THE PRIMES IN THE SMARANDACHE SYMMETRIC SEQUENCE

Maohua Le

Department of Mathematics, Zhanjiang Normal College
Zhanjiang, Guangdong, P.R.China.

Abstract. Let $S=\{s_n\}_{n=1}^{\infty}$ Be the Smarandache symmetric sequence. In this paper we prove that if n is an even integer and $n/2 \neq 1 \pmod 3$, then s_n is not a prime.

Let $S=\{s_n\}_{n=1}^{\infty}$ be the Smarandache symmetric sequence, where

(1) $s_1=1,\ s_2=11,\ s_3=121,\ s_4=1221,\ s_5=12321,\ s_6=123321,$
$s_7=1234321,\ s_8=12344321,\ \ldots$.

Smarandache asked how many primes are there among S? (See [1, Notions 3]). In this paper we prove the following result:

Theorem. If n is an iven integer and $n/2 \neq 1 \pmod 3$, then s_n is not a prime.

Proof. If n is an even integer, then n=2k, where k is a positive integer. We see from (1) that

(2) $s_n = \overline{12\ldots kk\ldots 21}$

It implies that

$$(3)\qquad s_n = 1*10^{t_1} + 2*10^{t_2} + \ldots + k*10^{t_k} + k*10^{t_{k+1}} + \ldots + 2*10^{t_{2k-1}} + 1*10^{t_{2k}},$$

where $t_1, t_2, \ldots, t_{2k}$ are nonnegative integers. Since $10^t \equiv 1 \pmod 3$ for any nonnegative integer t, we get from (3) that

$$(4)\qquad s_n \equiv 1+2+\ldots+k+k+\ldots+2+1 \equiv k(k+1) \pmod 3.$$

If $k \not\equiv 1 \pmod 3$, then either $k \equiv 0 \pmod 3$ or $k \equiv 2 \pmod 3$.
In both cases, we have $k(k+1) \equiv 0 \pmod 3$ and $3 \mid s_n$ by (4).
Thus, s_n is not a prime. The theorem is proved.

Reference

1. Dumitrescu and Seleacu, Some Notions and Questions In Number Theory, Erhus Univ. Press, Glendale, 1994.