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On Smarandache Filter of a Smarandache BH-Algebra

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Abstract. In this paper, The notion of a Smarandache filter of a Smarandache BH-Algebra is introduced, some theorems and examples are investigated and discussed to explain properties of this notion. A necessary and sufficient condition is derived for every Smarandache filter of a Smarandache BH-Algebra to become a filter. Finally, the relationships between this notion and Smarandache ideal are established

Keywords. BCK-algebra, BCH-algebra, BH-algebra, Smarandache BHalgebra.

1. Introduction

A new algebraic structure called BCK-algebra was introduced by Y.Imai and K.Iseki in 1966[1]. At the same year another algebraic structure called BCI-algebra which was a generalization of a BCK-algebra was given by K.Iseki[2]. In 1983, Q.P.Hu and X.Li introduced the notion of a BCH- algebra which was a generalization of BCK/BCI -algebras [3]. In1991, C. S. Hoo introduced the notions of an ideal, a closed ideal and a filter in a BCI-algebra [4]. A BH- algebra is an algebraic structure introduced by Y.B.Jun et al in 1998 which was a generalization of BCH/BCI/BCK-algebras [5]. The notions of a Smarandache BCI-algebra, Smarandache ideal of a Smarandache BCI-algebra are given by Y.B.Jun in 2005 [6]. A.B.Saeid and A.Namdar introduced the notion of a Smarandache BCH-algebra and Smarandache ideal of Smarandache BCH-algebra in 2009 [7]. In 2012, H.H.Abbass and H.A.Dahham discussed the concept of completely closed filter of a BH-algebra, and completely closed filter with respect to an element of BH-algebra[8]. In 2013, H. H. Abbass and S. J. Mohammed introduced notions of the Smarandache BH-algebra, Smarandache (ideal, closed ideal, fantastic ideal, completely closed ideal) of a Smarandache BH-algebra[9]. In this paper, the notion of Smarandache filter of a Smarandache BH-Algebra is introduced.

2. Preliminaries

In this section, some basic concepts about a BCI-algeba, a BCK-algebra,a BCH-algebra, a BH-algeba, a Smarandache BH-algebra, and a Smarandach ideal of a BH-algebra are viewed.

Definition 2.1. [10]. *A BCI-algebra is an algebra* $(X, *, 0)$ *, where* X *is a nonempty set,* $*$ *is a binary operation and* 0 *is a constant, satisfying the following axioms:for all* $x, y, z \in X$*:*

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i. $((x * y) * (x * z)) * (z * y) = 0$,

ii. $(x * (x * y)) * y = 0$,

iii. $x * x = 0$,

iv. $x * y = 0$ *and* $y * x = 0$ *imply* $x = y$.

Definition 2.2. [10] *BCK-algebra is a BCI-algebra satisfying the axiom:* $0 * x = 0$ *for all* $x \in X$.

Definition 2.3. [5] *A BH-algebra is a nonempty set* X *with a constant* 0 *and a binary operation* $*$ *satisfying the following conditions:*

i. $x * x = 0, \forall x \in X$.

ii. $x * y = 0$ *and* $y * x = 0$ *imply* $x = y, \forall x, y \in X$.

iii. $x * 0 = x, \forall x \in X$.

Remark 2.4. [5]

i. *Every BCK-algebra is a BCI-algebra.*

ii. *Every BCK-algebra is a* $BCH\backslash$ *BH-algebra.*

Definition 2.5. [12]
A BH-algebra is said to be normal BH-algebra if it satisfying the following conditions:

i. $0 * (x * y) = (0 * x) * (0 * y), \quad \forall x,\ y \in X$

ii. $(x * y) * x = 0 * y, \quad \forall x,\ y \in X$

iii. $(x * (x * y)) * y = 0 \quad \forall x,\ y \in X$

Definition 2.6. [13]. *A subset* R *of a BH-algebra* X *is said to be regular if it satisfies:* $(\forall x \in R)(\forall y \in X)(x * y \in R \Rightarrow y \in R)$

Definition 2.7. [5]
Let I be a nonempty subset of a BH-algebra X. Then I is called an ideal of X if it satisfies:

(i.) $0 \in I$.

(ii.) $x * y \in I$ and $y \in I \Longrightarrow x \in I, \forall x \in X$.

Definition 2.8. [9] *A Smarandache BH-algebra is defined to be a BH-algebra* X *in which there exists a proper subset* Q *of* X *such that*

i. $0 \in Q$ *and* $|Q| \geq 2$.

ii. Q *is a BCK-algebra under the operation of* X.

