SMARANDACHE FRIENDLY NUMBERS AND A FEW MORE SEQUENCES

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If the sum of any set of consecutive terms of a sequence = the product of the first and the last number of the set then this pair is called a **Smarndache Friendly Pair** with respect to the sequence.

{1} SMARANDACHE FRIENDLY NATURAL NUMBER PAIRS:

e.g. Consider the natural number sequence

1, 2, 3, 4, 5, 6, 7, . . .

then the Smarandache friendly pairs are

(1,1), (3, 6), (15,35), (85, 204), . . .etc.

as $3 + 4 + 5 + 6 = 18 = 3 \times 6$

$15 + 16 + 17 + \ldots + 33 + 34 + 35 = 525 = 15 \times 35$ etc.

There exist infinitely many such pairs. This is evident from the fact that if (m, n) is a friendly pair then so is the pair (2n+m, 5n +2m -1). **Ref [1].**

{2} SMARANDACHE FRIENDLY PRIME PAIRS:

Consider the prime number sequence

2, 3, 5, 7, 11, 13, 17, 23, 29, . . .

we have $2 + 3 + 5 = 10 = 2 \times 5$, Hence (2, 5) is a friendly prime pair.

$3 + 5 + 7 + 11 + 13 = 39 = 3 \times 13$, (3,13) is a friendly prime pair.

$5 + 7 + 11 + \ldots + 23 + 29 + 31 = 155 = 5 \times 31$, (5, 31) is a friendly prime pair.

Similarly (7, 53) is also a Smarandache friendly prime pair. In a friendly prime pair (p, q) we define **q** as the big brother of **p**.

Open Problems: (1) Are there infinitely many friendly prime pairs?

2. Are there big brothers for every prime?

{3} SMARANDACHE UNDER-FRIENDLY PAIR:

If the sum of any set of consecutive terms of a sequence is a **divisor** of the product of the first and the last number of the set then this pair is called a **Smarndache under- Friendly Pair** with respect to the sequence.

{4} SMARANDACHE OVER-FRIENDLY PAIR:

If the sum of any set of consecutive terms of a sequence is a **multiple** of the product of the first and the last number of the set then this pair is called a **Smarndache Over- Friendly Pair** with respect to the sequence.

{5} SMARANDACHE SIGMA DIVISOR PRIME SEQUENCE:

The sequence of primes p_n , which satisfy the following congruence.

$$\sum_{r=1}^{n-1} p_r \equiv 0 \pmod{p_n}$$

2, 5, 71, . . .

5 divides 10, and 71 divides 568 = 2 + 3 + 5 + . . . + 67

Problems: (1) Is the above sequence infinite?

Conjecture: Every prime divides at least one such cumulative sum.

{6} SMARANDACHE SMALLEST NUMBER WITH 'n' DIVISORS SEQUENCE:

1, 2, 4, 6, 16, 12, 64, 24, 36, 48, 1024, . . .

d(1) = 1, d(2) = 2, d(4) = 3, d(6) = 4, d(16) = 5, d(12) = 6 etc. , **d(T_n) = n. , where T_n is smallest such number.**

It is evident $T_p = 2^{p-1}$, if p is a prime.

The sequence T_n +1 is

2, 3, 5, 7, 17, 13, 65, 25, 37, 49, 1025, . . .

Conjectures: (1) The above sequence contains infinitely many primes.

(2) The only Mersenne's prime it contains is 7.

(3) The above sequence contains infinitely many perfect squares.

{7} SMARANDACHE INTEGER PART k^n SEQUENCE (SIPS) :

****In this sequence k is a non integer.** For example:

(i) SMARANDACHE INTEGER PART π^n SEQUENCE:

$[\pi^1]$, $[\pi^2]$, $[\pi^3]$, $[\pi^4]$, . . .

3, 9, 31, 97, . . .

(ii) SMARANDACHE INTEGER PART e^n SEQUENCE:

$[e^1], [e^2], [e^3], [e^4], \ldots$

2, 7, 20, 54, 148, 403, . . .

Conjecture: Every SIPS contains infinitely many primes.

{8} Smarandache Summable Divisor Pairs (SSDP):

Pair of numbers (m,n) which satisfy the following relation

$d(m) + d(n) = d(m + n)$

e.g. we have $d(2) + d(10) = d(12)$, $d(3) + d(5) = d(8)$, $d(4) + d(256) = d(260)$, $d(8) + d(22) = d(30)$, etc.

hence (2, 10), (3,5), (4, 256), (8, 22) are SSPDs.

Conjecture: (1)There are infinitely many SSDPs?

(2) For every integer m there exists a number n such that (m,n) is an SSDP.

{9} SMARANDACHE REIMANN ZETA SEQUENCE

6, 90, 945, 93555, 638512875, . . .

where Tn is given by the following relation of

$$z(s) = \sum_{n=1}^{\infty} n^{-s} = \pi^{2n} / T_n$$

Conjecture: No two terms of this sequence are relatively prime.

Consider the sequence obtained by incrementing each term by one

7, 91, 946, 9451, 93556, 638512876, . . .

Problem: How many primes does the above sequence contain?

{10} SMARANDACHE PRODUCT OF DIGITS SEQUENCE:

The n^{th} term of this sequence is defined as T_n = product of the digits of n.

1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 2, 4, 6, 8, 10, 12, . . .

{11} SMARANDACHE SIGMA PRODUCT OF DIGITS NATURAL SEQUENCE:

The n^{th} term of this sequence is defined as the sum of the products of all the numbers from 1 to n.

1, 3, 6, 10, 15, 21, 28, 36, 45, 45, 46, 48, 51, 55, 60, 66, 73, 81, 90, 90, 92, 96, . . .

Here we consider the terms of the sequence for some values of n.

For n = 9 we have $T_n = 45$, The sum of all the single digit numbers = 45

For n = 99 we have $T_n = 2070 = 45^2 + 45$..

Similarly we have $T_{999} = (T_9)^3 + (T_9)^2 + T_9 = 45^3 . 45^2 + 45 = (45^4 - 1) / (45 - 1) = (45^4 - 1) /44$

The above proposition can easily be proved.

This can be further generalized for a number system with base 'b' (b = 10 , the decimal system has already been considered.)

For a number system with base 'b' the $(b^r -1)^{th}$ term in the Smarandache sigma product of digits sequence is

$2[\{ b(b-1)/2\}^{r+1} - 1] /\{ b^2 - b -2\}$

Further Scope: The task ahead is to find the n^{th} term in the above sequence for an arbitrary value of n.

{12} SMARANDACHE SIGMA PRODUCT OF DIGITS ODD SEQUENCE:

1, 4, 9, 16, 25, 26, 29, 34, 41, 50, 52, 58, 68, 82, 100, 103, 112, 127, 148, . . .

It can be proved that for $n = 10^r -1$, T_n is the sum of the r terms of the Geometric progression with the first term as 25 and the common ratio as 45.

{13} SMARANDACHE SIGMA PRODUCT OF DIGITS EVEN SEQUENCE:

2, 6, 12, 20, 20, 22, 26, 32, 40, 40, 44, 52, 62, 78, 78, 84, 96, 114, 138, . . .

It can again be proved that for $n = 10^r -1$, T_n is the sum of the r terms of the Geometric progression with the first term as 20 and the common ratio as 45.

Open Problem: Are there infinitely many common members in {12} and {13} ?

Reference:

[1] Problem2/31, M&IQ ,3/99 Volume 9, Sept' 99, Bulgaria.