On Smarandache M-Semigroup

Hassan Rashed Yassein
University of Al-Qadisiya
College of Education
Department of Mathematics

Mazen Omran Karim
University of Al-Qadisiya
College of Education
Department of Mathematics

Abstract

In this research, we defined the term a smarandache M – semigroup (S-M-semigroup) and studied some basic properties.

Also defined smarandache fuzzy M-semigroup and some elementary properties about this concepts are discussed .

Introduction

In 1965 Zadeh introduced the concept of fuzzy set, in 1971 Rosenfeld formulated the term of fuzzy subgroup.In 1994 W.X.Gu , S.Y.Li and D.G.Chen studied fuzzy groups and gave some new concepts as M- fuzzy groups . In 1999 W.B.Vasantha introduced the concepts of smarandache semigroups . Smarandache fuzzy semigroups are studied in 2003 by W.B.Vasantha .

In this research , the concept of Smarandache M- fuzzy semigroup are given and its some elementary properties are discussed

1- Preliminaries

Definition(1.1) : A fuzzy set μ of a group G is called a fuzzy subgroup if

$$\mu(xy^{-1}) \geq \min\{\mu(x), \mu(y)\} \text{ for every } x,y \in G. \ [2]$$

Definition(1.2): A fuzzy subgroup μ of a group G is called a fuzzy normal subgroup if

$$\mu(xyx^{-1}) \geq \mu(y) \text{ for every } x,y \in G \quad .[2]$$

Definition(1.3) : A group with operators is an algebraic system consisting of a group G , set M and a function defined in the product M×G and having value in G such that , if ma denotes the elements in G determined by the element m of M , then

m(ab)=(ma)(mb) hold for all a,b in G ,m in M . [4]

We shall usually use the phrase "G is an M-group " to a group with operators .

Definition (1.4): If μ is a fuzzy set of G and t∈ [0,1] then $\mu_t = \{ x \in G \mid \mu(x) \geq t \}$ is called a t- level set μ . [1]

Definition (1.5): Let G and G' both be M – groups , f be a homomorphism from G onto G' , if f(mx)= mf(x) for every m ∈M , x∈X, then f is called a M – homomorphism . [4]

Definition (1.6): Let G be M – group and μ be a fuzzy subgroup of G if μ(mx) ≥ μ (x) for every x ∈ G , m∈M ,then μ is said to be a fuzzy subgroup with operators of G , we use the pharse μ is an M – fuzzy subgroup of G instead of a fuzzy subgroup with operators of G . [4]

Proposition (1.7): If μ is an M – fuzzy subgroup of G , then the following statements hold for every x,y ∈ G , m∈M : [4]

1- $\mu(m(xy)) \geq \mu(mx) \wedge \mu(my)$
2- $\mu(mx^{-1})) \geq \mu(x)$

Proposition (1.8): Let G and G' both M – groups and f an M- homomorphism from G onto G' , if μ' is an M – fuzzy subgroup of G' then $f^{-1}(\mu')$ is an M-fuzzy subgroup of G. [4]

Proposition (1.9): Let G and G' both M – groups and f an M- homomorphism from G onto G' if μ is an M – fuzzy subgroup of G then f(μ) is an M-fuzzy subgroup of G' . [1]

Definition (1.10): Let S be a semigroup , S is said to be a smarandache semigroup (S –semigroup) if S has a proper subset P such that P is a group under the operation of G . [2]

Definition (1.11): Let S be an S-semigroup . A fuzzy subset μ :S → [0,1] is said to be smarandache fuzzy semigroup (S- fuzzy semigroup) if μ restricted to at least one subset P of S which is a subgroup is a fuzzy subgroup . [3]

that is for all $x,y \in P \subset S$, $\mu(xy^{-1}) \geq \min \{ \mu(x), \mu(y) \}$.

this S- fuzzy semigroup is denoted by μ_p:P $\to$ [0,1] is fuzzy group .

Definition (1.12): A semigroup H with operators is an algebraic system consisting of a semigroup H , set M , and a function defined in the product M$\times$H and having values in H such that , if ma denotes the element in H determined by the element a in H and the element m in M , then m(ab)=(ma)(mb) , a,b $\in$H and m $\in$M then H is M – semigroup . [4]

Definition (1.13): Let f be a function from a set X to a set Y while μ is fuzzy set of X then the image f(μ) of μ is the fuzzy set f(μ) :Y $\to$ [0,1] defined by : [1]

$$f(\mu(y)) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{if } f^{-1}(y) = \phi \end{cases}$$

Definition (1.14): Let f be a function from a set X to a set Y while μ is fuzzy set of Y then the inverse image f^{-1} (μ) of μ under f is the fuzzy set f^{-1} (μ) :X $\to$ [0,1] defined by f^{-1} (μ)(x)= μ (f(x)) . [1]

2-The Main Results

In this section we shall define smarandache M – semigroup and smarandache fuzzy M-semigroup and given some its results .

Definition (2.1): Let H be M- semigroup . H is said to be a smarandache M – semigroup (S-M-semigroup) if H has a proper subset K such that K is M- group under the operation of H .

Definition (2.2):Let H be a S - M –semigroup. A fuzzy subset μ :H $\to$ [0,1] is said be smarandache fuzzy M-semigroup if μ restricted to at least one subset K of H which is subgroup is fuzzy subgroup .

