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Smarandache–Boolean–Near–Rings and Algorithms

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Abstract. In this paper we introduced Smarandache-2-algebraic structure of Boolean-near-ring namely Smarandache-Boolean-near-ring. A Smarandache-2-algebraic structure on a set N means a weak algebraic structure A_0 on N such that there exists a proper subset M of N, which is embedded with a stronger algebraic structure A_1 , stronger algebraic structure means satisfying more axioms, by proper subset one understands a subset different from the empty set, form the unit element if any, from the whole set. We define Smarandache-Boolean-near-ring and obtain some of its algorithms through Boolean-ring with left-ideals, direct summand, Boolean-*l*-algebra, Brouwerian algebra, Compatibility, maximal set and Polynomial Identities.

Keywords: Boolean-ring, Boolean-near-ring, Smarandache-Boolean-near-ring, left-ideal, direct summand, Boolean-*l*-algebra, Brouwerian algebra, Compatibility, maximal set and Polynomial Identities

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1. Introduction

In order that New notions are introduced in algebra to better study the congruence in number theory by Smarandache [4]. By <proper subset> of a set A we consider a set P included in A, and different from A, different form the empty set, and from the unit element in A – if any they rank the algebraic structures using an order relationship:They say that the algebraic structures S_1<< S_2 if: both are defined on the same set; all S_1 laws are also S_2 laws; all axioms of an S_1 law are accomplished by the corresponding S_2 law; S_2 law accomplish strictly more axioms that S_1 laws, or S_2 has more laws than S_1.

For example: Semi group <<Monoid<< group << ring<< field, or Semi group<< commutative semi group, ring<< unitary , ring etc. They define a General special structure to be a structure SM on a set A, different form a structure SN, such that a proper subset of A is an structure, where SM<< SN <<

2. Preliminaries

Definition 2.1. A left near-ring A is a system with two binary operations, addition and multiplication, such that

(i) the elements of A form a group (A,+) under addition,

(ii) the elements of A form a multiplicative semi-group,

(iii) $x(y + z) = xy + xz$, for all $x,y,z \in A$

In particular, if A contains a multiplicative semi-group S whose elements generate (A,+) and satisfy

(iv) $(x+y)s = xs + ys$, for all $x,y \in A$ and $s \in S$, then we say that A is a distributively generated near-ring.

Definition 2.2. A near-ring (B,+,.) is Boolean-Near-Ring if there exists a Boolean-ring $(A,+,\Lambda,1)$ with identity such that $\bullet$ is defined in terms of +, Λ and 1, and for any $b \in B$, b.b = b.

Definition 2.3. A near-ring (B,+,.) is said to be idempotent if $x^2 = x$, for all $x \in B$. If (B,+,.) is an idempotent ring, then for all $a, b \in B$, $a + a = 0$ and $a.b = b.a$

Definition 2.4. A Boolean-near-ring (B,+,.) is said to be Smarandache-Boolean-near-ring whose proper subset A is a Boolean-ring with respect to same induced operation of B.

Definition 2.5. (Alternative definition for S-Boolean-near-ring) If there exists a non-empty set A which is a Boolean-ring such that it superset B of A is a Boolean-near-ring with respect to the same induced operation, then B is called Smarandache-Boolean-near-ring. It can also written as S-Boolean-near-ring.

3. Algorithms

Left – Ideal: Clay and Lawver [2] have introduced the left-ideals of (B,+,.) in P(x) are the subgroups of the groups (P(x), +), where $P(x) = \{b \in B \,/\, b \wedge x = b\} = B_z$ is a maximal sub-z-ring. It also contained in an ideal. Let $A = I_0$. Nowto construct a set B as follows.

B contains a unique minimal ideal I_0 contained in all other non – zero ideals. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache -Boolean-near-ring.

Algorithm 3.1.

Step 1: Consider a Boolean-ring A

Step 2: Let $A = I_0$, be an ideal

Step 3: Let I_i, i= 0,1,2,3,…… be supersets of I_0.

