Scientia Magna
Vol. 4 (2008), No. 4, 12-14

On the Smarandache prime-digital subsequence sequences

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Abstract The main purpose of the paper is using the elementary method to study the properties of the Smarandache Prime-Digital Subsequence, and give an interesting limit Theorem. This solved a problem proposed by Charles.

Keywords Smarandache $SPDS$ subsequence, $\pi(n)$ function, limit.

§1. Introduction and results

For any positive integer n, the Smarandache Prime-Digital Subsequence ($SPDS$) is defined as follows:

A positive integer n is an element of $SPDS$, if it satisfies the following properties:

a) m is a prime.

b) All of the digits of m are prime, i.e, they are all elements of the set $\{2,\ 3,\ 5,\ 7\}$.

For example, the first few values of $SPDS$ are:

$2,\ 3,\ 5,\ 7,\ 23,\ 53,\ 73,\ 223,\ 227,\ 233,\ 257,\ 277,\ 377,\ 353,\ 373,\ 523, \cdots$.

This sequence was introduced by professor F.Smarandache in reference [1], where he asked us to studied its elementary properties. In reference [2], Charles Ashbacher had studied this problem, and obtained some interesting results. At the same time, he also proposed the following Conjecture and Unsolved problems.

Conjecture. Sequence $SPDS$ is a infinite set.

Unsolved problem 1. How many prime are there of the form

$$\underbrace{111\cdots111}_{k\ \ 1^{'}s}$$

where of course k is odd.

Unsolved problem 2.

$$\lim_{n\rightarrow\infty}\frac{SPDSN(n)}{\pi(n)}=0,$$

where $SPDSN(n)$ represent the number of elements of $SPDS$ that are less than or equal to n, and $\pi(n)$ denotes the number of all primes not exceeding n.

A short UBASIC program was run for all numbers up to $1,000,000$, and the counts were 78498 primes $< 1,000,000$; 587 members of $SPDS < 1,000,000$.

But at present, we still can not prove that $SPDS$ is a finite set, and we can not also solve Problem 1. In this paper, we will use the elementary method and analytic method to study Problem 2, and solved it completely. That is, we shall prove the following:

Theorem. Let $SPDSN(n)$ denotes the number of all elements of $SPDS$ that are less than or equal to n, and $\pi(n)$ denotes the number of all primes not exceeding n. Then we have the limit

$$\lim_{n\to\infty} \frac{SPDSN(n)}{\pi(n)} = 0.$$

It is clear that our Theorem solved the problem 2.

§2. Proof of the theorem

To complete the proof of our theorem, we need a simple Lemma which stated as follows:

Lemma. For every integer $n \geq 2$, we have the estimate

$$\frac{1}{6} \cdot \frac{n}{\ln n} < \pi(n) < 6 \cdot \frac{n}{\ln n}.$$

Proof. See reference [3].

Now we use this Lemma to prove our theorem. For any positive integer n, if digits of n in decimal notation are $A_{k-1},\ A_{k-2},\ \cdots,\ A_1,\ A_0$, then

$$n = A_{k-1}10^{k-1} + A_{k-2}10^{k-2} + \cdots + A_1 10^1 + A_0.$$

where $1 \leq A_i \leq 9$.
It is clear that

$$10^{k-1} \leq n \leq 10^k.$$

Therefore,

$$\lg n \leq k \leq \lg n + 1$$

or

$$k = \lg n + O(1).$$

Since

$$SPDNS(n) = \sum_{\substack{m\leq n\\ m\in SPDS}} 1 \leq 4^1 + 4^2 + 4^3 + \cdots + 4^k = \frac{4}{3}(4^k - 1) < 4^{k+1}$$

and

$$\frac{1}{6} \cdot \frac{n}{\ln n} < \pi(n) < 6 \cdot \frac{n}{\ln n}.$$

We have

$$0 \leq \frac{SPDSN(n)}{\pi(n)} \quad \leq \quad \frac{4^{k+1} \cdot 6 \cdot \ln n}{n} = \frac{4^{\lg n + O(1)} \cdot 6 \cdot \ln n}{n}.$$

Taking $x \to \infty$, we find that

$$\begin{aligned}
0 \le \lim_{x\to\infty} \frac{4^{\lg x + O(1)} \cdot 6 \cdot \ln x}{x} &\ll \lim_{x\to\infty} \frac{4^{\lg x} \cdot 6 \cdot \ln x}{x} = \lim_{x\to\infty} \frac{e^{\ln 4 \cdot \lg x} \cdot 6 \cdot \ln x}{x} \\
&= \lim_{x\to\infty} \frac{e^{\ln x \cdot \frac{\ln 4}{\ln 10}} \cdot 6 \cdot \ln x}{x} = \lim_{x\to\infty} \frac{x^{\frac{\ln 4}{\ln 10}} \cdot 6 \cdot \ln x}{x} \\
&= \lim_{x\to\infty} \frac{6 \cdot \ln x}{x^{(1-\frac{\ln 4}{\ln 10})}} = \lim_{x\to\infty} \frac{6}{(1-\frac{\ln 4}{\ln 10}) \cdot x^{(1-\frac{\ln 4}{\ln 10})}} \\
&= 0.
\end{aligned}$$

So from the properties of the limit we have

$$\lim_{n\to\infty} \frac{4^{\lg n + 1 + O(1)} \cdot 6 \cdot \ln n}{n} = 0.$$

Therefore,

$$\lim_{n\to\infty} \frac{SPDSN(n)}{\pi(n)} = 0.$$

This completes the proof of Theorem.

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