On The Algebraic Properties of Symbolic n-Plithogenic Matrices For n=5, n=6

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On The Algebraic Properties of Symbolic n-Plithogenic Matrices For n=5, n=6

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Abstract

The main goal of this paper is to study the properties of symbolic 5-plithogenic matrices and symbolic 6-plithogenic matrices with real entries, where an algebraic view of their properties and relations will be presented and discussed. Also, we present many theorems that concern the computing of their eigenvalues and eigenvectors and their connection with classical ordinary matrices. Many related examples will be provided to clarify the validity of our work.

Keywords: symbolic 5-plithogenic matrix; symbolic 6-plithogenic matrix; symbolic plithogenic eigenvalue; symbolic plithogenic eigenvector.

1. Introduction

Generalizing classical matrices into many new numerical systems was applied by many authors, where we can find the building of neutrosophic matrices [1], refined matrices [2], and split-complex matrices [3].

The connections between these generalizations and the classical systems of matrices were handled by many authors. For example, the problem of diagonalization [4], the Invertibility [5], and their applications in linear functions [6].

In [7], the concept of symbolic n-plithogenic algebraic structures was proposed by Smarandache, then it was used on a wide range by many researchers to generalize classical algebraic structures such as modules [8], spaces [9-10], equations [11], and number theory [12-13].

In [14], the concept of symbolic 2-plithogenic matrices was presented with many applications in the theory of algebraic equations and representing functions. Laterally, symbolic 3-plithogenic matrices and 4-plithogenic matrices were studied from many algebraic sides, especially those which are related to the diagonalization problem [15].

This has motivated us to define and study for the first time the symbolic 5-plithogenic square matrices and symbolic 6-plithogenic matrices. We present many effective algorithms for computing determinants, Invertibility, and eigenvalues.

For basic definitions about symbolic 2-plithogenic, 3-plihogenic, and 4-plithogenic square matrices, see [14-15].

Main Discussion

Definition:

The square symbolic 5-plithogenic matrix is defined as follows:

$A = A_0 + \sum_{i=1}^{5} A_i P_i$; $(A_i)_{n\times n}$ is square matrix of real entries.

Example.

Consider the symbolic 5-plithogenic matrix:

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} P_1 + \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} P_2 + \begin{pmatrix} 4 & -1 \\ -1 & 2 \end{pmatrix} P_3 + \begin{pmatrix} 5 & 5 \\ 0 & 0 \end{pmatrix} P_4 + \begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix} P_5.$$

Definition.

Let $A = A_0 + \sum_{i=1}^{5} A_i P_i$ be a symbolic 5-plithogenic matrix of size $n \times n$, hence:

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$$\det A = \det(A_0) + \left[\det\left(\sum_{i=0}^{1} A_i\right) - \det(A_0)\right] P_1 + \left[\det\left(\sum_{i=0}^{2} A_i\right) - \det\left(\sum_{i=0}^{1} A_i\right)\right] P_2 + \left[\det\left(\sum_{i=0}^{3} A_i\right) - \det\left(\sum_{i=0}^{2} A_i\right)\right] P_3 + \left[\det\left(\sum_{i=0}^{4} A_i\right) - \det\left(\sum_{i=0}^{3} A_i\right)\right] P_4 + \left[\det\left(\sum_{i=0}^{5} A_i\right) - \det\left(\sum_{i=0}^{4} A_i\right)\right] P_5$$

Theorem1.

Let $A = A_0 + \sum_{i=1}^{5} A_i P_i$ be a symbolic 5-plithogenic matrix of size $n \times n$, hence:

1. A is invertible if and only if $\det A$ is an invertible symbolic 5-plithogenic number.
2. $A^{-1} = A_0{}^{-1} + [(\sum_{i=0}^{1} A_i)^{-1} - A_0{}^{-1}]P_1 + [(\sum_{i=0}^{2} A_i)^{-1} - (\sum_{i=0}^{1} A_i)^{-1}]P_2 + [(\sum_{i=1}^{3} A_i)^{-1} - (\sum_{i=0}^{2} A_i)^{-1}]P_3 + [(\sum_{i=1}^{4} A_i)^{-1} - (\sum_{i=0}^{3} A_i)^{-1}]P_4 + \left[\left(\sum_{i=1}^{5} A_i\right)^{-1} - (\sum_{i=0}^{4} A_i)^{-1}\right] P_5$

Definition.

Let $t = t_0 + \sum_{i=1}^{5} t_i P_i$ be a symbolic 5-plithogenic real number and $A = A_0 + \sum_{i=1}^{5} A_i P_i$ be a symbolic 5-plithogenic square real matrix, then t is called symbolic 5-plithogenic eigen values if and only if $AX = tX$.
X is called symbolic 5-plithogenic eigen vector.

Theorem2.

Let $t = t_0 + \sum_{i=1}^{5} t_i P_i \in 5 - SP_R$, $X = X_0 + \sum_{i=1}^{5} X_i P_i$ be a symbolic 5-plithogeni real vector, then t is eigen value of $A = A_0 + \sum_{i=1}^{5} A_i P_i$ with X as the corresponding eigen vector if and only if:
$\sum_{i=0}^{j} t_i$ is eigen value of $\sum_{i=0}^{j} A_i$ with $\sum_{i=0}^{j} X_i$ as eigen vector with $0 \leq j \leq 5$.

Theorem3.

$$A^n = A_0{}^n + P_1\left[\left(\sum_{i=0}^{1} A_i\right)^n - A_0{}^n\right] + \left[\left(\sum_{i=0}^{2} A_i\right)^n - \left(\sum_{i=0}^{1} A_i\right)^n\right] P_2 + \left[\left(\sum_{i=1}^{3} A_i\right)^n - \left(\sum_{i=0}^{2} A_i\right)^n\right] P_3 + \left[\left(\sum_{i=1}^{4} A_i\right)^n - \left(\sum_{i=0}^{3} A_i\right)^n\right] P_4 + \left[\left(\sum_{i=1}^{5} A_i\right)^n - \left(\sum_{i=0}^{4} A_i\right)^n\right] P_5$$

Theorem4.