Step 4: Let $B = \bigcup I_0$

Step 5: Choose the sets Ij from I_i's subject to a,b and $c \in B$ such that

$$(a + b) .c + a .c + b .c = x \wedge c \in Ij \text{and } x \in B \text{ we have } P(x) \subseteq I$$

Step 6: Verify that $\bigcap Ij = I_0 \neq \{0\}$

Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Direct Summand

Clay and Lawver [2] has introduced the concept of direct summand. Let A be an ideal of B, then A is a direct summand if and only if A =P(x). Now to construct a set B as follows. B contains a unique minimal direct summand M_0 contained in all other non – zero direct summands. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.2.

Step 1: Consider a Boolean-ring A

Step 2: Let A =M_0, be a direct summand.

Step 3: Let M_i, i = 0,1,2,3,…… be supersets of M_0.

Step 4: Let B = $\bigcup M_i$

Step 5: Choose the sets Mj from M_i's subject to for all x $\in$ B such that M_0 is a direct summand we have $M_0 = P(x)$ and $B = P(x) + P(x^1)$, where $P(x)$ and $P(x^1)$ are ideals of B and $x, x^1 \in B$.

Step 6: Verify that $\bigcap Mj = M_0 \neq \{0\}$

Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Boolean-*l*-Algebra

Rao has introduced the notions of Boolean-*l*-algebraand lattice ordered groups. In [8] he proved A is a Boolean-ring if and only if A is a Boolean-*l*-algebra such that $x \leq a$ implies $x \bigcap (a\text{-}x) = 0$. He has established that the class of Boolean-*l*-algebra is a subclass of DRI semigroups also. Let A =I_0. Now to construct a set B as follows.
B contains a unique minimal Boolean-*l*-algebraI_0 contained in all other non – zero Boolean-*l*-algebras. According to G. Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.3.

Step 1: Consider a Boolean-ring A

Step 2: Let A =I_0, be a Boolean-l-algebra

Step 3: Let I_i , i = 0,1,2,3,…… be supersets of I_0.

Step 4: Let B = $\bigcup I_i$

Step 5: Choose the sets Ij from I_i's subject to for all i_{j1} , $i_{j2} \in$ Ij such that $i_{j1} \leq i_{j2}$ implies $i_{j1} \bigcap (i_{j2} - i_{j1}) = 0$

Step 6: Verify that $\bigcap Ij = I_0 \neq \{0\}$

Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Brouwerian Algebra

Rao has established that the class of Brouwerian algebras. Brouwerian algebras being a subclass of Boolean-l-algebras. If (B; -) is a Boolean-ring then (B; -) is a Boolean-l-algebra if and only if B is a Brouwerian such that that $x \leq a$ then $a = x \bigcup (a\text{-}x)$.

Let A be a Boolean – ring. Let A = M_0. Now to construct a set B as follows. B contains a unique minimal Brouwerian algebra contained in all other non – zero Brouwerian algebras.
According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.4.

Step 1: Consider a Boolean-ring A

Step 2: Let A $=M_0$

Step 3: Let M_i, i = 0,1,2,3,…… be the supersets of M_0.

Step 4: Let B = $\bigcup M_i$

Step 5: Choose the sets Mj from M_i's subject to for all x and a $\in$ B such that x $\leq$a then a = x $\bigcup$ (a-x).

Step 6: Verify that $\bigcap$ Mj = $M_0 \neq \{0\}$

Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Compatibility: A subset A of Boolean-near-ring B is said to be compatibility a $\sim b$ if $ab^2 = a^2b$. Let A $=I_0$. Now to construct a set B as follows. B contains a unique minimal compatibilityI_0 contained in all other non – zero compatibilities. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.5.

Step 1: Consider a Boolean-ring A

Step 2: Let A $=I_0$, be a compatibility

Step 3: Let I_i , i = 0,1,2,3,…… be the supersets of I_0.

