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# Smarandache Fuzzy Strong Ideal and Smarandache Fuzzy n-Fold Strong Ideal of a BH-Algebra

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**Abstract:** ***In this paper, we define the concepts of a Q-Smarandache n-fold strong ideal anda Q-Smarandache fuzzy (strong, n-fold strong) ideal of a BH- algebra .Also, we study some properties of these fuzzy ideals***

**Keywords**: BCK-algebra , BCI/BCH-algebras, BH-algebra , Smarandache BH-algebra, Q-Smarandache fuzzystrong ideal.

## 1. Introduction

In 1965, L. A. Zadeh introduced the notion of a fuzzy subset of a set as a method for representing uncertainty in real physical world [7] . In 1991, O. G. Xi applied the concept of fuzzy sets to the BCK-algebras [8] . In 1993, Y. B. Jun introduced the notion of closed fuzzy ideals in BCI -algebras[11].In 1999,Y.B.Jun introduced the notion of fuzzy closed ideal in BCH-algebras [13]. In 2001, Q. Zhang, E. H. Roh and Y. B. Jun studied the fuzzy theory in a BH-algebra [10] . In 2006,C.H. Park introduced the notion of an interval valued fuzzy BH-algebra in a BH-algebra [2].In 2009, A. B. Saeid and A. Namdar, introduced the notion of a Smarandache BCH-algebra and Q-Smarandache ideal of a Smarandache BCH-algebra [1].In 2012, H. H .Abbass introduced the notion of a Q-Smarandache fuzzy closed ideal with respect to an element of a Smarandache BCH-algebra [5].In the seam year, E.M. kim and S. S. Ahn defined the notion of a fuzzy (n-fold strong) ideal of a BH-algebra[3]. In 2013, E.M. kim and S. S. Ahn defined the notion of a fuzzy (strong) ideal of a BH-algebra[4].In the same year, H. H. Abbass and S. J. Mohammed introduced the Q-Smarandache fuzzy completely closed ideal with respect to an element of a BH-algebra[6] . In this paper, we define the concepts of Q-Smarandachen-fold strong ideal anda Q-Smarandache fuzzy (strong, n-fold strong) ideal of a Smarandache BH- algebra .Also, we study some properties of these fuzzy ideals

## 2. Preliminaries

In this section, we give some basic concept about a BCK-algebra ,a BCI-algebra ,a BH-algebra , a BH*-algebra,a normal BH-algebra, fuzzy strong ideal, fuzzyn-fold strong ideala Smarandache BH-algebra,( Q-Smarandache ideal, Q-Smarandache fuzzyclosedideal, Q-Smarandache fuzzy completely closed ideal and Q-Smarandache fuzzy ideal of BH-algebra

**Definition 1 (see[11]).**A **BCI-algebra** is an algebra (X,*,0), where X is a nonempty set, "*" is a binary operation and 0 is a constant, satisfying the following axioms : for all x, y, z∈ X:

i. (( x * y) * ( x * z) ) *( z * y)= 0, ii. (x*(x*y))*y = 0,
iii. x * x = 0,
iv. x * y =0 and y * x = 0 ⇒ x = y.

**Definition 2 (see [8]).** A **BCK-algebra** is a BCI-algebra satisfying the axiom

v. 0 * x = 0 for all x ∈X.

**Definition 3(see[9]).** A **BH-algebra** is a nonempty set X with a constant 0 and a binary operation "*" satisfying the following conditions:i.x*x=0, ∀ x∈X
ii. x*0 =x, ∀ x∈X.
iii. x*y=0 and y*x =0⇒ x = y,:for all x,y∈ X.

**Definition4.(see[4]).**A BH-algebraX is called a **BH*-algebra**if (x*y)*x=0 for all x,y∈X

**Definition5.(see[9]).** A BH-algebra X is said to be a **normalBH-algebra** if it satisfying the following condition:
i. 0*(x*y)= (0*x)*(0*y) ,∀x,y∈X .
ii. (x*y)*x = 0* y ,∀x,y∈X.
iii. (x*(x*y))*y = 0 , ∀x,y∈ X.