Definition (2.3): Let H be a S-M- semigroup . A fuzzy subset μ :H $\to$ [0,1] is said to be smarandache fuzzy M-semi group if restricted to at least one subset K of H which is M- subgroup is fuzzy M- subgroup

Definition (2.4): Let S and S$'$ be any two S- semigroup . A map φ from S to S$'$ is said to be S- semigroup homomorphism if φ restricted to a subgroup A$\subset$S $\to$ A$'\subset$S$'$ is a group homomorphism .

Definition (2.5): Let H and H' be any two S-M- semigroup . A map φ from H to H' is said to be S-M- semigroup homomorphism if φ restricted to a M- subgroup

$A \subset S \rightarrow A' \subset S'$ is M- homomorphism .

Proposition (2.6): If μ is S- fuzzy M-semigroup of S-M- semigroup then :

1) $\mu_K(m(xy)) \geq \min\{\mu_K(mx), \mu_k(my)\}$
2) $\mu_k(mx^{-1}) \geq \mu_k(x)$

For all $x \in M$, $x,y \in K$

Proof : μ is S- fuzzy M-semigroup

Then there exist subset K of H which is M- subgroup such μ restricted of K which is fuzzy M- subgroup

i.e . $\mu_K : K \rightarrow [0,1]$, M- fuzzy subgroup

for all $x,y \in K$, $m \in M$, it is clear that

1) $\mu_K(m(xy)) \geq \mu_K((mx)(my))$

$\geq \min\{\mu_K(mx), \mu_k(my)\}$

2) $\mu_k(mx^{-1}) = \mu_k(mx)^{-1}$

$\geq \mu_K(mx)$

$\geq \mu_K(mx)$ ■

Proposition (2.7): Let G be S- semigroup , μ fuzzy set of G , then μ is an S- fuzzy M-semigroup

of G iff $\forall t \in [0,1]$, μ_t is an S-M- semigroup $\mu_t \neq \emptyset$.

Proof : It is clear μ_t is semigroup of G while $\mu_t \neq \emptyset$ holds .

for any $x \in \mu_t$, $m \in M$

$\mu(mx) \geq \mu(x) \geq t$

hence mx in μ_t , hence μ_t is an M- semigroup of G .

since S-fuzzy M- semigroup $\exists K \subset G$ subgroup $\ni \mu_t : K \rightarrow [0,1]$

fuzzy M- subgroup.

$\mu_{K_t} = \{ x \in K \mid \mu_K(x) \geq t \}$.

It is clear μ_{K_t} is group .

hence μ_t S-M- semigroup .

Conversely ,

Since μ_t S-M- semigroup then there exists aproper subset K of G such that K is M-subgroup .

If there exists $x \in K$, $m \in M$ such that $\mu_K(mx) < \mu_K(x)$.

let $t = \frac{1}{2}(\mu_K(mx) + \mu_K(x))$

then $\mu_K(x) > t > \mu_K(mx)$

$mx \notin \mu_{K_t}$ so here emerges a contradiction .

$\mu_K(mx) \geq \mu_K(x)$ always holds for any $x \in K$,$m \in M$.

μ_K is M- fuzzy subgroup

hence μ is S – fuzzy M- subgroup . ■

Proposition(2.8): Let H and H' both be S-M- semigroup and f as S-M- semigroup homomorphism from H onto H' . if μ' is an S- fuzzy M- semigroup

of H' then $f^{-1}(\mu')$is an S- fuzzy M-semigroup of H.

Proof:

Since $f : H \rightarrow H'$ is as S-M- semigroup homomorphism then f restricted to M- subgroup .

$A \subset S \rightarrow A' \subset S'$ is M- homomorphism ,

$f^{-1}(\mu)_A : A \rightarrow [0,1]$ such that A M-subgroup ,

For any $m \in M$, $x \in A$

$$f^{-1}(\mu)_A(mx) = \mu'_A(f(mx))$$

$$= \mu'_A\, m(f(x)) \geq \mu'_A(f(x))$$

$$= f^{-1}(\mu')(x)$$

$f^{-1}(\mu')$ is S- fuzzy M- semigroup ■

Proposition(2.9): Let H and H' both be S-M- semigroups and f as S-M- semigroup homomorphism from H onto H' . if μ is an S- fuzzy M- semigroup of H then f(μ) is an S- fuzzy M-semigroup of H' .

Proof:

Since $f: H \to H'$ is as S-M- semigroup homomorphism then f restricted to M- subgroup .

$A \subset S \to A' \subset S'$ is M- homomorphism

$f(\mu)_{A'} : A' \to [0,1]$ such that A' M-subgroup ,

For any $m \in M$, $y \in A'$

$$
\begin{aligned}
f(\mu)(my) &= \sup \mu(x) \quad , x \in f^{-1}(my) \\
&= \sup \mu(x) \quad , f(x)=my \\
&\geq \sup \mu(mx') , f(mx')=mx , mx' \in H \\
&= \sup \mu(x') \quad , mf(x')=my , mx' \in H \\
&\geq \sup \mu(x') \quad , f(x')=y , x' \in H \\
&= f(\mu)(y)
\end{aligned}
$$

hence $f^{-1}(\mu')$ is S- fuzzy M- semigroup ■

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