On Slightly Smarandache Fuzzy Semiring Structure Homogeneous Spaces

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Abstract : ***In this disquisition, slightly homogeneous spaces in ordinary topological spaces are extended to Smarandache fuzzy semiring structure spaces. The notions of $\mathcal{S}$-homogeneous, $\mathcal{S}_s$-homogeneous spaces are introduced and their properties are established. In this connection, the concepts of $\mathcal{S}$-homogeneous component and $\mathcal{S}_s$-homogeneous component of S-fuzzy semiring structure spaces are introduced and their properties are discussed. Further, the notions of $\mathcal{S}_a$-homogeneous spaces are introduced and the relation between $\mathcal{S}_s$-homogeneous and $\mathcal{S}_a$-homogeneous spaces is studied.***

Keywords: S-fuzzy semiring, $\mathcal{S}$-continuous, $\mathcal{S}$-homogeneous, $\mathcal{S}_s$-continuous, $\mathcal{S}_s$-homogeneous and $\mathcal{S}_a$-homogeneous spaces.

I. INTRODUCTION

The notion of homogeneous spaces is prominent in general topology. Sierpinski [8] introduced the notion of homogeneous spaces. Some extensions of homogeneous concepts like strong locally homogeneous and local prehomogeneous spaces are studied in [5] and [2] respectively. Homogeneous components are preserved under homeomorphisms and it is essential in homogeneity research. A. Fora and S. Al Ghour [6] generalized homogeneous spaces in classical sense to fuzzy topological spaces. Many mathematicians studied slightly continuous functions. The concept of slightly continuous functions was established in [10]. With the succour of [10] and [8], S. Al Ghour and N. Al Khatib introduced and studied the notion of slight homogeneous spaces in [3]. Recently, the fuzzification of algebraic structures plays an eminent role in many disciplines of Mathematics and Engineering. In [11], Smarandache fuzzy semirings were introduced and studied.

In this disquisition, the concept of Smarandache fuzzy semiring structure spaces is introduced and studied. The notion of slightly fuzzy continuous functions is studied in S-fuzzy semiring structure spaces as $\mathcal{S}$-continuous functions.

Also the concepts of $\mathcal{S}$-homogeneous, $\mathcal{S}_s$-homogeneous spaces are introduced and their properties are established. Also the concepts of $\mathcal{S}$-homogeneous component and $\mathcal{S}_s$-homogeneous component of S-fuzzy semiring structure spaces are introduced and some of their interesting properties are investigated. Further, the notions of $\mathcal{S}_a$-homogeneous spaces are introduced and the relation between $\mathcal{S}_s$-homogeneous and $\mathcal{S}_a$-homogeneous spaces is studied.

II PRELIMINARIES

Definition 2.1. [11] The Smarandache semiring which will be denoted as **S-semiring** is defined to be a semiring S such that a proper subset B of S is a semifield with respect to the same induced operations.

Definition 2.2. [11] A fuzzy subset μ of a S-semiring S is called a Smarandache fuzzy semiring (**S-fuzzy semiring**) relative to $P \subset S$ where P is a field if for all $x,\ y \in P$,

$$\mu(x+y) \geq min(\mu(x), \mu(y)) \text{ and}$$
$$\mu(xy) \geq min(\mu(x), \mu(y)).$$

Thus every S-fuzzy semiring μ will be associated with a semifield P contained in S. Further, μ need not be a S-fuzzy semiring relative to all fuzzy subsets μ on a S-semiring S.

Definition 2.3. [7] A fuzzy set in X is called a **fuzzy point** iff it takes the value 0 for all $y \in X$ except one, say, $x \in X$. If its value at x is λ $(0 < \lambda \leq 1)$ we denote this fuzzy point by x_λ, where the point x is called its support.

Definition 2.4. [1] A fuzzy point x_α is said to be contained in a fuzzy set μ or μ is said to contain x_λ if $\alpha \leq \mu(x)$. We denote it by $x_\alpha \leq \mu$.

Definition 2.5. [4] Let (X, T) be a fuzzy topological space and Y be an ordinary subset of X. Then $T_Y = \{\lambda/Y | \lambda \in T\}$ is a fuzzy topology on Y and is called the induced or relative fuzzy topology. The pair (Y, T_Y) is called a **fuzzy subspace** of (X, T): (Y, T_Y) is called a fuzzy open/ fuzzy closed/ fuzzy β-open fuzzy subspace if the characteristic function of Y viz, χ_Y is fuzzy open/ fuzzy closed/ fuzzy β-open respectively.

Definition 2.6. [10] A mapping $f : X \to Y$ is said to be **slightly continuous** if for each point $x \in X$ and each clopen neighborhood V of $f(x)$, there exists a neighborhood U of x such that $f(U) \subset V$.

Definition 2.7. [9] A mapping $f : X \to Y$ is said to be **almost continuous** at a point $x \in X$, if for every neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(U) \subset intcl\ V$.

Definition 2.8. [3] A topological space (X, τ) is said to be **slightly homogeneous** if for any two points $x, y \in X$, there exists $f \in \mathrm{SH}(X, \tau)$ such that $f(x) = y$. Here the group of all slight homeomorphisms from a space (X, τ) onto itself is denoted by $\mathrm{SH}(X, \tau)$.

Definition 2.9. [3] Let (X, τ) be a topological space. We define the equivalence relation $\tilde{s}$ on X as follows. For $x_1, x_2 \in X$, $x_1\ \tilde{s}\ x_2$ if there exists $f \in \mathrm{SH}(X, \tau)$ such that $f(x_1) = x_2$. A subset of a topological space (X, τ), which has the form $\mathrm{SC}_x = \{y \in X: x\tilde{s}y\}$ is called the **slightly homogeneous component** of X at x.

III SLIGHTLY S-FUZZY SEMIRING STRUCTURE CONTINUOUS FUNCTIONS

In this section, the concepts of S-fuzzy semiring structure spaces, $\mathcal{S}$-continuous function, $\mathcal{S}$-homeomorphism, $\mathcal{S}_s$-continuous function and $\mathcal{S}_s$-homeomorphism are introduced and some properties are discussed.