Let $A = A_0 + \sum_{i=1}^{5} A_i P_i$ be a square 5-plithogenic invertible real matrix, then:
1). $\det(A^{-1}) = (\det A)^{-1}$
2). $\det A^t = \det A$
3). $\det(A.B) = \det A . \det B \,; B = B_0 + \sum_{i=1}^{5} B_i P_i$.

Definition.

Let $A = A_0 + \sum_{i=1}^{5} A_i P_i$ be a symbolic 5-plithogenic real square matrix, then:
A is called orthogonal if and only if $A^t = A^{-1}$.

Theorem5.

A is orthogonal if and only if $\sum_{i=0}^{j} A_i \,;\ 0 \leq j \leq 5$ is orthogonal.

Definition.

Let $A = A_0 + \sum_{i=1}^{5} A_i P_i$ be a symbolic 5-plithogenic complex square matrix, then A is called Hermit matrix if $A^* = (\bar{A})^t = A^{-1}$.

Theorem6.

A is Hermit matrix if and only if $\sum_{i=0}^{j} A_i \,;\ 0 \leq j \leq 5$ is Hermit matrix.

Proof of theorem1.

1). Let $A = A_0 + \sum_{i=1}^{5} A_i P_i$, then A is invertible if and only if there exists $B = B_0 + \sum_{i=1}^{5} B_i P_i$ such that:
$A \times A = U_{n\times n}$, hence:

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$$\begin{cases} A_0B_0 = U_{n\times n} \\ \sum_{i=0}^{1} A_i \sum_{i=0}^{1} B_i - A_0B_0 = O_{n\times n} \\ \sum_{i=0}^{2} A_i \sum_{i=0}^{2} B_i - \sum_{i=0}^{1} A_i \sum_{i=0}^{1} B_i = O_{n\times n} \\ \sum_{i=0}^{3} A_i \sum_{i=0}^{3} B_i - \sum_{i=0}^{2} A_i \sum_{i=0}^{2} B_i = O_{n\times n} \\ \sum_{i=0}^{4} A_i \sum_{i=0}^{4} B_i - \sum_{i=0}^{3} A_i \sum_{i=0}^{3} B_i = O_{n\times n} \\ \sum_{i=0}^{5} A_i \sum_{i=0}^{5} B_i - \sum_{i=0}^{4} A_i \sum_{i=0}^{4} B_i = O_{n\times n} \end{cases}$$

This implies that:

$$\begin{cases} A_0B_0 = U_{n\times n} \\ \sum_{i=0}^{j} A_i \sum_{i=0}^{j} B_i = U_{n\times n} \end{cases} ;\ 1 \le j \le 5$$

Hence $\det\left(\sum_{i=0}^{j} A_i\right) \neq 0$ for all $1 \le j \le 5$, so that $\det(A)$ is invertible in $5 - SP_R$.

2). It holds directly from the previous statement as follows:

$\sum_{i=0}^{j} B_i = \left(\sum_{i=0}^{j} A_i\right)^{-1}$ for $1 \le j \le 5$, hence:

$$A^{-1} = {A_0}^{-1} + P_1\left[\left(\sum_{i=0}^{1} A_i\right)^{-1} - {A_0}^{-1}\right] + \left[\left(\sum_{i=0}^{2} A_i\right)^{-1} - \left(\sum_{i=0}^{1} A_i\right)^{-1}\right]P_2 + \left[\left(\sum_{i=1}^{3} A_i\right)^{-1} - \left(\sum_{i=0}^{2} A_i\right)^{-1}\right]P_3 + \left[\left(\sum_{i=1}^{4} A_i\right)^{-1} - \left(\sum_{i=0}^{3} A_i\right)^{-1}\right]P_4 + \left[\left(\sum_{i=1}^{5} A_i\right)^{-1} - \left(\sum_{i=0}^{4} A_i\right)^{-1}\right]P_5$$

Proof of theorem2.

It is clear that t is an eigen value of A with X as an eigen vector if and only if:

$A.X = t.X$, which is equivalent to:

$$\begin{cases} A_0X_0 = t_0X_0 \\ \sum_{i=0}^{j} A_i \sum_{i=0}^{j} X_i = \sum_{i=0}^{j} t_i \sum_{i=0}^{j} X_i \end{cases} ;\ 1 \le j \le 5$$

Which is equivalent to the following statement:

$\sum_{i=0}^{j} t_i$ is an eigen value of $\sum_{i=0}^{j} A_i$ with $\sum_{i=0}^{j} X_i$ as an eigen vector for all $1 \le j \le 5$.

Proof of theorem3.

It holds directly as a special case of natural powers in symbolic 5-plithogenic rings, see [ ].

Proof of theorem4.