Step 4: Let B $=\bigcup I_i$

Step 5:Choose the sets Ij from I_i's subject to for all a , b $\in$ A such that $ab^2 = a^2b \in$ Ij

Step 6: Verify that $\bigcap$ Ij = $I_0 \neq \{0\}$

Step 7: If step (6) is true, then we write B is a Smarandache-boolean near-ring.

Maximal Set: Let B be a Boolean-near-ring and let A = (….., a, b, c, ……) be a set of pairwise compatible elements of an associate ring R. Let A be maximal in the sense that each element of A is compatible with every other element of A and no other such elements may be found in R. Then A is called maximal compatible set or a maximal set. Let A $=I_0$. Now to construct a set B as follows. B contains a unique minimal maximal setI_0 contained in all other non – zero maximal sets. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.6.

Step 1 : Consider a Boolean-ring A

Step 2 : Let A $=I_0$, be a maximal set

Step 3 : Let I_i , i = 0,1,2,3,…… be the supersets of IStep 4 :

Let B = $\bigcup I_i$

Step 5 : Choose the sets Ij from I_i's subject to for all a , b $\in$ Ij such that $a \vee b = a + b - 2a^0b = (a \bigcup b) - (a \bigcap b)$ and $a \wedge b = a^0b = ab^0 = a \bigcap b \in$ Ij, for all a , b $\in$ Ij

Step 6 : Verify that $\bigcap$ Ij = $I_0 \neq \{0\}$

Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity: Given two numbers $m > n \geq 1$, a ring B is said to be (m,n) – Boolean if $x^m = x^n$, for all x in B. Let A =I_0. Now to construct a set B as follows.
B contains a unique minimal Polynoimal identity I_0 contained in all other non – zero Polynoimal identities. According to G. Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.7.

Step 1 : Consider a Boolean-ring A
Step 2 : Let A =I_0
Step 3 : Let I_i , i = 0,1,2,3,…… be the supersets of I_0.
Step 4 : Let B = $I \bigcup_i$
Step 5 : Choose the sets Ij from I_i's subject to for all m, n $\in$ B and for all x $\in$ B such that $x^m = x^n \in$ Ij
Step 6 : Verify that $\bigcap$ Ij = $I_0 \neq \{0\}$
Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity: Let m and n be two positive integers such that $x^{2^{n+1}+2^n} = x$, for all x in B. Let A =M_0. Now to construct a set B as follows. B contains a unique minimal Polynomial identity M_0 contained in all other non – zero Polynomial identities. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.8.

Step 1 : Consider a Boolean-ring A
Step 2 : Let A =M_0
Step 3 : Let M_i, i = 0,1,2,3,…… be the supersets ofM_0.
Step 4 : Let B = $\bigcup M_i$
Step 5 : Choose the sets Mj from M_i's subject to for all two positive integers m and n $\in$ B and for all x$\in$ M_j such that $x^m = x^n$ and $x^{2^{n+1}+2^n} = x, \in M_j$
Step 6 : Verify that $\bigcap$ Mj = $M_0 \neq \{0\}$
Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity: Let m and q be two fixed positive integers and $x^{2^{q(m+1)}+2^m} = x$, for all x in B. Then B is known as a Smarandache-boolean-near-ring.

Let A =P_0.

Now to construct a set B as follows. B contains a unique minimal Polynomial identity P_0 contained in all other non – zero Polynoimal identities. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.9.

Step 1 : Consider a Boolean-ring A

Step 2 : Let A =P_0

Step 3 : Let P_i , i = 0,1,2,3,…… be the supersets of $P_{0.}$

Step 4 : Let B = $\bigcup P_i$

Step 5 : Choose the sets Pj from P_i's subject to for all two positive integers m and q such that $x^{2^{q(m+1)}+2^m} = x, \in$ P_j and for all x∈ P_j

Step 6 : Verify that $\bigcap$ Pj = $P_0 \neq \{0\}$

Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity: Let m and n be two positive integers such that $x^{2^{n+1}+2^n} = x,$ for all x in B. Let A =M_0.