Definition 3.1. Let S be a S-semiring. A family $\mathcal{S}$ of S-fuzzy semirings on S is said to be Smarandache fuzzy semiring structure (briefly S-fuzzy semiring structure) on S if it satisfies the following conditions:

i. $0_S, 1_S \in \mathcal{S}$,
ii If $\lambda_1, \lambda_2 \in \mathcal{S}$, then $\lambda_1 \wedge \lambda_2 \in \mathcal{S}$,
iii If $\lambda_i \in \mathcal{S}$ for each $i \in$ I, then $\vee \lambda_i \in \mathcal{S}$.

And the ordered pair $(S, \mathcal{S})$ is said to be a **S-fuzzy semiring structure space**. Every member of $\mathcal{S}$ is said to be a S-fuzzy open semiring and the complement of a S-fuzzy open semiring is said to be an anti-fuzzy open semiring (or S-fuzzy closed semiring).

Example 3.1. Let $S = \{0, 1, 2\}$ be a set of integers modulo 3 with two binary operations as follows :

.	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

Hence $(S, ., +)$ is a S-semiring. Let λ, μ be two S-fuzzy semirings on S defined as follows : $\lambda(0) = 0.2$, $\lambda(1) = 0.3$ and $\lambda(2) = 0.4$, $\mu(0) = 0.4$, $\mu(1) = 0.5$ and $\mu(2) = 0.6$. Then $\mathcal{S} = \{0_s, 1_s, \lambda, \mu\}$ is a S-fuzzy semiring structure on S and the pair $(S, \mathcal{S})$ is a S-fuzzy semiring structure space.

Definition 3.2. Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space. Let $\lambda \in I^S$ be a S-fuzzy semiring. Then the **S-fuzzy semiring interior** of λ is defined and denoted as SFR$int(\lambda) = \vee\{\mu : \mu \leq \lambda$ and μ is a S-fuzzy open semiring$\}$.

Definition 3.3. Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space. Let $\lambda \in I^S$ be a S-fuzzy semiring. Then the **S-fuzzy semiring closure** of λ is defined and denoted as SFR$cl(\lambda) = \wedge\{\mu : \mu \geq \lambda$ and μ is a S-fuzzy closed semiring$\}$.

Definition 3.4. Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces. A function $f : (S_1, \mathcal{S}_1) \rightarrow (S_2, \mathcal{S}_2)$ is said to be S-fuzzy semiring structure continuous (simply **$\mathcal{S}$-continuous**) if for every fuzzy point $x_\lambda \in$ FSP(S_1) and every S-fuzzy open semiring μ of $(S_2, \mathcal{S}_2)$ with $f(x_\lambda) \leq \mu$, there exists a S-fuzzy open semiring γ in $(S_1, \mathcal{S}_1)$ with $x_\lambda \leq \gamma$ such that $f(\gamma) \leq \mu$.

Definition 3.5. Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces. A function $f : (S_1, \mathcal{S}_1) \rightarrow (S_2, \mathcal{S}_2)$ is said to be a S-fuzzy semiring structure homeomorphism (simply **$\mathcal{S}$-homeomorphism**) if f is bijective and both f, f^{-1} are $\mathcal{S}$-continuous.

Notation 3.1. The family of all $\mathcal{S}$-homeomorphisms from a S-fuzzy semiring structure space $(S, \mathcal{S})$ onto itself is denoted by FH(S, $\mathcal{S}$). Let S be a S-semiring.

Definition 3.6. Let S be a S-semiring. A **fuzzy point** x_λ on S is a fuzzy set and is defined as

$$x_\lambda(x_0) = \begin{cases} \lambda, & if\ x = x_0 , \\ 0, & if\ x \neq x_0 , \end{cases}$$

for all $x \in S$, where $0 < \lambda \leq 1$. x_λ is said to have support x and value λ. If the fuzzy point x_λ satisfies the conditions of a S-fuzzy semiring on S, then it is called S-fuzzy semiring point on S. The collection of all S-fuzzy semiring points on S is denoted by FSP(S).

Definition 3.7. Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces. A function $f : (S_1, \mathcal{S}_1) \rightarrow (S_2, \mathcal{S}_2)$ is said to be slightly S-fuzzy semiring structure continuous (simply **$\mathcal{S}_s$-continuous**) if for every fuzzy point $x_\lambda \in$ FSP(S_1) and every S-fuzzy clopen semiring μ of $(S_2, \mathcal{S}_2)$ with $f(x_\lambda) \leq \mu$, there exists a S-fuzzy open semiring γ in $(S_1, \mathcal{S}_1)$ with $x_\lambda \leq \gamma$ such that $f(\gamma) \leq \mu$.

Proposition 3.1. Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces. For a function $f : (S_1, \mathcal{S}_1) \rightarrow (S_2, \mathcal{S}_2)$, the following are equivalent :

(i) f is $\mathcal{S}_s$-continuous.

(ii) For every S-fuzzy clopen semiring μ of $(S_2, \mathcal{S}_2)$, $f^{-1}(\mu)$ is a S-fuzzy open semiring in $(S_1, \mathcal{S}_1)$.

(iii) For every S-fuzzy clopen semiring μ of $(S_2, \mathcal{S}_2)$, $f^{-1}(\mu)$ is a S-fuzzy clopen semiring of $(S_1, \mathcal{S}_1)$.

Proof : **(i)⇒(ii)** Let γ be a S-fuzzy clopen semiring of $(S_2, \mathcal{S}_2)$. Let $x_\lambda \in$ FSP(S_1) such that $x_\lambda \leq f^{-1}(\gamma)$. Since $f(x_\lambda) \leq \gamma$, by (i) there exists a S-fuzzy open semiring μ_{x_λ} in $(S_1, \mathcal{S}_1)$ with $x_\lambda \leq \mu_{x_\lambda}$ such that $f(\mu_{x_\lambda}) \leq \gamma$. This implies $\mu_{x_\lambda} \leq f^{-1}(\gamma)$. Also $f^{-1}(\gamma) = \vee_{x_\lambda \leq f^{-1}(\gamma)} \mu_{x_\lambda}$. Since arbitrary union of S-fuzzy open semirings is a S-fuzzy open semiring, $f^{-1}(\gamma)$ is a S-fuzzy open semiring in $(S_1, \mathcal{S}_1)$.