1). $detA^{-1} = det\left({A_0}^{-1}\right) + P_1\left[det\left(\sum_{i=0}^{1} A_i\right)^{-1} - det\left({A_0}^{-1}\right)\right] + \left[det\left(\sum_{i=0}^{2} A_i\right)^{-1} - det\left(\sum_{i=0}^{1} A_i\right)^{-1}\right]P_2 + \left[det\left(\sum_{i=1}^{3} A_i\right)^{-1} - det\left(\sum_{i=0}^{2} A_i\right)^{-1}\right]P_3 + \left[det\left(\sum_{i=1}^{4} A_i\right)^{-1} - det\left(\sum_{i=0}^{3} A_i\right)^{-1}\right]P_4 + \left[det\left(\sum_{i=1}^{5} A_i\right)^{-1} - det\left(\sum_{i=0}^{4} A_i\right)^{-1}\right]P_5 = (detA)^{-1}.$

2). $A^t = {A_0}^t + {A_1}^tP_1 + {A_2}^tP_2 + {A_3}^tP_3 + {A_4}^tP_4 + {A_5}^tP_5.$

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$$detA^t = det(A_0{}^t) + \left[det\left(\sum_{i=0}^{1} A_i{}^t\right) - det(A_0{}^t)\right]P_1 + \left[det\left(\sum_{i=0}^{2} A_i{}^t\right) - det\left(\sum_{i=0}^{1} A_i{}^t\right)\right]P_2$$
$$+ \left[det\left(\sum_{i=0}^{3} A_i{}^t\right) - det\left(\sum_{i=0}^{2} A_i{}^t\right)\right]P_3 + \left[det\left(\sum_{i=0}^{4} A_i{}^t\right) - det\left(\sum_{i=0}^{3} A_i{}^t\right)\right]P_4$$
$$+ \left[det\left(\sum_{i=0}^{5} A_i{}^t\right) - det\left(\sum_{i=0}^{4} A_i{}^t\right)\right]P_5$$
$$= \det(A_0) + \left[\det\left(\sum_{i=0}^{1} A_i\right) - \det(A_0)\right]P_1 + \left[\det\left(\sum_{i=0}^{2} A_i\right) - \det\left(\sum_{i=0}^{1} A_i\right)\right]P_2$$
$$+ \left[\det\left(\sum_{i=0}^{3} A_i\right) - \det\left(\sum_{i=0}^{2} A_i\right)\right]P_3 + \left[\det\left(\sum_{i=0}^{4} A_i\right) - \det\left(\sum_{i=0}^{3} A_i\right)\right]P_4$$
$$+ \left[\det\left(\sum_{i=0}^{5} A_i\right) - \det\left(\sum_{i=0}^{4} A_i\right)\right]P_5 = detA$$

3). we have:

$A.B = A_0B_0 + [\sum_{i=0}^{1} A_i \sum_{i=0}^{1} B_i - A_0B_0]P_1 + [\sum_{i=0}^{2} A_i \sum_{i=0}^{2} B_i - \sum_{i=0}^{1} A_i \sum_{i=0}^{1} B_i]P_2 + [\sum_{i=0}^{3} A_i \sum_{i=0}^{3} B_i - \sum_{i=0}^{2} A_i \sum_{i=0}^{2} B_i]P_3 + [\sum_{i=0}^{4} A_i \sum_{i=0}^{4} B_i - \sum_{i=0}^{3} A_i \sum_{i=0}^{3} B_i]P_4 + [\sum_{i=0}^{5} A_i \sum_{i=0}^{5} B_i - \sum_{i=0}^{4} A_i \sum_{i=0}^{4} B_i]P_5.$

$det(A.B) = det(A_0B_0) + [det(\sum_{i=0}^{1} A_i \sum_{i=0}^{1} B_i) - det(A_0B_0)]P_1 + [det(\sum_{i=0}^{2} A_i \sum_{i=0}^{2} B_i) - det(\sum_{i=0}^{1} A_i \sum_{i=0}^{1} B_i)]P_2 + [det(\sum_{i=0}^{3} A_i \sum_{i=0}^{3} B_i) - det(\sum_{i=0}^{2} A_i \sum_{i=0}^{2} B_i)]P_3 + [det(\sum_{i=0}^{4} A_i \sum_{i=0}^{4} B_i) - det(\sum_{i=0}^{3} A_i \sum_{i=0}^{3} B_i)]P_4 + [det(\sum_{i=0}^{5} A_i \sum_{i=0}^{5} B_i) - det\ (\sum_{i=0}^{4} A_i \sum_{i=0}^{4} B_i)]P_5 = det(A_0)det(B_0) + [det(\sum_{i=0}^{j} A_i).det(\sum_{i=0}^{j} B_i) - det(\sum_{i=1}^{j-1} A_{i-1}).det(\sum_{i=1}^{j-1} B_{i-1})]P_i = det(A)det(B); 1 \le j \le 5.$

Proof of theorem5.

A is orthogonal if and only if $A^t = A^{-1}$, hence:

$A_0{}^t + \sum_{i=1}^{5} A_i{}^t P_i = A_0{}^{-1} + [(\sum_{i=0}^{1} A_i)^{-1} - A_0{}^{-1}]P_1 + [(\sum_{i=0}^{2} A_i)^{-1} - (\sum_{i=0}^{1} A_i)^{-1}]P_2 + [(\sum_{i=1}^{3} A_i)^{-1} - (\sum_{i=0}^{2} A_i)^{-1}]P_3 + [(\sum_{i=1}^{4} A_i)^{-1} - (\sum_{i=0}^{3} A_i)^{-1}]P_4 + \left[(\sum_{i=1}^{5} A_i)^{-1} - (\sum_{i=0}^{4} A_i)^{-1}\right]P_5$, thus:

$$\begin{cases} A_0{}^t = A_0{}^{-1} \\ A_1{}^t = \left(\sum_{i=0}^{1} A_i\right)^{-1} - A_0{}^{-1} \\ A_2{}^t = \left(\sum_{i=0}^{2} A_i\right)^{-1} - \left(\sum_{i=0}^{1} A_i\right)^{-1} \\ A_3{}^t = \left(\sum_{i=0}^{3} A_i\right)^{-1} - \left(\sum_{i=0}^{2} A_i\right)^{-1} \\ A_4{}^t = \left(\sum_{i=0}^{4} A_i\right)^{-1} - \left(\sum_{i=0}^{3} A_i\right)^{-1} \\ A_5{}^t = \left(\sum_{i=0}^{5} A_i\right)^{-1} - \left(\sum_{i=0}^{4} A_i\right)^{-1} \end{cases}$$

