91	344	(19501)_c339
92	348	(7)(31)(571)(811)_c340
93	352	$(3)^2$(7)_c350
94	356	(251)(79427)_c348
95	360	(7)_c359
96	364	$(3)^2(7)^2$_c361
97	368	(7559)_c364
98	372	(7)(1129)(4703)(63367)
99	376	$(3)^5$(7)_c372
100	381	NO Answer, Yet

Table 1. Smarandache Symmetric Sequence of the first order.

n	*Digits*	*Factorisation*
2	4	(3)(11)(37)
3	6	(3)(11)(37)(101)
4	8	(11)(41)(101)(271)
5	10	(3)(7)(11)(13)(37)(41)(271)
6	12	(3)(7)(11)(13)(37)(239)(4649)
7	14	(11)(73)(137)(239)(4649)
8	16	$(3)^2$(11)(37)(73)(101)(137)(333667)
9	18	$(3)^2$(11)(37)(41)(271)(333667)(9091)
10	**22**	**PRIME**
11	26	(3)(43)(97)(548687)(1798162193492119)
12	30	(3)(11)(31)(37)(61)(92869187)(575752909227253)
13	34	(109)(3391)(3631)(919886914249704430301189)
14	38	(3)(41)(271)(9091)(290971)(1400164978896215684979l7)
15	42	(3)(37)(661)(1682637405802185215073413380233484451)
16	46	No Answer Yet
17	50	$(3)^2$(1371742101123468126835130190683490346790109739369)
18	54	$(3)^2$(37)(1301)(333667)(6038161)_c36
19	58	(41)(271)(9091)_c50
20	62	(3)(11)(97)_c58
21	66	(3)(37)(983)_c61
22	70	(67)(773)(2383749861966990503207452683288838257844397322240377925143576831)
23	74	(3)(11)(7691)_c68
24	78	(3)(37)(41)(43)(271)(9091)(165857)_c61
25	82	(227)(2287)(33871)_c71
26	86	$(3)^3$(163)(5711)_c78
27	90	$(3)^3$(31)(37)(333667)_c80
28	94	(146273)(608521)_c83
29	98	(3)(41)(271)(9091)(407407407407441077407747474744141750841417542087508417508414141414141077441077407407407407)
30	102	(3)(37)(5167)_c96
31	106	$(11)^3$(4673)_c99
32	110	(3)(43)(1021)_c?
33	114	(3)(37)(881)_c109
34	118	(11)(41)(271)(9091)_c109
35	122	$(3)^2$(3209)_c117
36	126	$(3)^2$(37)(333667)(68697367)_c110
37	130	No Answer Yet
38	134	(3)(1913)(12007)(58417)(597269)_c115
39	138	(3)(37)(41)(271)(347)(9091)(23473)_c121
40	142	No Answer Yet
41	146	(3)(156841)_c140
42	150	(3)(11)(31)(37)(61)_c143
43	154	(71)(5087)_c?
44	158	$(3)^2$(41)(271)(9091)_c149
45	162	$(3)^2$(11)(37)(43)(333667)_c151
46	166	No Answer Yet
47	170	(3)_c169
48	174	(3)(37)(173)(60373)_c165
49	178	(41)(271)(929)(34613)(9091)_c162
50	182	(3)(167)(1789)(9923)(159652607)_c163
51	186	(3)(37)(1847)_C180
52	190	No Answer Yet
53	194	$(3)^3$(11)(43)(26539)_c185

54	198	$(3)^3$(37)(41)(151)(271)(347)(463)(9091)(333667)_c174
55	202	(67)_c200
56	206	(3)_c205
57	210	(3)(37)_c208
58	214	(59)(109)_c210
59	218	(3)$(11)^2$(41)(59)(271)(9091)_c205
60	222	(3)(37)(8837)_c216
61	226	$(11)^2$(17)(197)(631)_c217
62	230	$(3)^4$(19)(72617)_c222
63	234	$(3)^2$(37)(333667)_c226
64	238	(41)(89)(271)(9091)(63857)(6813559)_c216
65	242	(3)(2665891)_c235
66	246	(3)(37)_c244
67	250	(1307)_c246
68	254	(3)(43)(107)(8147)(3373)(37313)_c237
69	258	(3)(17)(37)(41)(271)(1637)(9091)(4802689)_c236
70	262	(11)(109)(21647107)
71	266	$(3)^2$(19)_c263
72	270	$(3)^2$(11)(37)(333667)(1099081)_c254
73	274	No Answer Yet
74	278	(3)(41)(271)(1481)(9091)_c266
75	282	(3)(37)(17827)(26713)_c271
76	286	No Answer Yet
77	290	(3)$(17)^2$(337)(8087)(341659)_c275
78	294	(3)(37)_c292
79	298	(41)(271)(9091)(10651)(98887)_c281
80	302	$(3)^2$(19)_c299
81	306	$(3)^6$(11)(37)(333667)_c295
82	310	No Answer Yet
83	314	(3)(11)(41543)(48473)(69991)_c298
84	318	(3)(37)(41)(271)(9091)_c308
85	322	(17)(2203)(19433)_c313
86	326	(3)(89)(193)_c321
87	330	(3)(37)(59)_c326
88	334	(59)(67)_c330
89	338	$(3)^3$(19)(41)(43)(271)(9091)_c325
90	342	$(3)^2$(37)(333667)_c334
91	346	No Answer Yet
92	350	(3)(11)(18859)_c344
93	354	(3)(17)(37)(1109)(1307)_c344
94	358	(11)(41)(271)(9091)_c349
95	362	(3)_c361
96	366	(3)(37)(373)(169649)(24201949)_c348
97	370	(113)(163)(457)(7411)_c359
98	374	$(3)^2$(19)(572597)_c366
99	378	$(3)^2$(37)(41)(271)(499)(593)(333667)(9091)_c?
100	384	(89)_c382

Table 2. Smarandache Symmetric Sequence of the second order.