(ii)⇒(iii) Let γ be a S-fuzzy clopen semiring of $(S_2, \mathcal{S}_2)$. Then $1_{S_2} - \gamma$ is a S-fuzzy clopen semiring of $(S_2, \mathcal{S}_2)$. By (ii), $f^{-1}(1_{S_2} - \gamma) = 1_{S_1} - f^{-1}(\gamma)$ is a S-fuzzy open semiring in $(S_1, \mathcal{S}_1)$. Hence $f^{-1}(\gamma)$ is a S-fuzzy closed semiring of $(S_1, \mathcal{S}_1)$. But by (ii), $f^{-1}(\gamma)$ is a S-fuzzy open semiring in $(S_1, \mathcal{S}_1)$. Hence $f^{-1}(\gamma)$ is a S-fuzzy clopen semiring of $(S_1, \mathcal{S}_1)$.

(iii)⇒(i) Let γ be a clopen semiring of $(S_2, \mathcal{S}_2)$. Let $x_\lambda \in$ FSP(S_1) such that $f(x_\lambda) \leq \gamma$. By (iii), $f^{-1}(\gamma)$ is S-fuzzy clopen semiring of $(S_1, \mathcal{S}_1)$. Let $\mu = f^{-1}(\gamma)$. Hence $f(\mu) \leq \gamma$. Thus f is $\mathcal{S}_s$-continuous.

Definition 3.8. Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces. A function $f : (S_1, \mathcal{S}_1) \rightarrow (S_2, \mathcal{S}_2)$ is said to be a slightly S-fuzzy semiring structure homeomorphism (simply **$\mathcal{S}_s$-homeomorphism**) if f is bijective and both f, f^{-1} are $\mathcal{S}_s$-continuous.

Notation 3.2. The family of all $\mathcal{S}_s$-homeomorphisms from a S-fuzzy semiring structure space $(S, \mathcal{S})$ onto itself is denoted by SFSH$(S, \mathcal{S})$.

Definition 3.9. The S-fuzzy semiring structure spaces $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ are called slightly S-fuzzy semiring structure homeomorphic (simply **$\mathcal{S}_s$-homeomorphic**) if and only if there exists a $\mathcal{S}_s$-homeomorphism $f : (S_1, \mathcal{S}_1) \rightarrow (S_2, \mathcal{S}_2)$.

Definition 3.10. Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces. Then $(S_1, \mathcal{S}_1)$ is said to have S-fuzzy semiring topological property if and only if every S-fuzzy semiring structure space $(S_2, \mathcal{S}_2)$ $\mathcal{S}$-homeomorphic to $(S_1, \mathcal{S}_1)$ also has the same property.

Definition 3.11. Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces. Then $(S_1, \mathcal{S}_1)$ is said to have slightly S-fuzzy semiring topological property if and only if every S-fuzzy semiring structure space $(S_2, \mathcal{S}_2)$ $\mathcal{S}_s$-homeomorphic to $(S_1, \mathcal{S}_1)$ also has the same property.

Definition 3.12. A S-fuzzy semiring structure space (S, $\mathcal{S}$) is said to be **S-fuzzy semiring structure connected** if it has no proper S-fuzzy clopen semirings.
(A S-fuzzy semiring $\lambda \in \mathrm{I}^S$ is said to be proper if $\lambda \neq 0_S$ and $\lambda \neq 1_S$).

Proposition 3.2. Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces and $(S_1, \mathcal{S}_1)$ be S-fuzzy semiring structure connected. If $f : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$ is $\mathcal{S}_s$-continuous, then $(S_2, \mathcal{S}_2)$ is S-fuzzy semiring structure connected.
Proof : Assume that $(S_2, \mathcal{S}_2)$ is not S-fuzzy semiring structure connected. Let λ be a proper S-fuzzy clopen semiring of $(S_2, \mathcal{S}_2)$. Since f is $\mathcal{S}_s$-continuous, by Proposition 3.1, $f^{-1}(\lambda)$ is a proper S-fuzzy clopen semiring of $(S_1, \mathcal{S}_1)$ which is a contradiction, since $(S_1, \mathcal{S}_1)$ is S-fuzzy semiring structure connected. Hence $(S_2, \mathcal{S}_2)$ is S-fuzzy semiring structure connected.

Proposition 3.3. Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces and $(S_2, \mathcal{S}_2)$ be S-fuzzy semiring structure connected. Then $f : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$ is $\mathcal{S}_s$-continuous.
Proof : Since $(S_2, \mathcal{S}_2)$ is S-fuzzy semiring structure connected, the only S-fuzzy clopen semirings are 0_{S_2} and 1_{S_2}. Hence $f^{-1}(0_{S_2})$ and $f^{-1}(1_{S_2})$ are both S-fuzzy clopen semirings of $(S_1, \mathcal{S}_1)$. Hence by Proposition 3.1, f is $\mathcal{S}_s$-continuous.

Proposition 3.4. Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces. If $f : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$ is bijective such that $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ are both S-fuzzy semiring structure connected, then f is a $\mathcal{S}_s$-homeomorphism.
Proof : It is enough to prove that both f and f^{-1} are $\mathcal{S}_s$-continuous. Since $(S_2, \mathcal{S}_2)$ is S-fuzzy semiring structure connected, the only S-fuzzy clopen semirings are 0_{S_2} and 1_{S_2}. Hence $f^{-1}(0_{S_2})$ and $f^{-1}(1_{S_2})$ are both S-fuzzy clopen semirings of $(S_1, \mathcal{S}_1)$. Hence by Proposition 3.1, f is $\mathcal{S}_s$-continuous. Similarly, it can be proved that $f^{-1} : (S_2, \mathcal{S}_2) \to (S_1, \mathcal{S}_1)$ is $\mathcal{S}_s$-continuous. Hence f is a $\mathcal{S}_s$-homeomorphism.

Proposition 3.5. Every $\mathcal{S}$-continuous function is $\mathcal{S}_s$-continuous.

Proof : Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces. Let $f : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$ be $\mathcal{S}$-continuous. Let $x_\lambda \in \mathrm{FSP}(S_1)$ be a fuzzy point and let μ be a S-fuzzy clopen semiring of $(S_2, \mathcal{S}_2)$ with $f(x_\lambda) \leq \mu$. Since f is $\mathcal{S}$-continuous, there exists a S-fuzzy open semiring γ in $(S_1, \mathcal{S}_1)$ with $x_\lambda \leq \gamma$ such that $f(\gamma) \leq \mu$. Hence f is $\mathcal{S}_s$-continuous.