This implies that:

$$\begin{cases} A_0{}^t = A_0{}^{-1} \\ \sum_{i=0}^{1} A_i{}^t = (\sum_{i=0}^{1} A_i)^{-1} \\ \sum_{i=0}^{2} A_i{}^t = (\sum_{i=0}^{2} A_i)^{-1} \\ \sum_{i=0}^{3} A_i{}^t = (\sum_{i=0}^{3} A_i)^{-1} \\ \sum_{i=0}^{4} A_i{}^t = (\sum_{i=0}^{4} A_i)^{-1} \\ \sum_{i=0}^{5} A_i{}^t = (\sum_{i=0}^{5} A_i)^{-1} \end{cases}$$, so that our proof is complete.

Theorem6 can be proven by a similar argument of theorem5.

Example.

Consider the symbolic 5-plithogenic real 2×2 matrix:

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$$A=\begin{pmatrix}2+P_1-2P_2+4P_3-P_4-2P_5 & 1+P_2+P_3-2P_4+4P_5\\ 1+P_1-P_2+2P_3-6P_4+4P_5 & 1+2P_2-P_3-3P_4+4P_5\end{pmatrix}$$
$$=\begin{pmatrix}2&1\\1&1\end{pmatrix}+\begin{pmatrix}1&0\\1&0\end{pmatrix}P_1+\begin{pmatrix}-2&1\\-1&2\end{pmatrix}P_2+\begin{pmatrix}4&1\\2&-1\end{pmatrix}P_3+\begin{pmatrix}-1&-2\\-6&-3\end{pmatrix}P_4+\begin{pmatrix}-2&4\\4&4\end{pmatrix}P_5$$

We have:

$$A_0=\begin{pmatrix}2&1\\1&1\end{pmatrix},\sum_{i=0}^{1}A_i=A_0+A_1=\begin{pmatrix}3&1\\2&1\end{pmatrix},\sum_{i=0}^{2}A_i=A_0+A_1+A_2=\begin{pmatrix}1&2\\1&3\end{pmatrix},\sum_{i=0}^{3}A_i=A_0+A_1+A_2+A_3$$
$$=\begin{pmatrix}5&3\\3&2\end{pmatrix},\sum_{i=0}^{4}A_i=A_0+A_1+A_2+A_3+A_4=\begin{pmatrix}4&1\\-3&-1\end{pmatrix},\sum_{i=0}^{5}A_i=\begin{pmatrix}2&5\\1&3\end{pmatrix}$$

On the other hand, we have:

$$\det(A_0)=1,\det\left(\sum_{i=0}^{1}A_i\right)=1,\det\left(\sum_{i=0}^{2}A_i\right)=1,\det\left(\sum_{i=0}^{3}A_i\right)=1,\det\left(\sum_{i=0}^{4}A_i\right)=-1,\det\left(\sum_{i=0}^{5}A_i\right)=1$$

Hence $\sum_{i=0}^{j}A_i$ is invertible for all $0\le j\le 5$, thus A is invertible.

$${A_0}^{-1}=\begin{pmatrix}1&-1\\-1&2\end{pmatrix},\left(\sum_{i=0}^{1}A_i\right)^{-1}=\begin{pmatrix}1&-1\\-2&3\end{pmatrix},\left(\sum_{i=0}^{2}A_i\right)^{-1}=\begin{pmatrix}3&-2\\-1&1\end{pmatrix},\left(\sum_{i=0}^{3}A_i\right)^{-1}$$
$$=\begin{pmatrix}2&-3\\-3&5\end{pmatrix},\left(\sum_{i=0}^{4}A_i\right)^{-1}=\begin{pmatrix}1&1\\-3&-4\end{pmatrix},\left(\sum_{i=0}^{5}A_i\right)^{-1}=\begin{pmatrix}3&-5\\-1&2\end{pmatrix}$$

This means that:

$$A^{-1}=\begin{pmatrix}1&-1\\-1&2\end{pmatrix}+\begin{pmatrix}0&0\\-1&1\end{pmatrix}P_1+\begin{pmatrix}2&-1\\1&-2\end{pmatrix}P_2+\begin{pmatrix}-1&-1\\-2&4\end{pmatrix}P_3+\begin{pmatrix}-1&4\\0&-9\end{pmatrix}P_4+\begin{pmatrix}2&-6\\-2&6\end{pmatrix}P_5$$
$$=\begin{pmatrix}1+2P_2-P_3-P_4+2P_5 & -1-P_2-P_3+4P_4-6P_5\\ -1-P_1+P_2-2P_3+2P_5 & 2+P_1-2P_2+4P_3-9P_4+6P_5\end{pmatrix}$$

Example on theorem2.

Consider the matrix:

$$A=\begin{pmatrix}2-P_1+3P_2-5P_3+3P_4-4P_5 & P_2-P_3+P_4-P_5\\ P_1-P_2+3P_3-3P_4+P_5 & 3-P_1+P_2-2P_3+2P_4-P_5\end{pmatrix}$$
$$=\begin{pmatrix}2&0\\0&3\end{pmatrix}+\begin{pmatrix}-1&0\\1&-1\end{pmatrix}P_1+\begin{pmatrix}3&1\\-1&1\end{pmatrix}P_2+\begin{pmatrix}-5&-1\\3&-2\end{pmatrix}P_3+\begin{pmatrix}3&1\\-3&2\end{pmatrix}P_4+\begin{pmatrix}-4&-1\\1&-1\end{pmatrix}P_5$$

The eigen values of A_0 are $\{2,3\}$, with the following eigen vectors$\{X_0=(1,0),\acute{X}_0=(0,1)\}$.

