SMARANDACHE PARTITION TYPE AND OTHER SEQUENCES*

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ABSTRACT

Thanks to C. Dumitrescu and Dr. V. Seleacu of the University of Craiova, Department of Mathematics, I became familiar with some of the Smarandache Sequences. I list some of them, as well as questions related to them. Now I'm working in a few conjectures involving these sequences.

Examples of Smarandache Partition type sequences:

A. 1, 1, 1, 2, 2, 2, 2, 3, 4, 4, ...

(How many times is n written as a sum of non-null squares, disregarding the order of the terms:
for example:

$$\begin{aligned}9 &= 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2\\ &= 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 2^2\\ &= 1^2 + 2^2 + 2^2\\ &= 3^2,\end{aligned}$$

therefore ns(9) = 4.)

B. 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, ...

(How many times is n written as a sum of non-null cubes, disregarding the order of the terms:
for example:

$$\begin{aligned}9 &= 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3\\ &= 1^3 + 2^3,\end{aligned}$$

therefore, nc(9) = 2.)

C. General-partition type sequence:

Let f be an arithmetic function and R a relation among numbers.
(How many times can n be written under the form:

$$n = R(f(n_1), f(n_2), \ldots, f(n_k))$$

for some k and $n_1, n_2, \ldots, n_k$ such that

$$n_1 + n_2 + \ldots + n_k = n?\}$$

Examples of other sequences:

(1) Smarandache Anti-symmetric sequence:

11, 1212, 123123, 12341234, 1234512345, 123456123456, 12345671234567, 1234567812345678, 123456789, 123456789, 1234567891012345678910, 1234567891011, 1234567891011, ...

(2) Smarandache Triangular base:

1, 2, 10, 11, 12, 100, 101, 102, 110, 1000, 1001, 1002, 1010, 1011, 10000, 10001, 10002, 10010, 10011, 10012, 100000, 100001, 100002, 100010, 100011, 100012, 100100, 1000000, 1000001, 1000002, 1000010, 1000011, 1000012, 1000100, ...

(Numbers written in the triangular base, defined as follows:

$t(n) = (n(n+1))/2$, for $n \geq 1$.)

(3) Smarandache Double factorial base:

1, 10, 100, 101, 110, 200, 201, 1000, 1001, 1010, 1100, 1101, 1110, 1200, 10000, 10001, 10010, 10100, 10101, 10110, 10200, 10201, 11000, 11001, 11010, 11100, 11101, 11110, 11200, 11201, 12000, ...

(Numbers written in the double factorial base, defined as follows:

$df(n) = n!!$)

(4) Smarandache Non-multiplicative sequence:

General definition: Let $m_1, m_2, ..., m_k$ be the first k terms of the sequence, where $k \geq 2$;

then m_i, for $i >= k+1$, is the smallest number not equal to the product of k previous distinct terms.

(5) Smarandache Non-arithmetic progression:

1, 2, 4, 5, 10, 11, 13, 14, 28, 29, 31, 32, 37, 38, 40, 41, 64, ...

General definition: if m_1 , m_2 , are the first two terms of the sequence,

then m_k, for $k \geq 3$, is the smallest number such that no 3-term arithmetic

progression is in the sequence.

In our case the first two terms are 1, respectively 2.

Generalization: same initial conditions, but no i-term arithmetic progression in the sequence (for a given $i \geq 3$).

(6) Smarandache Prime product sequence:

2, 7, 31, 211, 2311, 30031, 510511, 9699691, 223092871, 6469693231, 200560490131, 7420738134811, 304250263527211, ...

$P_n = 1 + p_1 p_2 \dots p_k$, where p_k is the k-th prime.

Question: How many of them are prime?

(7) Smarandache Square product sequence:

2, 5, 37, 577, 14401, 518401, 25401601, 1625702401, 131681894401, 13168189440001, 1593350922240001, ...

$S_k = 1 + s_1 s_2 \dots s_k$, where s_k is the k-th square number.

Question: How many of them are prime?

(8) Smarandache Cubic product sequence:

2, 9, 217, 13825, 1728001, 373248001, 128024064001, 65548320768001, ...

$C_k = 1 + c_1 c_2 \dots c_k$, where c_k is the k-th cubic number.

Question: How many of them are prime?

(9) Smarandache Factorial product sequence:

2, 3, 13, 289, 34561, 24883201, 125411328001, 5056584744960001, ...

$F_k = 1 + f_1 f_2 \dots f_k$, where f_k is the k-th factorial number.

Question: How many of them are prime?

(10) Smarandache U-product sequence {generalization}:

Let u_n, $n \geq 1$, be a positive integer sequence. Then we define a U-sequence as follows:

$Un = 1 + u_1 u_2 \dots u_n$.

(11) Smarandache Non-geometric progression.

1, 2, 3, 5, 6, 7, 8, 10, 11, 13, 14, 15, 16, 17, 19, 21, 22, 23, 24, 26, 27, 29, 30, 31, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 45, 47, 48, 50, 51, 53, . . .

General definition: if m_1, m_2, are the first two terms of the sequence, then m_k, for $k \geq 3$, is the smallest number such that no 3-term geometric progression is in the sequence. In our case the first two terms are 1, respectively 2.

(12) Smarandache Unary sequence:

11, 111, 11111, 1111111, 11111111111, 1111111111111, 11111111111111111,
1111111111111111111, 11111111111111111111111,
11111111111111111111111111111, 1111111111111111111111111111111, ...

u(n) = 11...1, p_n digits of "1", where p_n is the n-th prime.

The old question: are there are infinite number of primes belonging to the sequence?

(13) Smarandache No-prime-digit sequence:

1, 4, 6, 8, 9, 10. 11, 1, 1, 14, 1, 16. 1, 18, 19, 0, 1, 4, 6, 8, 9,
0, 1, 4, 6, 8, 9, 40, 41, 42, 4, 44, 4, 46, 48, 49, 0, ...

(Take out all prime digits of n.)

(14) Smarandache No-square-digit-sequence.

2, 3, 5, 6, 7, 8, 2, 3, 5, 6, 7, 8, 2, 2, 22, 23, 2, 25, 26, 27, 28,
2, 3, 3, 32, 33, 3, 35, 36, 37, 38, 3, 2, 3, 5, 6, 7, 8, 5, 5, 52, 53,
5, 55, 56, 57, 58, 5, 6, 6, 62, ...

(Take out all square digits of n.)

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