Corollary 3.1. Every $\mathcal{S}$-homeomorphism is a $\mathcal{S}_s$-homeomorphism.

Definition 3.13. Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space and let $P \subseteq S$. Then the collection

$$\mathcal{S}_P = \{\lambda|_P = \lambda \wedge \chi_P : \lambda \in \mathcal{S}\}$$

is a S-fuzzy semiring structure on P. The ordered pair $(P, \mathcal{S}_P)$ is called a **S-fuzzy semiring subspace** of $(S, \mathcal{S})$. $(P, \mathcal{S}_P)$ is called a S-fuzzy semiring open(resp. closed) subspace if the characteristic function χ_P of P is a S-fuzzy open(resp. closed) semiring in $(S, \mathcal{S})$.

Proposition 3.6. Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space and $P \subseteq S$. Let χ_P be a S-fuzzy clopen semiring. If $f_1 \in \mathrm{SFSH}(P, \mathcal{S}_P)$ and $f_2 \in \mathrm{SFSH}(S - P, \mathcal{S}_{S-P})$ and a function $f : (S, \mathcal{S}) \to (S, \mathcal{S})$ is defined by

$$f(a) = \begin{cases} f_1(a), & if\ a \in P, \\ f_2(a), & if\ a \in S - P, \end{cases}$$

then $f \in \mathrm{SFSH}(S, \mathcal{S})$.
Proof : Let γ be a S-fuzzy clopen semiring of $(S, \mathcal{S})$. Then

$$f^{-1}(\gamma) = f^{-1}(\gamma \wedge \chi_P) \vee f^{-1}(\gamma \wedge \chi_{S-P}).$$

Thus $f^{-1}(\gamma)$ is a S-fuzzy clopen semiring. By Proposition 3.1, f is $\mathcal{S}_s$-continuous. Similarly it can be proved that f^{-1} is $\mathcal{S}_s$-continuous. Therefore $f \in \mathrm{SFSH}(S, \mathcal{S})$, since f is bijective.

Proposition 3.7. Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces and $P \subseteq S_1$. If $f : (S_1, \mathcal{S}_1) \rightarrow (S_2, \mathcal{S}_2)$ is a $\mathcal{S}_s$-homeomorphism, then the restriction function on P, $f|_P : (P, (\mathcal{S}_1)_P) \rightarrow (f(\mathrm{P}), (\mathcal{S}_2)_{f(P)})$ is a $\mathcal{S}_s$-homeomorphism.
Proof : Let γ be a S-fuzzy clopen semiring of $f(P)$. Then γ is a S-fuzzy clopen semiring of $(S_2, \mathcal{S}_2)$. Since f is a $\mathcal{S}_s$-homeomorphism, $f^{-1}(\gamma)$ is S-fuzzy clopen semiring of $(S_1, \mathcal{S}_1)$. So it follows that $f^{-1}(\gamma)$ is a S-fuzzy clopen semiring of $(P, (\mathcal{S}_1)_P)$, proving that $f|_P$ is $\mathcal{S}_s$-continuous. Similarly, it can be proved that $(f|_P)^{-1}$ is $\mathcal{S}_s$-continuous. Since f is a $\mathcal{S}_s$-homeomorphism, $f|_P : P \rightarrow f(\mathrm{P})$ is bijective and hence $f|_P$ is a $\mathcal{S}_s$-homeomorphism.

Proposition 3.8. Let $(S_1, \mathcal{S}_1)$ be a S-fuzzy semiring structure space and $(S_2, \mathcal{S}_2)$ has a S-fuzzy semiring structure base consisting of S-fuzzy clopen semirings. If $f : (S_1, \mathcal{S}_1) \rightarrow (S_2, \mathcal{S}_2)$ is $\mathcal{S}_s$-continuous, then f is $\mathcal{S}$-continuous.
Proof : Let $x_\lambda \in \mathrm{FSP}(S_1)$ and let μ be a S-fuzzy open semiring of $(S_2, \mathcal{S}_2)$ such that $f(x_\lambda) \leq \mu$. Since $(S_2, \mathcal{S}_2)$ has a S-fuzzy semiring structure base consisting of S-fuzzy clopen semirings, there exists a S-fuzzy clopen semiring γ with $f(x_\lambda) \leq \gamma$ such that $\gamma \leq \mu$. Since f is $\mathcal{S}_s$-continuous, there exists a S-fuzzy open semiring δ in $(S_1, \mathcal{S}_1)$ with $x_\lambda \leq \delta$ such that $f(\delta) \leq \gamma \leq \mu$. Hence f is $\mathcal{S}$-continuous.

Corollary 3.2. Let $f : (S_1, \mathcal{S}_1) \rightarrow (S_2, \mathcal{S}_2)$ be $\mathcal{S}_s$-homomorphism. If $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ are both having S-fuzzy semiring structure bases consisting of S-fuzzy clopen semirings, then f is $\mathcal{S}$-homomorphism.

IV SLIGHTLY S-FUZZY SEMIRING STRUCTURE HOMOGENEOUS SPACES

In this section, the properties of $\mathcal{S}$-homogeneous and $\mathcal{S}_s$-homogeneous spaces are studied. Also the notions of $\mathcal{S}$-homogeneous component and $\mathcal{S}_s$-homogeneous component of S-fuzzy semiring structure spaces are introduced and their properties are discussed.

Definition 4.1. Let $(S, \mathcal{S})$ be a Smarandache fuzzy semiring structure space. Then $(S, \mathcal{S})$ is said to be S-fuzzy semiring structure homogeneous (simply **$\mathcal{S}$-homogeneous**) if for any two points s_1, s_2 in S, there exists a $\mathcal{S}$-homoeomorphism $f \in \mathrm{FH}(S, \mathcal{S})$ such that $f(s_1) = s_2$.

Definition 4.2. A S-fuzzy semiring structure space $(S, \mathcal{S})$ is said to be slightly Smarandache fuzzy semiring structure homogeneous (simply **$\mathcal{S}_s$-homogeneous**) if for any two points s_1, s_2 in S, there exists a $\mathcal{S}_s$-homoeomorphism $f \in \mathrm{SFSH}(S, \mathcal{S})$ such that $f(s_1) = s_2$.