The eigen values of $\sum_{i=0}^{1}A_i$ are $\{1,2\}$, with the following eigen vectors$\{X_1=(1,-1),\acute{X}_1=(0,1)\}$.

The eigen values of $\sum_{i=0}^{2}A_i$ are $\{4,3\}$, with the following eigen vectors$\{X_2=(1,0),\acute{X}_2=(1,-1)\}$.

The eigen values of $\sum_{i=0}^{3}A_i$ are $\{-1,1\}$, with the following eigen vectors$\left\{X_3=\left(1,-\frac{3}{2}\right),\acute{X}_3=(0,1)\right\}$.

The eigen values of $\sum_{i=0}^{4}A_i$ are $\{2,3\}$, with the following eigen vectors$\{X_4=(1,0),\acute{X}_4=(1,1)\}$.

The eigen values of $\sum_{i=0}^{5}A_i$ are $\{-2,2\}$, with the following eigen vectors$\left\{X_5=\left(1,-\frac{1}{4}\right),\acute{X}_5=(1,4)\right\}$.

We get that A has 2^6 eigen values.

For example:

$t_0=2+(1-2)P_1+(4-1)P_2+(-1-4)P_3+(2+1)P_4+(-2-2)P_5=2-P_1+3P_2-5P_3+3P_4-4P_5$

Is an eigenvalue of A with the following eigen value vector:

$$X_0=(1,0)+[(1,-1)-(1,0)]P_1+[(1,0)-(1,-1)]P_2+\left[\left(1,-\frac{3}{2}\right)-(1,0)\right]P_3+\left[(1,0)-\left(1,-\frac{3}{2}\right)\right]P_4$$
$$+\left[\left(1,-\frac{1}{4}\right)-(1,0)\right]P_5$$
$$=(1,0)+(0,-1)P_1+(0,1)P_2+\left(0,-\frac{1}{2}\right)P_3+\left(0,\frac{3}{2}\right)P_4+\left(0,-\frac{1}{4}\right)P_5$$

Another eigen value is:

$t_0=2+(2-3)P_1+(3-2)P_2+(1-3)P_3+(3-1)P_4+(2-3)P_5=3-P_1+P_2-2P_3+2P_4-P_5$

With the following eigen vector:

$$X_1=(0,1)+[(0,1)-(0,1)]P_1+[(1,-1)-(0,1)]P_2+[(0,1)-(1,-1)]P_3+[(1,1)-(0,1)]P_4$$
$$+[(1,4)-(1,1)]P_5=(0,1)+(0,0)P_1+(1,-2)P_2+(-1,2)P_3+(1,0)P_4+(0,3)P_5$$

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Example on theorem4.

Consider:

$$A=\begin{pmatrix} 2-P_1+3P_2-5P_3+3P_4-4P_5 & P_2-P_3+P_4-P_5 \\ P_1-P_2+3P_3-3P_4+P_5 & 3-P_1+P_2-2P_3+2P_4-P_5 \end{pmatrix}$$

$$=\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}+\begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}P_1+\begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}P_2+\begin{pmatrix} -5 & -1 \\ 3 & -2 \end{pmatrix}P_3+\begin{pmatrix} 3 & 1 \\ -3 & 2 \end{pmatrix}P_4+\begin{pmatrix} -4 & -1 \\ 1 & -1 \end{pmatrix}P_5$$

$$\det(A_0)=6, \det\left(\sum_{i=0}^{1}A_i\right)=2, \det\left(\sum_{i=0}^{2}A_i\right)=12, \det\left(\sum_{i=0}^{3}A_i\right)=-1, \det\left(\sum_{i=0}^{4}A_i\right)=6, \det\left(\sum_{i=0}^{5}A_i\right)=-4$$

$$\det A=6+(2-6)P_1+(12-2)P_2+(-1-12)P_3+(6+1)P_4+(-4-6)P_5$$

$$=6-4P_1+10P_2-13P_3+7P_4-10P_5$$

$$(detA)^{-1}=\frac{1}{6}+\left(\frac{1}{2}-\frac{1}{6}\right)P_1+\left(\frac{1}{12}-\frac{1}{2}\right)P_2+\left(-1-\frac{1}{12}\right)P_3+\left(\frac{1}{6}+1\right)P_4+\left(\frac{1}{4}-\frac{1}{6}\right)P_5=\frac{1}{6}+\frac{1}{3}P_1-\frac{5}{12}P_2-\frac{13}{12}P_3+\frac{7}{6}P_4-\frac{5}{12}P_5=detA^{-1}.$$

Symbolic 6-plithogenic matrices

Definition:

The square symbolic 6-plithogenic matrix is defined as follows:

$A=A_0+\sum_{i=1}^{6}A_iP_i$; $(A_i)_{n\times n}$ is square matrix of real entries.

Example.

Consider the symbolic 6-plithogenic matrix:

$$A=\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}+\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}P_1+\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}P_2+\begin{pmatrix} 4 & -1 \\ -1 & 2 \end{pmatrix}P_3+\begin{pmatrix} 5 & 5 \\ 0 & 0 \end{pmatrix}P_4+\begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}P_5+\begin{pmatrix} 1 & 1 \\ -1 & -2 \end{pmatrix}P_6.$$

Definition.