Definition 2.9. [13]. *A Smarandache BH-algebra* X *is called a Smarandache medial BH-algebra if* $x * (x * y) = y, \forall x, y \in Q$

Definition 2.10. [9]. *A nonempty subset* I *of a Smarandache BH-algebra* X *is called a Smarandache ideal of* X*, if it satisfies:*

(J_1) $0 \in I$.

(J_2) $\forall\, y \in I$ *and* $x * y \in I \Longrightarrow x \in I, \forall\, x \in Q$.

Definition 2.11. [13]. *A subset I of a Smarandache BH-algebra X is called a Smarandache commutative ideal of X if it satisfies* J_1 *and*
(J_3). $(x * y) * z \in I$ *and* $z \in I \Rightarrow x * (y * (y * x)) \in I\ \ \forall\, x, y\ \in Q$ *and* $z \in X$

Definition 2.12. [13].*A Smarandache ideal I of a Smarandache BH-algebra X is called a Smarandache normal ideal of X if* $x * (x * y) \in I$ *implies* $y * (y * x) \in I, \forall x, y \in Q$.

Definition 2.13. [8] *A filter of a BH-algebra X is a non-empty subset F of X such that:*

(F_1) *If* $x \in F$ *and* $y \in F$ *then* $y * (y * x) \in F$ *and* $x * (x * y) \in F$.

(F_2) *If* $x \in F$ *and* $x * y = 0$ *then* $y \in F\ \forall\, y \in X$

Theorem 2.14. [9]. *Let X be a Smarandache BH-algebra and let I be a regular subset of X such that I is a subset of Q . If I is a Smarandache ideal of X then I is a filter of X.*

3. Main results

In this section, the concept of a Smarandache filter of a Smarandache BH-algebra is introduced, some properties of this concept are studied .

Definition 3.1. *A non-empty subset F of a Smarandache BH-algebra X is called a Smarandache filter of X, if it satisfies* (F_1) *and*
(F_3) *If* $x \in F$ *and* $x * y = 0$ *then* $y \in F\ \forall\, y \in Q$.

Example 3.2. .
Consider the Smarandache BH-algebra $X = \{0, 1, 2\}$ *with the binary operation* $'*'$ *defined by the following table:*

*	0	1	2
0	0	0	0
1	1	0	2
2	2	0	0

where $Q = \{0, 2\}$ *is a BCK-algebra. The subset* $F = \{1, 2\}$ *is Smarandache filter of X*

Remark 3.3. *If X is a Smarandache BH-algebra. Then* $\{0\}$ *and X are Smarandache filters of X, called trivial Smarandache filters of X. A Smarandache filter F of X is called a proper Smarandache filter of X if* $F \neq X$.

Proposition 3.4. *Let X be a Smarandache BH-algebra. Then every filter of X is a Smarandache filter of X.*

Proof. *Is obvious. Since* $Q \subseteq X$ *and F is a filter of X.*

Example 3.5. *The convers of proposition (3.4) is not correct in general as in the following example. Consider* $X = \{0, 1, 2, 3, 4\}$ *with binary operation* $''*''$ *defined by the following table:*

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	2
2	2	2	0	2	0
3	3	1	3	0	3
4	4	4	4	4	0

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where $Q = \{0, 2\}$. The subset $F = \{0, 1, 2\}$ is a Smarandache filter of X but it is not a filter. Since $0 \in F$, $3 \in X$ and $0 * 3 = 0$ but $3 \notin F$

Proposition 3.6. *Consider the Smarandache BH-algebra X=R the set of real number with binary operation $''*''$ defined by* $x * y = \begin{cases} x & if \quad x \neq y \quad and \quad x \in Z, \quad y \in R^{+} \\ 0 & if \quad x = 0 \quad and \quad y \in Z^{-} \\ x - y & otherwise \end{cases}$
where Q=Z the set of integers is a BCK-algebra. The subset $F = Z^{+} \bigcup \{0\}$ is the set a non negative integers is a Smarandache filter of X, but it is not a filter of X, since $0 \in F, \sqrt{2} \in R$ and $0 * \sqrt{2} = 0$ but $\sqrt{2} \notin F$

Proposition 3.7. *Let X be a Smarandache BH-algebra , and Q_1, Q_2 be a BCK-algebra, which are properly contained in X, such that $Q_1 \subseteq Q_2$. Then every Q_2-Smarandache filter is a Q_1-Smarandache filter of X.*

Proof. Let $x, y \in F$ then $y * (y * x) \in F$ and $x * (x * y) \in F$ by F_1 Now,let $x \in F$ and $x * y = 0$, $y \in Q_1$. Since $Q_1 \subseteq Q_2$ and F is a Q_2-Smarandache filter of X then $y \in F$. Therefore, F is a Q_1-Smarandache filter of X.