Proposition 4.1. Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space. If $(S, \mathcal{S})$ is $\mathcal{S}$-connected, then $(S, \mathcal{S})$ is $\mathcal{S}_s$-homogeneous.
Proof : Let $(S, \mathcal{S})$ be $\mathcal{S}$-connected and let $s_1, s_2 \in S$. Let a function $f : (S, \mathcal{S}) \rightarrow (S, \mathcal{S})$ be defined by $f(s_1) = s_2$, $f(s_2) = s_1$ and $f(s) = s$ for all $s \in S - \{s_1, s_2\}$. It is clear that f is bijective. Then by Proposition 3.4, $f \in \mathrm{SFSH}(S, \mathcal{S})$. Hence $(S, \mathcal{S})$ is $\mathcal{S}_s$-homogeneous.

Proposition 4.2. Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space. If $(S, \mathcal{S})$ is $\mathcal{S}$-homogeneous, then $(S, \mathcal{S})$ is $\mathcal{S}_s$-homogeneous.
Proof : The Proof follows from the Corollary 3.1.

Proposition 4.3. Being $\mathcal{S}_s$-homogeneous is a slightly S-fuzzy semiring topological property.
Proof : Let $(S_1, \mathcal{S}_1)$ be a $\mathcal{S}_s$-homogeneous space and let $(S_2, \mathcal{S}_2)$ be any S-fuzzy semiring structure space which is $\mathcal{S}_s$-homeomorphic to $(S_1, \mathcal{S}_1)$. Let $s_1, s_2 \in S_1$ and let $q_1, q_2 \in S_2$. Let the function $f_1 : (S_1, \mathcal{S}_1) \rightarrow (S_2, \mathcal{S}_2)$ be a $\mathcal{S}_s$-homeomorphism such that $f_1(s_1) = q_1$ and $f_1(s_2) = q_2$. Since $(S_1, \mathcal{S}_1)$ is $\mathcal{S}_s$-homogeneous, there exists a $\mathcal{S}_s$-homeomorphism $f_2 : (S_1, \mathcal{S}_1) \rightarrow (S_1, \mathcal{S}_1)$ such that $f_2(s_1) = s_2$. Let a function $f_3 : (S_2, \mathcal{S}_2) \rightarrow (S_2, \mathcal{S}_2)$ be defined by $f_3(q) = (f_1 o f_2 o f_1^{-1})(q)$. Then it can be verified that f_3 is a $\mathcal{S}_s$-homeomorphism and $f_3(q_1) = q_2$. Hence $(S_2, \mathcal{S}_2)$ is $\mathcal{S}_s$-homogeneous.

Corollary 4.1. Being $\mathcal{S}_s$-homogeneous is a S-fuzzy semiring topological property.

Proposition 4.4. Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space. If $(S, \mathcal{S})$ has a S-fuzzy semiring structure base consisting of S-fuzzy clopen semirings and $\mathcal{S}_s$-homogeneous, then f is $\mathcal{S}$-homogeneous.
Proof : The Proof follows from the Corollary 3.2.

Definition 4.3. A S-fuzzy semiring structure space $(S, \mathcal{S})$ is said to be a S-fuzzy semiring structure extremally disconnected (simply **$\mathcal{S}$-extremally disconnected**) space if S-fuzzy semiring closure of every S-fuzzy open semiring is S-fuzzy open semiring.

Definition 4.4. Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space. A subfamily $\mathcal{B}$ of $\mathcal{S}$ is called a **S-fuzzy semiring structure base** for $\mathcal{S}$ if each member of $\mathcal{S}$ is a union of some members of $\mathcal{B}$.

Definition 4.5. Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces. The **S-fuzzy semiring product** of $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ is the cartesian product $(S_1, \mathcal{S}_1) \times (S_2, \mathcal{S}_2)$ of S-fuzzy semirings in $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ together with the S-fuzzy semiring structure $\mathcal{S}_1 \times \mathcal{S}_2$ generated by the family $\{\mathcal{P}_1^{-1}(\lambda_i), \mathcal{P}_2^{-1}(\mu_j) \mid \lambda_i \in \mathcal{S}_1, \mu_j \in \mathcal{S}_2$, where $\mathcal{P}_1$ and $\mathcal{P}_2$ are projections of $(S_1, \mathcal{S}_1) \times (S_2, \mathcal{S}_2)$ onto $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ respectively$\}$. Because $\mathcal{P}_1^{-1}(\lambda_i) = \lambda_i \times 1$, $\mathcal{P}_2^{-1}(\mu_j) = 1\times \mu_j$ and $(\lambda_i \times 1) \wedge (1\times \mu_j) = \lambda_i \times \mu_j$; the family $\mathcal{B} = \{\lambda_i \times \mu_j \mid \lambda_i \in \mathcal{S}_1, \mu_j \in \mathcal{S}_2 \}$ forms a S-fuzzy semiring structure base for the S-fuzzy semiring product structure $\mathcal{S}_1 \times \mathcal{S}_2$ on $S_1 \times S_2$.

Proposition 4.5. The S-fuzzy semiring product of two $\mathcal{S}$-extremally disconnected $\mathcal{S}_s$-homogeneous spaces is $\mathcal{S}_s$-homogeneous.

Proof : Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be $\mathcal{S}$-extremally disconnected $\mathcal{S}_s$-homogeneous. Let $(a_1, b_1), (a_2, b_2) \in S_1 \times S_2$. Then $a_1, a_2 \in S_1$ and $b_1, b_2 \in S_2$. Since $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ are $\mathcal{S}_s$-homogeneous, there exist $f \in$ SFSH$(S_1, \mathcal{S}_1)$ and $g \in$ SFSH$(S_2, \mathcal{S}_2)$ such that $f(a_1) = a_2$, $g(b_1) = b_2$. Let $h : (S_1 \times S_2, \mathcal{S}_1 \times \mathcal{S}_2) \to (S_1 \times S_2, \mathcal{S}_1 \times \mathcal{S}_2)$ be defined by $h(a,b) = (f \times g)(a, b) = (f(a), g(b))$.