Let $A=A_0+\sum_{i=1}^{6}A_iP_i$ be a symbolic 6-plithogenic matrix of size $n\times n$, hence:

$$\det A=\det(A_0)+\left[\det\left(\sum_{i=0}^{1}A_i\right)-\det(A_0)\right]P_1+\left[\det\left(\sum_{i=0}^{2}A_i\right)-\det\left(\sum_{i=0}^{1}A_i\right)\right]P_2$$
$$+\left[\det\left(\sum_{i=0}^{3}A_i\right)-\det\left(\sum_{i=0}^{2}A_i\right)\right]P_3+\left[\det\left(\sum_{i=0}^{4}A_i\right)-\det\left(\sum_{i=0}^{3}A_i\right)\right]P_4$$
$$+\left[\det\left(\sum_{i=0}^{5}A_i\right)-\det\left(\sum_{i=0}^{4}A_i\right)\right]P_5+\left[\det\left(\sum_{i=0}^{6}A_i\right)-\det\left(\sum_{i=0}^{5}A_i\right)\right]P_6$$

Theorem1.

Let $A=A_0+\sum_{i=1}^{6}A_iP_i$ be a symbolic 6-plithogenic matrix of size $n\times n$, hence:

1. A is invertible if and only if $\det A$ is an invertible symbolic 6-plithogenic number.
2. $A^{-1}={A_0}^{-1}+\left[(\sum_{i=0}^{1}A_i)^{-1}-{A_0}^{-1}\right]P_1+[(\sum_{i=0}^{2}A_i)^{-1}-(\sum_{i=0}^{1}A_i)^{-1}]P_2+[(\sum_{i=1}^{3}A_i)^{-1}-(\sum_{i=0}^{2}A_i)^{-1}]P_3+[(\sum_{i=1}^{4}A_i)^{-1}-(\sum_{i=0}^{3}A_i)^{-1}]P_4+\left[(\sum_{i=1}^{5}A_i)^{-1}-(\sum_{i=0}^{4}A_i)^{-1}\right]P_5+\left[(\sum_{i=1}^{6}A_i)^{-1}-(\sum_{i=0}^{5}A_i)^{-1}\right]P_6$

Definition.

Let $t=t_0+\sum_{i=1}^{6}t_iP_i$ be a symbolic 6-plithogenic real number and $A=A_0+\sum_{i=1}^{6}A_iP_i$ be a symbolic 6-plithogenic square real matrix, then t is called symbolic 6-plithogenic eigen values if and only if $AX=tX$.

X is called symbolic 6-plithogenic eigenvector.

Theorem2.

Let $t=t_0+\sum_{i=1}^{6}t_iP_i\in 6-SP_R$, $X=X_0+\sum_{i=1}^{6}X_iP_i$ be a symbolic 6-plithogeni real vector, then t is eigen value of $A=A_0+\sum_{i=1}^{6}A_iP_i$ with X as the corresponding eigen vector if and only if:

$\sum_{i=0}^{j}t_i$ is eigen value of $\sum_{i=0}^{j}A_i$ with $\sum_{i=0}^{j}X_i$ as eigen vector with $0\le j\le 6$.

Theorem3.

$$A^n={A_0}^n+P_1\left[\left(\sum_{i=0}^{1}A_i\right)^n-{A_0}^n\right]+\left[\left(\sum_{i=0}^{2}A_i\right)^n-\left(\sum_{i=0}^{1}A_i\right)^n\right]P_2+\left[\left(\sum_{i=1}^{3}A_i\right)^n-\left(\sum_{i=0}^{2}A_i\right)^n\right]P_3$$
$$+\left[\left(\sum_{i=1}^{4}A_i\right)^n-\left(\sum_{i=0}^{3}A_i\right)^n\right]P_4+\left[\left(\sum_{i=1}^{5}A_i\right)^n-\left(\sum_{i=0}^{4}A_i\right)^n\right]P_5+\left[\left(\sum_{i=1}^{6}A_i\right)^n-\left(\sum_{i=0}^{5}A_i\right)^n\right]P_6$$

Theorem4.

Let $A=A_0+\sum_{i=1}^{6}A_iP_i$ be a square 6-plithogenic invertible real matrix, then:

1). $\det(A^{-1})=(\det A)^{-1}$

2). $\det A^t=\det A$

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3). $\det(A.B) = \det A . \det B\,; B = B_0 + \sum_{i=1}^{6} B_i P_i$.

Definition.

Let $A = A_0 + \sum_{i=1}^{6} A_i P_i$ be a symbolic 6-plithogenic real square matrix, then:
A is called orthogonal if and only if $A^t = A^{-1}$.

Theorem5.

A is orthogonal if and only if $\sum_{i=0}^{j} A_i\,;\ 0 \leq j \leq 6$ is orthogonal.

Definition.

Let $A = A_0 + \sum_{i=1}^{6} A_i P_i$ be a symbolic 5-plithogenic complex square matrix, then A is called Hermit matrix if $A^* = (\bar{A})^t = A^{-1}$.

Theorem6.

A is Hermit matrix if and only if $\sum_{i=0}^{j} A_i\,;\ 0 \leq j \leq 6$ is Hermit matrix.

Proof of theorem1.

1). Let $A = A_0 + \sum_{i=1}^{6} A_i P_i$, then A is invertible if and only if there exists $B = B_0 + \sum_{i=1}^{6} B_i P_i$ such that:
$A \times A = U_{n\times n}$, hence:

$$\begin{cases} A_0 B_0 = U_{n\times n} \\ \sum_{i=0}^{1} A_i \sum_{i=0}^{1} B_i - A_0 B_0 = O_{n\times n} \\ \sum_{i=0}^{2} A_i \sum_{i=0}^{2} B_i - \sum_{i=0}^{1} A_i \sum_{i=0}^{1} B_i = O_{n\times n} \\ \sum_{i=0}^{3} A_i \sum_{i=0}^{3} B_i - \sum_{i=0}^{2} A_i \sum_{i=0}^{2} B_i = O_{n\times n} \\ \sum_{i=0}^{4} A_i \sum_{i=0}^{4} B_i - \sum_{i=0}^{3} A_i \sum_{i=0}^{3} B_i = O_{n\times n} \\ \sum_{i=0}^{5} A_i \sum_{i=0}^{5} B_i - \sum_{i=0}^{4} A_i \sum_{i=0}^{4} B_i = O_{n\times n} \\ \sum_{i=0}^{6} A_i \sum_{i=0}^{6} B_i - \sum_{i=0}^{5} A_i \sum_{i=0}^{5} B_i = O_{n\times n} \end{cases}$$