**Definition 6.(see[5])** . Let X be a BH-algebra.Then the set $X_+$={x∈X:0*x=0} is called the **BCA-part** of X .

**Remark 1(see[6]).**Let X and Y be BH-algebras. A mapping f : X→Y is called a **homomorphism** if f(x*y)=f(x)*` f(y) for all x, y∈X. A homomorphism f is called a **monomorphism** (resp., **epimorphism**) if it is injective (resp**., surjective**). For any homomorphism f:X →Y, the set { x∈X : f(x)=0'} is called the **kernel** of f, denoted by Ker(f), and the set { f(x) : x∈X} is called the **image** of f, denoted by Im(f). Notice that f(0)=0' for all homomorphism f.

**Definition7 (see[3])**. Let X be a BH-algebraand n be a positive integer**.** A nonempty subset I of X is called a ***n-fold strong ideal of X*** if it satisfies: i. 0 ∈ I , ii. ∀y ∈I and (x* y)*$z^n$∈I ⇒x*$z^n$∈I, ∀ x ,z ∈X.

**Definition 8.(see[6]).**A Smarandache BH-**algebra** is defined to be a BH-algebra X in which there exists a proper subset Q of X such that
i. 0 ∈ Q and |Q| ≥ 2 .
ii. Q is a BCK-algebra under the operation of X.

**Definition9 (see[6])**. Let X be a Smarandache BH-algebra **.** A nonempty subset I of X is called a ***Smarandache strong ideal of X related toQ*** ( or briefly, ***Q- Smarandache strong ideal*** of X) if it satisfies: i. 0 ∈ I , ii. ∀y ∈I and (x* y)*z ∈I ⇒x*z ∈I, ∀ x ,z ∈Q.

**Definition 10(see[7]).** Let $\mu$ be a fuzzy set in X, for all t∈[0,1]. The set $\mu_t$={x∈X, $\mu$(x) ≥ t} is called a ***level subset of*** $\mu$.

**Definition11(see [4]).**Let A and B be any two sets, μ be any fuzzy set in A and $f$ : A → B be any function. Set$f^{-1}$(y) = {x ∈A| $f$ (x) = y} for y ∈B. The fuzzy set ν in B defined by
$\nu(y) = \begin{cases} \sup\{\mu(x) \mid x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$
for all $y$ ∈$B$, is called the ***image*** of $\mu$ under $f$ and is denoted by $f$ ($\mu$).

**Definition 12(see[ 4].**Let $A$ and $B$ be any two sets, $f$ : $A$ → $B$ be any function andν be any fuzzy set in $f$ (A). The fuzzy set μ in A defined by: μ(x) = ν( $f$ (x)) for all x ∈X is called the ***preimage*** of ν under $f$ and is denoted by $f^{-1}$(ν).

**Definition13.(see[4].**A fuzzy set $\mu$ in a BH-algebra X is called a fuzzy strong ideal of X if i.For all x∈X, $\mu$(0) ≥ $\mu$(x).
ii.$\mu$ (x ∗z) ≥ min{$\mu$ ((x ∗y) ∗z) ,$\mu$ (y)}, ∀x; y ∈X.

**Definition14.(see[5].**A fuzzy set $\mu$ in a BH-algebra X is called a fuzzy n-fold strong idealof X if i. For all x∈X, (0) ≥ $\mu$(x).
ii. $\mu$ (x ∗$z^n$) ≥ min{$\mu$ ((x ∗y) ∗$z^n$); $\mu$ (y)}, ∀x, y ∈X.

**Definition15(see [6]).**A fuzzy subset $\mu$ of a Smarandache BH-algebra X is said to be a ***Q-Smarandachefuzzy ideal***if and only if :
i. For all x∈X, $\mu$(0) ≥ $\mu$(x).
ii.For all x ∈ Q, y∈X, $\mu$(x) ≥ min{ $\mu$(x*y), $\mu$(y)}.
Is said to be ***closed*** if (0*x) ≥ $\mu$(x), for all x∈X.

**Definition16(see[6]).**Let X be a Smarandache BH-algebra and $\mu$ be a Q-Smarandache fuzzy ideal of X. Then $\mu$ is called ***a Q-Smarandachefuzzy completely closed ideal*** if $\mu$(x*y) ≥ min{ $\mu$(x), $\mu$(y)},∀x,y∈X.