Now to prove h is $\mathcal{S}_s$-continuous. Let $(x_\delta \times x_\sigma) \in$ FSP$(S_1 \times S_2)$ and let γ be a S-fuzzy clopen semiring of $(S_1 \times S_2, \mathcal{S}_1 \times \mathcal{S}_2)$ such that $h(x_\delta \times x_\sigma) \le \gamma$. Since γ is a S-fuzzy open semiring of $(S_1 \times S_2, \mathcal{S}_1 \times \mathcal{S}_2)$, there exist S-fuzzy open semirings μ_1 and μ_2 in $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ respectively such that

$$h(x_\delta \times x_\sigma) = (f \times g)(x_\delta \times x_\sigma) \le f(x_\delta) \times g(x_\sigma) \le \mu_1 \times \mu_2 = \gamma.$$

This implies that $f(x_\delta) \le$ SFR$cl(\mu_1)$ and $g(x_\sigma) \le$ SFR$cl(\mu_2)$. Since $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ are $\mathcal{S}$-extremally diconnected spaces, SFR$cl(\mu_1)$ and SFR$cl(\mu_2)$ are S-fuzzy clopen semirings of $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ respectively. Since f and g are $\mathcal{S}_s$-continuous, there exist S-fuzzy open semirings $\lambda_1 \in \mathcal{S}_1$ and $\lambda_2 \in \mathcal{S}_2$ such that $x_\delta \le \lambda_1$, $x_\sigma \le \lambda_2$ and $f(\lambda_1) \le$ SFR$cl(\mu_1)$ and $g(\lambda_2) \le$ SFR$cl(\mu_2)$. Therefore $x_\delta \times x_\sigma \le \lambda_1 \times \lambda_2$ where $\lambda_1 \times \lambda_2$ is a S-fuzzy open semiring in $(S_1 \times S_2, \mathcal{S}_1 \times \mathcal{S}_2)$ and

$$h(\lambda_1 \times \lambda_2) \le \text{SFR}cl(\mu_1) \times \text{SFR}cl(\mu_2) = \text{SFR}cl(\mu_1 \times \mu_2) = \text{SFR}cl(\gamma) = \gamma.$$

Hence h is $\mathcal{S}_s$-continuous. Similarly it can be proved that $h^{-1} : (S_1 \times S_2, \mathcal{S}_1 \times \mathcal{S}_2) \to (S_1 \times S_2, \mathcal{S}_1 \times \mathcal{S}_2)$ defined by $h^{-1}(a,b) = (f^{-1}(a), g^{-1}(b))$ is $\mathcal{S}_s$-continuous. It is clear that h is bijective and $h(a_1, b_1) = (a_2, b_2)$. Hence the proof.

Definition 4.6. Let $\{(S_i, \mathcal{S}_i), i \in$ I $\}$ be a family of pairwise disjoint S-fuzzy semiring structure spaces. Define $S = \bigcup_{i\in I} S_i$ and $\mathcal{S} = \{\lambda \in \mathrm{I}^S \mid \lambda \wedge S_i \in \mathcal{S}_i$ for all $i \in$ I $\}$. Then $\mathcal{S}$ is a S-fuzzy semiring structure on S and is called the sum S-fuzzy semiring structure on S. The corresponding pair $(S, \mathcal{S})$ is called the **sum S-fuzzy semiring structure space** $(S_i, \mathcal{S}_i)$, $i \in$ I.

Proposition 4.6. Let $\{(S_i, \mathcal{S}_i) : i \in$ I$\}$ be a family of pairwise disjoint S-fuzzy semiring structure spaces. If $(S_i, \mathcal{S}_i)$ is $\mathcal{S}_s$-homeomorphic to $(S_j, \mathcal{S}_j)$ for all $i, j \in$ I, then the sum S-fuzzy semiring structure space $\{(S_i, \mathcal{S}_i)$: $i \in$ I$\}$ is $\mathcal{S}_s$-homogeneous.

Proof : Let $a, b \in \bigcup_{i\in I} S_i$. Let $S = \bigcup_{i\in I} S_i$ and $(S, \mathcal{S})$ is the sum $\mathcal{S}$-fuzzy semiring structure space$(S_i, \mathcal{S}_i)$. Then there are two cases.

Case(i)

Let $a, b \in S_j$ for $j \in$ I. Since $(S_j, \mathcal{S}_j)$ is $\mathcal{S}_s$-homogeneous, there exists $f_j \in$ SFSH$(S_j, \mathcal{S}_j)$ such that $f_j(a) = b$. Let a function $g : (S, \mathcal{S}) \to (S, \mathcal{S})$ be defined by $g(p) = \begin{cases} f_j(p), & if\ p \in S_j, \\ p, & if\ p \in S - S_j. \end{cases}$

Then it is clear that $g(a) = b$. By Proposition 3, g is $\mathcal{S}_s$-homeomorphism.

Case(ii)

Let $a \in S_k$ and $b \in S_j$ for $k, j \in$ I and $k \neq j$. Since $(S_k, \mathcal{S}_k)$ is $\mathcal{S}_s$-homeomorphic to $(S_j, \mathcal{S}_j)$, there exists a $\mathcal{S}_s$-homeomorphism $h : (S_k, \mathcal{S}_k) \to (S_j, \mathcal{S}_j)$. Since $(S_j, \mathcal{S}_j)$ is $\mathcal{S}_s$-homogeneous, there exist $g \in$ SFSH$(S_j, \mathcal{S}_j)$ such that $g(h(a)) = b$.

Let $f : (S, \mathcal{S}) \rightarrow (S, \mathcal{S})$ be defined by

$$f(p) = \begin{cases} (goh)(p), & if\ p \in S_k, \\ (goh)^{-1}(p), & if\ p \in S_j, \\ p, & if\ p \in S - (S_k \cup S_j). \end{cases}$$

It is clear that f is bijective and $f(a) = b$ and by Proposition 3, f is $\mathcal{S}_s$-homeomorphism. Hence the sum of S-fuzzy semiring structure spaces $\{(S_i, \mathcal{S}_i): i \in \mathrm{I}\}$ is $\mathcal{S}_s$-homogeneous.

Definition 4.7. Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space. The equivalence relation $\tilde{\mathcal{S}}$ on S is defined as follows : for $a, b \in S$, $a\ \tilde{\mathcal{S}}\ b$ if and only if there exists $f \in$ FH$(S, \mathcal{S})$ such that $f(a) =$ b. A S-fuzzy semiring of $(S, \mathcal{S})$ is called the **$\mathcal{S}$-homogeneous component** of $(S, \mathcal{S})$ at a if it has the form FC$_a^{\mathcal{S}} = \{b \in$ S $: a\ \tilde{\mathcal{S}}\ b\}$.

