Problems

Edited by

Charles Ashbacher
Decisionmark
200 2nd Ave. SE
Cedar Rapids, IA 52401
FAX (319) 365-5694
e-mail 71603.522@compuserve.com

Welcome to the inaugural version of what is to be a regular feature in **Smarandache Notions**! Our goal is to present interesting and challenging problems in all areas and at all levels of difficulty with the only limits being good taste. Readers are encouraged to submit new problems and solutions to the editor at one of the addresses given above. All solvers will be acknowledged in a future issue. Please submit a solution along with your proposals if you have one. If there is no solution and the editor deems it appropriate, that problem may appear in the companion column of unsolved problems. Feel free to submit computer related problems and use computers in your work. Programs can also be submitted as part of the solution. While the editor is fluent in many programming languages, be cautious in submitting programs as solutions. Wading through several pages of obtuse program to determine if the submitter has done it right is not the editors idea of a good time. Make sure you explain things in detail.

If no solution is currently available, the problem will be flagged with an asterisk*. The deadline for submission of solutions will generally be six months after the date appearing on that issue. Regardless of deadline, no problem is ever officially closed in the sense that new insights or approaches are always welcome. If you submit a problem or solution and wish to guarantee a reply, please include a self-addressed envelope or postcard with appropriate stamps attached. Suggestions for improvement or modification are also welcome at any time. All proposals in this initial offering are by the editor.

The Smarandache function S(n) is defined in the following way

For $n \geq 1$, $S(n) = m$ is the smallest nonnegative integer such that n evenly divides m factorial.

New Problems

1) The Euler phi function $\phi(n)$ is defined to the number of positive integers not exceeding n that are relatively prime to n.

a) Prove that there are no solutions to the equation

$$\phi(S(n)) = n$$

b) Prove that there are no solutions to the equation

$$S(\phi(n)) = n$$

c) Prove that there are an infinite number of solutions to the equation

$$n - \phi(S(n)) = 1$$

d) Prove that for every odd prime p, there is a number n such that

$$n - \phi(S(n)) = p+1$$

2) This problem was proposed in **Canadian Mathematical Bulletin** by P. Erdos and was listed as unsolved in the book **Index to Mathematical Problems 1980-1984** edited by Stanley Rabinowitz and published by MathPro Press.

Prove that for infinitely many n

$$\phi(n) < \phi(n - \phi(n)).$$

3) The following appeared as unsolved problem (21) in **Unsolved Problems Related To Smarandache Function**, edited by R. Muller and published by Number Theory Publishing Company.

Are there m,n,k non-null positive integers, $m,n \neq 1$ for which

$$S(mn) = m^k * S(n)?$$

Find a solution.

4) The following appeared as unsolved problem (22) in **Unsolved Problems Related to Smarandache Function**, edited by R. Muller and published by Number Theory Publishing Company.

Is it possible to find two distinct numbers k and n such that

$$\log_{S(k^n)} S(n^k)$$

is an integer?

Find two integers n and k that satisfy these conditions.

5) Solve the following doubly true Russian alphametic

ДВА	2
ДВА	2
ТРИ	3
-------	--
СЕМЬ	7

where 2 divides ДВА, 3 divides ТРИ and 7 divides СЕМЬ.

Can anyone come up with a similar Romanian alphametic?

6) Prove the Smarandache Divisibility Theorem

If a and m are integers and $m > 0$, then

$$(a^m - a)(m - 1)!$$

is divisible by m.

Which was problem (126) in **Some Notions and Questions in Number Theory**, published by Erhus University Press.

7) Let $D = \{0,1,2,3,4,5,6,7,8,9\}$. For any number $1 \leq n \leq 10$, we can take n unique digits from D and form a number, leading zero not allowed. Let P_n be the set of all numbers that can be formed by choosing n unique digits from D. If 1 is not considered prime, which of the sets P_n contains the largest percentage of primes?

This problem is similar to unsolved problem 3 part (a) that appeared in **Only Problems, Not Solutions**, by Florentin Smarandache.

*8) The following four problems are all motivated by unsolved problem 3 part (b) that appeared in **Only Problems, Not Solutions**, by Florentin Smarandache.

a) Find the smallest integer n such that n! contains all 10 decimal digits.

b) Find the smallest integer n such that the n-th prime contains all 10 decimal digits.

c) Find the smallest integer n such that n^n contains all 10 decimal digits.

d) Find the smallest integer n such that n! contains one digit 10 times. What is that digit?