This implies that:

$$\begin{cases} A_0 B_0 = U_{n\times n} \\ \sum_{i=0}^{j} A_i \sum_{i=0}^{j} B_i = U_{n\times n} \end{cases} ;\ 1 \leq j \leq 6$$

Hence $\det\left(\sum_{i=0}^{j} A_i\right) \neq 0$ for all $1 \leq j \leq 6$, so that $\det(A)$ is invertible in $6 - SP_R$.

2). It holds directly from the previous statement as follows:

$\sum_{i=0}^{j} B_i = \left(\sum_{i=0}^{j} A_i\right)^{-1}$ for $1 \leq j \leq 6$, hence:

$$\begin{aligned} A^{-1} = {A_0}^{-1} + P_1\left[\left(\sum_{i=0}^{1} A_i\right)^{-1} - {A_0}^{-1}\right] + \left[\left(\sum_{i=0}^{2} A_i\right)^{-1} - \left(\sum_{i=0}^{1} A_i\right)^{-1}\right] P_2 + \left[\left(\sum_{i=1}^{3} A_i\right)^{-1} - \left(\sum_{i=0}^{2} A_i\right)^{-1}\right] P_3 \\ + \left[\left(\sum_{i=1}^{4} A_i\right)^{-1} - \left(\sum_{i=0}^{3} A_i\right)^{-1}\right] P_4 + \left[\left(\sum_{i=1}^{5} A_i\right)^{-1} - \left(\sum_{i=0}^{4} A_i\right)^{-1}\right] P_5 \\ + \left[\left(\sum_{i=1}^{6} A_i\right)^{-1} - \left(\sum_{i=0}^{5} A_i\right)^{-1}\right] P_6 \end{aligned}$$

Proof of theorem2.

It is clear that t is an eigen value of A with X as an eigen vector if and only if:
$A.X = t.X$, which is equivalent to:

$$\begin{cases} A_0 X_0 = t_0 X_0 \\ \sum_{i=0}^{j} A_i \sum_{i=0}^{j} X_i = \sum_{i=0}^{j} t_i \sum_{i=0}^{j} X_i \end{cases} ;\ 1 \leq j \leq 6$$

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Which is equivalent to the following statement:
$\sum_{i=0}^{j} t_i$ is an eigen value of $\sum_{i=0}^{j} A_i$ with $\sum_{i=0}^{j} X_i$ as an eigen vector for all $1 \le j \le 6$.

Proof of theorem3.

It holds directly as a special case of natural powers in symbolic 6-plithogenic rings.

Proof of theorem4.

1). $detA^{-1} = det(A_0{}^{-1}) + P_1[det(\sum_{i=0}^{1} A_i)^{-1} - det(A_0{}^{-1})] + [det(\sum_{i=0}^{2} A_i)^{-1} - det(\sum_{i=0}^{1} A_i)^{-1}]P_2 + [det(\sum_{i=1}^{3} A_i)^{-1} - det(\sum_{i=0}^{2} A_i)^{-1}]P_3 + [det(\sum_{i=1}^{4} A_i)^{-1} - det(\sum_{i=0}^{3} A_i)^{-1}]P_4 + \left[det(\sum_{i=1}^{5} A_i)^{-1} - det(\sum_{i=0}^{4} A_i)^{-1}\right] P_5 + \left[det(\sum_{i=1}^{6} A_i)^{-1} - det(\sum_{i=0}^{5} A_i)^{-1}\right] P_6 = (detA)^{-1}$.

2). $A^t = A_0{}^t + A_1{}^t P_1 + A_2{}^t P_2 + A_3{}^t P_3 + A_4{}^t P_4 + A_5{}^t P_5 + A_6{}^t P_6$.

$detA^t = det(A_0{}^t) + [det(\sum_{i=0}^{1} A_i{}^t) - det(A_0{}^t)]P_1 + [det(\sum_{i=0}^{2} A_i{}^t) - det(\sum_{i=0}^{1} A_i{}^t)]P_2 + [det(\sum_{i=0}^{3} A_i{}^t) - det(\sum_{i=0}^{2} A_i{}^t)]P_3 + [det(\sum_{i=0}^{4} A_i{}^t) - det(\sum_{i=0}^{3} A_i{}^t)]P_4 + [det(\sum_{i=0}^{5} A_i{}^t) - det\ (\sum_{i=0}^{4} A_i{}^t)]P_5 + [det(\sum_{i=0}^{6} A_i{}^t) - det\ (\sum_{i=0}^{5} A_i{}^t)]P_6 = \det(A_0) + [\det(\sum_{i=0}^{1} A_i) - \det(A_0)]P_1 + [\det(\sum_{i=0}^{2} A_i) - \det(\sum_{i=0}^{1} A_i)]P_2 + [\det(\sum_{i=0}^{3} A_i) - \det(\sum_{i=0}^{2} A_i)]P_3 + [\det(\sum_{i=0}^{4} A_i) - \det(\sum_{i=0}^{3} A_i)]P_4 + [\det(\sum_{i=0}^{5} A_i) - \det(\sum_{i=0}^{4} A_i)]P_5 + [\det(\sum_{i=0}^{6} A_i) - \det(\sum_{i=0}^{5} A_i)]P_6 = detA$.