Definition 4.8. Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space. The equivalence relation $\tilde{\mathcal{S}}$ on S is defined as follows : for $a, b \in S$, $a\ \tilde{\mathcal{S}}\ b$ if and only if there exists $f \in$ SFSH$(S, \mathcal{S})$ such that $f(a) =$ b. A S-fuzzy semiring of $(S, \mathcal{S})$ is called the **$\mathcal{S}_s$-homogeneous component** of $(S, \mathcal{S})$ at a if it has the form FSC$_a^{\mathcal{S}} = \{b \in S : a\ \tilde{\mathcal{S}}\ b\}$.

Proposition 4.7. Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space. Let FSC$_a^{\mathcal{S}}$ be a $\mathcal{S}_s$-homogeneous component. If $(S, \mathcal{S})$ is $\mathcal{S}_s$-homogeneous, then it has exactly one $\mathcal{S}_s$-homogeneous component and vice versa.
Proof : The Proof is obvious from Definition 4.8.

Proposition 4.8. Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space. Let FC$_a^{\mathcal{S}}$ and FSC$_a^{\mathcal{S}}$ be $\mathcal{S}$-homogeneous component of $(S, \mathcal{S})$ at a and $\mathcal{S}_s$-homogeneous component of $(S, \mathcal{S})$ at a. Then FC$_a^{\mathcal{S}} \subseteq$ FSC$_a^{\mathcal{S}}$ for all $a \in S$.
Proof : Since every $\mathcal{S}$-homeomorphism is a $\mathcal{S}_s$ homeomorphism, it is clear that FC$_a^{\mathcal{S}} \subseteq$ FSC$_a^{\mathcal{S}}$ for all $a \in S$.

Proposition 4.9. Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space. Let FSC$_a^{\mathcal{S}}$ be a $\mathcal{S}$-homogeneous component of S at a. If $f \in$ SFSH$(S, \mathcal{S})$, then $f($FSC$_a^{\mathcal{S}}) =$ FSC$_a^{\mathcal{S}}$.
Proof : The Proof follows from the Definition 4.8.
Proposition 4.10 Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space and FSC$_a^{\mathcal{S}}$ be a $\mathcal{S}_s$-homogeneous component. Let $\chi_{FSC_a^{\mathcal{S}}}$ be a S-fuzzy clopen semiring. Then (FSC$_a^{\mathcal{S}}$, $\mathcal{S}_{FSC_a^{\mathcal{S}}}$) is $\mathcal{S}_s$-homogeneous.
Proof : Let $s_1, s_2 \in$ FSC$_a^{\mathcal{S}}$. Then there exist $g_1, g_2 \in$ SFSH$(S, \mathcal{S})$ such that $g_1(s_1) = a$ and $g_2(a) = s_2$. Let $g : (S, \mathcal{S}) \rightarrow (S, \mathcal{S})$ be defined by $g = g_2\ o\ g_1$. This implies that $g \in$ SFSH$(S, \mathcal{S})$. By Proposition 3.7, and Proposition 4.9, it follows that $g|_{FSC_a^{\mathcal{S}}} \in$ SFSH(FSC$_a^{\mathcal{S}}$, $\mathcal{S}_{FSC_a^{\mathcal{S}}}$) such that $(g|_{FSC_a^{\mathcal{S}}})(s_1) = (s_2)$.

Proposition 4.11 Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space and FSC$_a^{\mathcal{S}}$ be a $\mathcal{S}_s$-homogeneous component. Let $P \subseteq S$ and χ_p be a S-fuzzy clopen semiring. If $(P, \mathcal{S}_P)$ is a $\mathcal{S}_s$-homogeneous subspace of $(S, \mathcal{S})$ such that $P \cap$ FSC$_a^{\mathcal{S}} \neq \emptyset$, then $P \subseteq$ FSC$_a^{\mathcal{S}}$.
Proof : Let $x \in P$ and $y \in P \cap$ FSC$_a^{\mathcal{S}}$. Since $(P, \mathcal{S}_P)$ is a $\mathcal{S}_s$-homogeneous subspace, there exists $g \in$ SFSH$(P, \mathcal{S}_P)$ such that $g(x) = y$. Let $f : (S, \mathcal{S}) \rightarrow (S, \mathcal{S})$ be defined by

$$f(a) = \begin{cases} g(a), & if\ a \in P, \\ a, & if\ a \in S - P. \end{cases}$$

By Proposition 3.6, it follows that $f \in$ SFSH$(S, \mathcal{S})$. Also since $f(x) = y$, $f(x) \in$ FSC$_a^{\mathcal{S}}$ and so $x \in f^{-1}($FSC$_a^{\mathcal{S}}) =$ FSC$_a^{\mathcal{S}}$. Hence $P \subseteq$ FSC$_a^{\mathcal{S}}$.

V. ALMOST S-FUZZY SEMIRING STRUCTURE HOMOGENEOUS SPACES

In this section, the concepts of $\mathcal{S}_a$-homogeneous spaces are introduced. The relation between $\mathcal{S}$-homogeneous and $\mathcal{S}_a$-homogeneous, $\mathcal{S}_s$-homogeneous and $\mathcal{S}_a$-homogeneous spaces is studied.

Definition 5.1. Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces. A function $f : (S_1, \mathcal{S}_1) \rightarrow (S_2, \mathcal{S}_2)$ is said to be almost S-fuzzy semiring structure continuous (simply **$\mathcal{S}_a$-continuous**) if for every fuzzy point $x_\lambda \in$ FSP(S_1) and every S-fuzzy open semiring μ of $(S_2, \mathcal{S}_2)$ with $f(x_\lambda) \leq \mu$, there exists a S-fuzzy open semiring γ in $(S_1, \mathcal{S}_1)$ with $x_\lambda \leq \gamma$ such that $f(\gamma) \leq$ SFR$int($SFR$cl(\mu))$.

Definition 5.2. Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces. A function $f : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$ is said to be almost S-fuzzy semiring structure homeomorphism (simply **$\mathcal{S}_a$-homeomorphism**) if f is bijective and both f, f^{-1} are $\mathcal{S}_a$-continuous.