Remark 3.8. *The convers of proposition (3.7) is not correct in general as in the following example. Consider the Smarandache BH-algebra $X = \{0, 1, 2, 3, 4\}$ with binary operation $''*''$ defined by the following table:*

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	1
2	2	2	0	2	0
3	3	1	3	0	3
4	4	4	4	4	0

where $Q_1 = \{0, 1\}$, $Q_2 = \{0, 1, 3\}$ are BCK-algebras and $Q_1 \subseteq Q_2$. $F = \{0, 1, 2\}$ is a Q_1-Smarandache filter of X, but it is not Q_2-Smarandache filter of X. Since $0 \in F$, $3 \in Q_2$ and $0 * 3 = 0$, but $3 \notin F$

Theorem 3.9. *Let X be a Smarandache medial BH-algebra. Then every a non-empty subset A of X is a Smarandache filter of X.*

Proof. Let A be a non-empty subset of X and $x, y \in A$. Then $x = y * (y * x)$ by Definition(2.9). Thus $y * (y * x) \in A$. Similarly, $x * (x * y) \in A$. Now, let $x \in A, x * y = 0$, $y \in Q$.Since X is a medial BH-algebra then $y = x * (x * y)$, imply that $y = x * 0$,by Definition(2.1)(iii)$x * 0 = x$. Thus $y = x$, so $y \in A$. Therefore, A is a Smarandache filter of X.

Proposition 3.10. *Let X be a Smarandache BH-algebra and let $\{F_i,\ i \in \lambda\}$ be a family of Smarandache filter of X.Then $\bigcap_{i \in \lambda} F_i$ is a Smarandache filter of X.*

Proof. Let $\{F_i,\ i \in \lambda\}$ be a family of Smarandache filter of X. To prove $\bigcap_{i \in \lambda} F_i$ is a Smarandache filter of X. Let $x, y \in \bigcap_{i \in \lambda} F_i$. Then $x, y \in F_i$, $\forall i \in \lambda$. Since F_i is a Smarandache filter of X, $\forall\, i \in \lambda$. Hence $y * (y * x)$, $x * (x * y) \in F_i \forall\, i \in \lambda$ by Definition(3.1)(F_1). Then $y * (y * x)$, $x * (x * y) \in \bigcap_{i \in \lambda} F_i$. Now, let $x \in \bigcap_{i \in \lambda} F_i$, $x * y = 0$ *and* $y \in Q$. Then $x \in F_i\ \forall\, i \in \lambda$. Since F_i is a Smarandache filter of X,$\forall\, i \in \lambda$, then $y \in F_i \forall\, i \in \lambda$ by Definition(3.1)(F_3). This means that $y \in \bigcap_{i \in \lambda} F_i$. Therefore, $\bigcap_{i \in \lambda} F_i$ is a Smarandache filter of X.

Remark 3.11. *The union of Smarandache filter of Smarandache BH-algebra X may be not a Smarandache filter as in the following example.*

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Example 3.12. *Consider the Smarandache BH-algebra* $X = \{0, 1, 2, 3, 4\}$ *with binary operation* $''*''$ *defined by the following table:*

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	0	0	0
2	2	4	0	0	1
3	3	2	3	0	1
4	4	4	1	4	0

Where $Q_1 = \{0, 2\}$. $F_1 = \{1, 2\}$ *and* $F_2 = \{2, 4\}$ *are two Smarandache filters of* X*, the union of the Smarandache filters is not a Smarandache filter of* X*. Since* $1, 4 \in F_1 \bigcup F_2$*, but* $4 * (4 * 1) = 0 \notin F_1 \bigcup F_2$

Proposition 3.13. *Let X be a Smarandache filter and let* $\{F_i,\ i \in \lambda\}$ *be a chain of Smarandache filter of X.Then* $\bigcup_{i\in\lambda} F_i$ *is a Smarandache filter of X.*