3). we have:

$A.B = A_0B_0 + [\sum_{i=0}^{1} A_i \sum_{i=0}^{1} B_i - A_0B_0]P_1 + [\sum_{i=0}^{2} A_i \sum_{i=0}^{2} B_i - \sum_{i=0}^{1} A_i \sum_{i=0}^{1} B_i]P_2 + [\sum_{i=0}^{3} A_i \sum_{i=0}^{3} B_i - \sum_{i=0}^{2} A_i \sum_{i=0}^{2} B_i]P_3 + [\sum_{i=0}^{4} A_i \sum_{i=0}^{4} B_i - \sum_{i=0}^{3} A_i \sum_{i=0}^{3} B_i]P_4 + [\sum_{i=0}^{5} A_i \sum_{i=0}^{5} B_i - \sum_{i=0}^{4} A_i \sum_{i=0}^{4} B_i]P_5 + [\sum_{i=0}^{6} A_i \sum_{i=0}^{6} B_i - \sum_{i=0}^{5} A_i \sum_{i=0}^{5} B_i]P_6$.

$det(A.B) = det(A_0B_0) + [det(\sum_{i=0}^{1} A_i \sum_{i=0}^{1} B_i) - det(A_0B_0)]P_1 + [det(\sum_{i=0}^{2} A_i \sum_{i=0}^{2} B_i) - det(\sum_{i=0}^{1} A_i \sum_{i=0}^{1} B_i)]P_2 + [det(\sum_{i=0}^{3} A_i \sum_{i=0}^{3} B_i) - det(\sum_{i=0}^{2} A_i \sum_{i=0}^{2} B_i)]P_3 + [det(\sum_{i=0}^{4} A_i \sum_{i=0}^{4} B_i) - det(\sum_{i=0}^{3} A_i \sum_{i=0}^{3} B_i)]P_4 + [det(\sum_{i=0}^{5} A_i \sum_{i=0}^{5} B_i) - det\ (\sum_{i=0}^{4} A_i \sum_{i=0}^{4} B_i)]P_5 + [det(\sum_{i=0}^{6} A_i \sum_{i=0}^{6} B_i) - det\ (\sum_{i=0}^{5} A_i \sum_{i=0}^{5} B_i)]P_6 = det(A_0)det(B_0) + [det(\sum_{i=0}^{j} A_i).det(\sum_{i=0}^{j} B_i) - det(\sum_{i=1}^{j-1} A_{i-1}).det(\sum_{i=1}^{j-1} B_{i-1})]P_i = det(A)det(B); 1 \le j \le 6$.

Proof of theorem5.

A is orthogonal if and only if $A^t = A^{-1}$, hence:

$A_0{}^t + \sum_{i=1}^{6} A_i{}^t P_i = A_0{}^{-1} + [(\sum_{i=0}^{1} A_i)^{-1} - A_0{}^{-1}]P_1 + [(\sum_{i=0}^{2} A_i)^{-1} - (\sum_{i=0}^{1} A_i)^{-1}]P_2 + [(\sum_{i=1}^{3} A_i)^{-1} - (\sum_{i=0}^{2} A_i)^{-1}]P_3 + [(\sum_{i=1}^{4} A_i)^{-1} - (\sum_{i=0}^{3} A_i)^{-1}]P_4 + \left[(\sum_{i=1}^{5} A_i)^{-1} - (\sum_{i=0}^{4} A_i)^{-1}\right] P_5 + \left[(\sum_{i=1}^{6} A_i)^{-1} - (\sum_{i=0}^{5} A_i)^{-1}\right] P_6$, thus:

$$\begin{cases} A_0{}^t = A_0{}^{-1} \\ A_1{}^t = \left(\sum_{i=0}^{1} A_i\right)^{-1} - A_0{}^{-1} \\ A_2{}^t = \left(\sum_{i=0}^{2} A_i\right)^{-1} - \left(\sum_{i=0}^{1} A_i\right)^{-1} \\ A_3{}^t = \left(\sum_{i=0}^{3} A_i\right)^{-1} - \left(\sum_{i=0}^{2} A_i\right)^{-1} \\ A_4{}^t = \left(\sum_{i=0}^{4} A_i\right)^{-1} - \left(\sum_{i=0}^{3} A_i\right)^{-1} \\ A_5{}^t = \left(\sum_{i=0}^{5} A_i\right)^{-1} - \left(\sum_{i=0}^{4} A_i\right)^{-1} \\ A_6{}^t = \left(\sum_{i=0}^{6} A_i\right)^{-1} - \left(\sum_{i=0}^{5} A_i\right)^{-1} \end{cases}$$

This implies that:

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$$\begin{cases} A_0{}^t = A_0{}^{-1} \\ \sum_{i=0}^{1} A_i{}^t = (\sum_{i=0}^{1} A_i)^{-1} \\ \sum_{i=0}^{2} A_i{}^t = (\sum_{i=0}^{2} A_i)^{-1} \\ \sum_{i=0}^{3} A_i{}^t = (\sum_{i=0}^{3} A_i)^{-1} \\ \sum_{i=0}^{4} A_i{}^t = (\sum_{i=0}^{4} A_i)^{-1} \\ \sum_{i=0}^{5} A_i{}^t = \left(\sum_{i=0}^{5} A_i\right)^{-1} \\ \sum_{i=0}^{6} A_i{}^t = (\sum_{i=0}^{6} A_i)^{-1} \end{cases}$$, so that our proof is complete.

Theorem6 can be proven by a similar argument of theorem5.

5. Conclusion

In this paper, we have studied for the first time the square symbolic 5-plithogenic and 6-plithogenic matrices, where we have present many effective algorithms for computing determinants, Invertibility, and eigenvalues. As a future research direction, we aim to study the diagonalization problem and the representation problem of symbolic 5-plithogenic and symbolic 6-plithogenic matrices.

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