THE SCIENCE OF LUCKY SCIENCES

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Abstract   A lucky science as defined by Smarandache is whereby the correct result to a mathematical equation is achieved by erroneous methods ([1]). For example, if asked 10+10, we might say 20, which happens to be correct in all positive integer bases except for base 2, in which case the answer should have been 100. As we probably did the sum in base 10, we have been lucky (however we would have been unlucky in base 2). This paper questions under which circumstances we may be lucky or unlucky.

§1. Introduction

A few more examples of lucky science in action.

In Smarandache Notions Journal 14 ([2]), an example is given of a lucky differentiation. If $g(x) = x^x$, then $g'(x) = nx^{n-1}$, so $g'(x) = n^n$. Then $f(x) = e^x$ is of this form, so $f'(e) = e^e$, which happens to be correct, but the method used is only valid for this example with x=e, i.e. given the vast majority of functions, this method fails to produce the correct example.

Another example given in [2] is $16/64 = 1/4$ - simply cancel the 6's.

But this does not work for $26/76$, or practically anything else.

Let a numerator is given by $n_1 \cdots n_a$, and a denominator by $d_1 \cdots d_b$, and $A = \{1, \cdots, a\}$ and $B = \{1, \cdots, b\}$, and A_6 is a subset of A such that a_i is 6, and similarly for B_6.

Then when does the cancelling of the sixes work? More generally when does cancelling of any given integer/integer set work? And even more generally, when does any erroneous method work?

§2. Smarandache Function

The Smarandache function $S(k)$ is defined as the lowest value such that k divides $S(k)!$ ([3]).

If we glibly say $S(k) = k$ for all k, this is our lucky method. We might even have a 'proof' of it! And we check that $S(1) = 1, S(2) = 2, S(3) =$

$3, S(4) = 4$ and $S(5) = 5$. So we can assume our method is good, and declare it to a bewildered professor who says 'but $S(6) = 3$'.

What went wrong? Our lucky method failed to be a truthful interpretation of the question, and hence it failed. However if in testing our hypothesis we considered only primes (every integer is a unique factorization of the primes after all), we would be correct.

So we can define a few terms;

Let E be a mathematical problem.

Let L be a lucky method on E, and let C be a correct method on E

Let $L(x)$ be the set of x such that $L(x)$ equals $C(x)$, i.e. the set of x for which the lucky method produces the correct result, and $L'(x)$ to be the set of x such that $L(x)$ does not equal $C(x)$, i.e. the set of x for which the lucky method fails. In the example in this section, E is the Smarandache numbers, and $L(x) = \{1, 4, primes\}$.

Examples revisited

§3. Differentiation

The derivation given in the introduction hardly ever works. If we consider $g'(x) = nx^{n-1}$, then we have differentiated with respect to x. x here is a real variable, and due to the normal criteria of continuity, g' is an accepted result.

$f(x) = e^x$ is a different function to $g(x)$, and this is the first step we make in determining L. x is still a real variable, but now it is an exponent, and so has been transformed, and hence behaves differently. Also, e is not just a number, it is a function;

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

From here we see that $d/dx\{xk/k!\} = kx^{k-1}/k! = (k-1)x^{k-1}$, and so $f'(x) = f(x) = e^e$.

Now $f(e) = e^e$, and $g(e) = e^e$, and $f'(e) = g'(e) = e^e$.

So L contains $\{e\}$.

But look at the region around e, i.e. between $e - d$ and $e + d$ for some (small) d.

$$g'(e + d) = (e + d)(e + d)^{e+d-1} = (e + d)^{e+d}$$

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However;

$$f'(e + d) = e^{e+d}$$

which is greater than $g'(e + d)$.

Similarly $f'(e - d)$ is always greater than $g'(e - d)$.

So when is $f'(x)$ equal to $g'(x)$?

Answer : When $e^x = nx^{n-1}$, i.e. $x = \{e\}$.

So if E is determine the differential of e^x, C is f, L is g, and $L(x) = \{e\}$.

§4. Bases

The question raised in the abstract of this paper is interesting - in which bases does a simple addition sum remain valid.

$20 + 20 = 40$ is true in base 5 and above, $27 + 31 = 60$ is valid is base 8 only.

Let us define E as to determine whether a sum $S = S_1 + S_2 = z$ is valid in a base. C is then the usual definition of addition. $L(S = z)$ is the set of bases such that the sum $S = z$ is valid.

Given any sum, base k addition is always valid - the 27 above in base 7 is another way of writing 30_7 (but in base 7 the correct answer is 61).

We construct a table of $27 + 31$ in the bases 2 to 10:

Base	Answer
2	1110
3	212
4	130
5	113
6	102
7	61
8	60
9	59
10	59
11	59

So $L(27 + 31 = 59) = \{9, 10, 11, 12, \cdots\}$

And $L(27 + 31 = z)$ for z greater than 59 is defined according to the table above (e.g. $L(27 + 31 = 60) = \{8\}$).

We can easily say that if S is valid in base k and base $k + 1$, then it is valid for all further bases, as in this case all the problematic carries have been absorbed by the base.

If we let K be the lowest base that $S = z$ is true and has no carries, we can define $L(S = z)$ as $\{K, K + 1, K + 2, \cdots\}$.

Let $Z = z$ in this case, and so furthermore, for each z greater than z, $L(s = z)$ is either empty or a single point.

§5. Fractions

This is the hardest problem yet to analyze.

Let E be the problem of reducing a fraction to it's simplest form. Then C is the problem of factoring the numerator and denominator, and removing common prime factors.

L_D is defined as cancelling a set of digits D from both the numerator and denominator, and $L_D(r)$ is the set such that the rational r is produced both by C and by L_D.

Let's see if we can construct such a number. Let's start with the obvious $1/2$.

We require the numerator to be twice that of the denominator. Trivially, let $D = \{3, 6\}$, then $1[3]/2[6]$ cancels to $1/2$, e.g. $13/26$, $1333/2666$ or $331/662$.

But for the case of D a single integer this is impossible.

Proof.   Let n be the numerator of any such fraction. Then we may generalize n as;

$$n = \sum_{\substack{i=0 \\ i \notin J}}^{N} d10^i + \sum_{j \in J} 10^j$$

for some $J-$ this holds the position of the 1's.

$$m = \sum_{\substack{i=0 \\ i \notin K}}^{M} d10^i + 2 \sum_{k \in K} 10^k$$

for some $K-$ this holds the position of the 2's.

We now require $n/m = 1/2$, or $2n = m$.

Note that d must equal 6, but $6 + 6$ produces a carry, and as the next component in the sum is either $1 + 1$ or $6 + 6$, we end up with a 3.

Hence $L_D(1/2)$ is the empty set for $|D| = 1$.

So why does $16/64$ work?

Potential fractions for $1/4$ can be expanded as above, but we find a solution quickly.

If $(10 + d)/10d + 4 = 1/4$, then $6d = 36$, so $d = 6$.

Hence $L_6(1/4)$ contains $16/64$.

General solutions to these equations is beyond the scope of this paper.

§**6. Summary**

Any erroneous method may produce correct answers for specific numbers. The science of lucky sciences develops this hit-and-miss scene into a mathematical system.

References

[1] Charles Ashbacher, Smarandache Lucky Science, http://www. gallup. unm. edu/ smarandache/lucky.htm

[2] Henry Bottomley, A lucky derivative, Smarandache Notions Journal **14** (2004), 352.

[3] Smarandache Numbers, A002034, OEIS.