Notation 5.1. The family of all $\mathcal{S}_a$-homeomorphisms from a S-fuzzy semiring structure space $(S, \mathcal{S})$ onto itself is denoted by SFAH$(S, \mathcal{S})$.

Proposition 5.1. Let $(S_1, \mathcal{S}_1)$ be a S-fuzzy semiring structure space and $(S_2, \mathcal{S}_2)$ be a $\mathcal{S}$-extremally disconnected space. If $f : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$ is $\mathcal{S}_s$- continuous, then f is $\mathcal{S}_a$-continuous.
Proof : Let $x_\lambda \in$ FSP(S_1) and let μ be a S-fuzzy open semiring of $(S_2, \mathcal{S}_2)$ such that $f(x_\lambda) \le \mu$. Since $(S_2, \mathcal{S}_2)$ is $\mathcal{S}$-extremally disconnected, SFR$\,cl(\mu)$ is S-fuzzy open semiring and hence S-fuzzy clopen semiring. Now $f(x_\lambda) \le$ SFR$cl(\mu)$. Since f is $\mathcal{S}_s$-continuous, there exists a S-fuzzy open semiring γ in $(S_1, \mathcal{S}_1)$ with $x_\lambda \le \gamma$ such that $f(\gamma) \le$ SFR$cl(\mu)$. Since F$\mathcal{S}cl(\mu)$ is S-fuzzy open semiring, $f(\gamma) \le$ SFRint(SFR$cl(\mu)$). Hence f is $\mathcal{S}_a$-continuous.

Corollary 5.1. Let $f : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$ be $\mathcal{S}_s$- homeomorphism. If $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ are both $\mathcal{S}$-extremally disconnected, then f is $\mathcal{S}_a$- homeomorphism.

Definition 5.3. A S-fuzzy semiring structure space $(S, \mathcal{S})$ is said to be almost S-fuzzy semiring structure homogeneous (simply **$\mathcal{S}_a$-homogeneous**) if for any two points s_1, s_2 in S, there exists a function $f \in$ SFAH(S, $\mathcal{S}$) such that $f(s_1) = s_2$.

Proposition 5.2. Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space. If $(S, \mathcal{S})$ is $\mathcal{S}$-extremally disconnected $\mathcal{S}_s$-homogeneous, then $(S, \mathcal{S})$ is $\mathcal{S}_a$-homogeneous.
Proof : The Proof follows from the Corollary 5.1.

Definition 5.4. A S-fuzzy semiring structure space $(S, \mathcal{S})$ is said to be S-fuzzy semiring structure semi-regular (**$\mathcal{S}$-semi-regular**) if for each fuzzy point $x_\lambda \in$ FSP(S) and each S-fuzzy open semiring μ such that $x_\lambda \le \mu$, there exists a S-fuzzy open semiring γ such that $x_\lambda \le \gamma \le$ SFRint(SFR$cl(\gamma)$) $\le \mu$.

Proposition 5.3. Let $(S_1, \mathcal{S}_1)$ be a S-fuzzy semiring structure space and $(S_2, \mathcal{S}_2)$ be a $\mathcal{S}$-semi-regular space. If $f : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$ is $\mathcal{S}_a$-continuous, then f is $\mathcal{S}$-continuous.
Proof : Let $x_\lambda \in$ FSP(S_1) and let μ be a S-fuzzy open semiring of $(S_2, \mathcal{S}_2)$ with $f(x_\lambda) \le \mu$. Since $(S_2, \mathcal{S}_2)$ is $\mathcal{S}$-semi-regular, there exists a S-fuzzy open semiring γ such that $f(x_\lambda) \le \gamma \le$ SFRint(SFR$cl(\gamma)$) $\le \mu$. Since f is $\mathcal{S}_a$-continuous, there exists a S-fuzzy open semiring δ in $(S_1, \mathcal{S}_1)$ with $x_\lambda \le \delta$ such that $f(x_\lambda) \le f(\delta) \le$ SFRint(SFR$cl(\gamma)$). Hence there exists a S-fuzzy open semiring δ in $(S_1, \mathcal{S}_1)$ with $x_\lambda \le \delta$ such that $f(\delta) \le \mu$. Therefore f is $\mathcal{S}$-continuous.

Corollary 5.2. Let $f : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$ be $\mathcal{S}_a$-homomorphism. If $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ are both $\mathcal{S}$-semi-regular spaces, then f is $\mathcal{S}$-homomorphism.

Proposition 5.4. Every $\mathcal{S}$-continuous function is $\mathcal{S}_a$-continuous.
Proof : Let $(S_1, \mathcal{S}_1)$ and $(S_2, \mathcal{S}_2)$ be any two S-fuzzy semiring structure spaces. Let $f : (S_1, \mathcal{S}_1) \to (S_2, \mathcal{S}_2)$ be $\mathcal{S}$-continuous. Let $x_\lambda \in$ FSP(S_1) and let μ be a S-fuzzy open semiring of $(S_2, \mathcal{S}_2)$ with $f(x_\lambda) \le \mu$. Since f is $\mathcal{S}$-continuous, there exists a S-fuzzy open semiring γ in $(S_1, \mathcal{S}_1)$ with $x_\lambda \le \gamma$ such that $f(\gamma) \le \mu$. This implies that $f(\gamma) \le$ SFRint(SFR$cl(\mu)$). Hence f is $\mathcal{S}_a$-continuous.

Corollary 5.3. Every $\mathcal{S}$-homeomorphism is a $\mathcal{S}_a$-homeomorphism.

Proposition 5.5. Let $(S, \mathcal{S})$ be a S-fuzzy semiring structure space. If $(S, \mathcal{S})$ is $\mathcal{S}$-homogeneous, then $(S, \mathcal{S})$ is $\mathcal{S}_a$-homogeneous.
Proof : The Proof follows from the Corollary 5.3.

VI. CONCLUSION

In this treatise, slightly homogeneous spaces in ordinary topological spaces are extended to Smarandache fuzzy semiring structure spaces. The relation between $\mathcal{S}$-homogeneous and $\mathcal{S}_s$-homogeneous, $\mathcal{S}$-homogeneous and $\mathcal{S}_a$-homogeneous, $\mathcal{S}_s$-homogeneous and $\mathcal{S}_a$-homogeneous spaces are discussed.

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