Now to construct a set B as follows. B contains a unique minimal Polynomial identity M_0 contained in all other non – zero Polynomial identities. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.10.

Step 1 : Consider a Boolean-ring A

Step 2 : Let A =M_0

Step 3 : Let M_i, i = 0,1,2,3,…… be the supersets of $M_{0.}$

Step 4 : Let B = $\bigcup M_i$

Step 5 : Choose the sets Mj from M_i's subject to for all two positive integers m and n ∈ B and for all x∈ M_j such that x^m = x^n and $x^{2^{n+1}+2^n} = x, \in M_j$

Step 6 : Verify that $\bigcap$ Mj = $M_0 \neq \{0\}$

Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity: Let m and n be two positive integers such that $x^{2^{n+1}+2^n} = x,$ for all x in B. Let A =M_0. Now to construct a set B as follows. B contains a unique minimal Polynomial identity M_0 contained in all other non – zero Polynomial identities. According to G. Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.11.

Step 1 : Consider a Boolean-ring A

Step 2 : Let A =M_0

Step 3 : Let M_i, i = 0,1,2,3,…… be the supersets of $M_{0.}$

Step 4 : Let B = $\bigcup M_i$

Step 5 : Choose the sets Mj from M_i's subject to for all two positive integers m and n ∈ B and for all x∈ M_j such that x^m = x^n and $x^{2^{n+1}+2^n} = x, \in M_j$

Step 6 : Verify that $\bigcap$ Mj = $M_0 \neq \{0\}$

Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity : Let m and n be two positive integers such that $x^{2^{n+1}+2^n} = x,$ for all x in B. Let A =M_0. Now to construct a set B as follows. B contains a unique minimal Polynomial identity M_0 contained in all other non – zero Polynomial identities. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.12.

Step 1 : Consider a Boolean-ring A

Step 2 : Let A =M_0

Step 3 : Let M_i, i = 0,1,2,3,…… be the supersets of M_0.

Step 4 : Let B = $\bigcup M_i$

Step 5 : Choose the sets Mj from M_i's subject to for all two positive integers m and n ∈ B and for all x∈ M_j such that x^m = x^n and $x^{2^{n+1}+2^n} = x, \in M_j$

Step 6 : Verify that $\bigcap$ Mj = $M_0 \neq \{0\}$

Step 7: If step (6) is true, then we write B is a Smarandache-boolean-near-ring.

Polynomial Identity: Let B be a Boolean-near-ring and let m, q and r be fixed positive integers with r < m+1 such that $x^{2^{q(m+1)+r}+2^m} = x,$ for all x in B and $x^{2^{r+1}} = x,$ then B is Smarandache-Boolean-near-ring. Let A = M_0 . Now to construct a set B as follows. B contains a unique minimal Polynomial identity M_0 contained in all other non – zero Polynomial identities. According to Pilz [4, Theorem (1.60 (d))], B is Boolean-near-ring. Now by definition, B is a Smarandache-boolean-near-ring.

Algorithm 3.13.

Step 1 : Consider a Boolean-ring A

Step 2 : Let A =M_0

Step 3 : Let M_i, i = 0,1,2,3,…… be the supersets of M_0.

Step 4 : Let B = $\bigcup M_i$

Step 5 : Choose the sets Mj from M_i's subject to for all two positive integers m, q and r be three fixed positive integers with r < m+1 and for all x ∈ M_j such that $x^{2^{q(m+1)+r}+2^m} = x,$ and $x^{2^{r+1}} = x, \in M_j$

Step 6 : Verify that $\bigcap$ Mj = $M_0 \neq \{0\}$

Step 7: If step (6) is true, then we write B is a Smarandache-Boolean-near-ring.

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