Proof. Let $\{F_i,\ i \in \lambda\}$ be a chain of Smarandache filter of X and $x, y \in \bigcup_{i\in\lambda} F_i$, $\forall i \in \lambda$. Then there exist F_j, $F_k \in \{F_i\}_{i\in\lambda}$ such that $x \in F_j$ and $y \in F_k$. So, either $F_j \subseteq F_k$ or $F_k \subseteq F_j$. If $F_j \subseteq F_k$, then $x \in F_k$ and $y \in F_k$. Since F_k is a Smarandache filter of X,then $y * (y * x) \in F_k$ and $x * (x * y) \in F_k$, by Definition(3.1)(F_1). Similarly, if $F_k \subseteq F_j$. Then $y * (y * x)$, $x * (x * y) \in \bigcup_{i\in\lambda} F_i$. Now Let $x \in \bigcup_{i\in\lambda} F_i$ such that $x * y = 0$ and $y \in Q$. Then there exists $j \in \lambda$ such that $x \in F_j$.Since F_j is a Smarandache filter of X, hence $y \in F_j$ by Definition(3.1)(F_3). Thus $y \in \bigcup_{i\in\lambda} F_i$. Therefore, $\bigcup_{i\in\lambda} F_i$ is a Smarandache filter of X.

Theorem 3.14. *Let X be a Smarandache BH-algebra, and F be a Smarandache filter of X such that* $x * y \neq 0$*, for all* $y \notin F$ *and* $x \in F$*. Then F is a filter of X.*

Proof. Let F be a Smarandache filter of X such that $y \in X$ and $x \in F$,
Let $x, y \in F$ Since F is a Smarandache filter of X *it follows that* $y * (y * x), x * (x * y) \in F$ *by* F_1. Now, let $x \in F, x * y = 0$, Then there are two cases.
Case 1: If $y \in Q$ *imply then* $y \in F$*by* F_2
Case 2: If $y \notin Q$ then either $y \notin F$ *or* $y \in F$ suppose $y \notin F$, then $x * y \neq 0$,by hypothesis , this a contradiction. Thus $y \in F$. Therefore , F is a filter of X

Theorem 3.15. *Let X be a Smarandache normal BH-algebra, and let I be a regular subset of X. If I is an ideal, then I is a Smarandache filter of X.*

Proof. Let I be an ideal of X and $x, y \in I$. From I_1 we have $0 \in I$. By Definition2.5(iii)$(x*(x*y))*y = 0 \in I$. So, I_2 follows that $(x * (x * y)) \in I$, similarly $y * (y * x) \in I$. Let $x \in I$, $x * y = 0$, $y \in Q$. Then $x*y \in I$, $x \in I$, $y \in X[Q \subseteq X]$. Since I is a regular subset of X. Thus $y \in I$. *Therefore, I is a Smarandache filter of X.*

Proposition 3.16. *let X be a Smarandache medial BH-algebra X, and let I be a Smarandache ideal of X, such that* $Q \subseteq I$*. Then I is a Smarandache commutative ideal of X if and only if I is a Smarandache filter of X.*

Proof. Let I be a Smarandache commutative ideal of X and $x, y \in I$. Since X is a Smarandache medial BH-algebra , by Definition(2.9) we get $y = y * (y * x) \in I$ and $y = x * (x * y) \in I$. Now, Let $x \in I$, $x * y = 0$, and $y \in Q$. *X is a Smarandache medial BH-algebrait follows that* $y = x * (x * y) = x * 0$ implies that $y = x$. Hence $y \in I$ Therefore, I is a Smarandache filter of X. *Conversely, let I be a Smarandache filter of X. From Definition* 2.8(i) $0 \in Q$. Since $Q \subseteq I$ then $0 \in I$. **Now**, *let* $x, y \in Q, z \in I$,*such that* $(x*y)*z \in I$, Since $x * x = 0$, *it follows that* $x * (y * (y * x)) = 0 \in I$ [Since X is a Smarandache medial BH-algebra]. Therefore, I is a Smarandache commutative ideal of X.

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Corollary 3.16.1. *Let X be a Smarandache BH-algebra and let I be a regular subset of X such that I is a subset of Q . If I is a Smarandache ideal of X, then I is a Smarandache filter of X.*

Proof. It is directly from Theorem 2.14 and proposition3.4. Smarandache filter of X.

Proposition 3.17. *Let X be a Smarandache BH-algebra, and let F be a Smarandache filter of X, such that $Q \subseteq I$. Then F is Smarandache normal ideal of X.*

Proof. Let F be a Smarandache filter of X, *since* $0 \in Q$ *and* $Q \subseteq F$, implies that $0 \in F$. Now, let $x * y \in F$ and $y \in F$, $x \in Q$[Since $Q \subseteq F$] *we get* $x \in F$, [By Definition 2.10(ii)]*it follows that F is a Smarandache ideal of X.*
Now, let $x, y \in Q$ such that $x * (x * y) \in F$[Since $Q \subseteq F$ and F is a Smarandache filter of X by Definition3.1(i)]*we get* $y * (y * x) \in F$. Therefore, F is a Smarandache normal ideal of X.

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