**Proposition1(see[6]).**Let X be a BH*-algebra ,and $\mu$ be a Q-Smarandache fuzzy ideal. Then
i. $\mu$ is a Q-Smarandache fuzzy closed ideal of X .
ii. $\mu$ is a Q-Smarandache fuzzy completely closed ideal of X, if X*X/{0} ⊆Q

## 3. The Main Results

In this paper, we give the concepts a Q-Smarandache n-fold strong ideal and a Q-Smarandache fuzzy (strong, n-fold strong) ideal of a BH- algebra .Also, we give some properties of these fuzzy ideals

**Definition 1** .A fuzzy subset of a BH-algebra X is called a Q-***Smarandache* fuzzy *strong* ideal** ,iff i. μ (0) ≥μ (x) ) ∀x∈X
ii. μ (x*z) ≥ min{ μ ((x*y)*z), μ (y)},∀x,z∈Q.

**Example 1:** The set X={0,1,2,3} with the following operation table

| * | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| **0** | 0 | 0 | 2 | 2 |
| **1** | 1 | 0 | 1 | 2 |
| **2** | 2 | 2 | 0 | 0 |
| **3** | 3 | 2 | 1 | 0 |

is a BH-algebra Q={0,1} is a BCK-algebra .Then (X,*,0) is a Smarandache BH-algebra.The fuzzy set $\mu$ which is defined by:
$\mu(x) = \begin{cases} 0.5 & x = 0,3 \\ 0.4 & x = 1,2 \end{cases}$

is a Q-Smarandache fuzzy strong ideal , since: i.$\mu$(0) = 0.5 ≥$\mu$(x) ∀x∈X, ii. $\mu$ (x*z) ≥ min{ $\mu$((x*y)*z), $\mu$(y)},∀x, z∈Q.

But the fuzzy set$\mu(x) = \begin{cases} 0.5 & x = 0,2,3 \\ 0.4 & x = 1 \end{cases}$
is not a Q-Smarandache fuzzy strong ideal since$\mu$(1*0)=$\mu$(1)=0.4 < min{ $\mu$(1*3)*0), $\mu$(3)}=0.5

**Proposition**1.Every Q-Smarandache fuzzy strong ideal of a Smarandache BH-algebra X is a Q-Smarandache fuzzy ideal of X.

**Proof :**Let $\mu$ be a fuzzy strong ideal of X .
i. Let x ∈ X ⇒ (0) ≥$\mu$(x) .
[By definition 1(i)]
ii. let x, z ∈X and y ∈ X ⇒x,z∈ Q
⇒$\mu$(x*z) ≥ min{$\mu$((x*y)*z) , $\mu$(y)}
[By definition 1(ii)]
When z=0⇒$\mu$(x*0) ≥ min{$\mu$((x*y)*0) , $\mu$(y)}⇒$\mu$(x) ≥ min{$\mu$(x*y) , $\mu$(y)}
⇒$\mu$ is a Q-Smarandache fuzzy ideal of X. ■

**Proposition2.** Let $Q_1$ and $Q_2$ be a BCK-algebras contained in a Smarandache BH- algebra X and $Q_1$⊆$Q_2$. Let $\mu$ be $Q_2$-Smarandache fuzzy strong ideal of X then $\mu$ is a $Q_1$-Smarandache fuzzy strong ideal of X .

**Proof :**Let $\mu$ be a $Q_2$-Smarandache fuzzy strong ideal of X .
i. Let x ∈ X ⇒ $\mu$(0) ≥$\mu$(x) .
[Since $\mu$ is a $Q_2$ -Smarandache fuzzy strong ideal. By definition 1(i)]
ii. Let x,z∈ $Q_1$,y∈X ⇒x,z∈ $Q_2$
⇒ $\mu$(x*z) ≥ min{ $\mu$((x*y)*z) ,$\mu$(y)}
[Since $\mu$ is a $Q_2$ -Smarandache fuzzy strong ideal. By definition 1(ii)]
⇒$\mu$ is a $Q_1$-Smarandache fuzzy strong ideal of X. ■

**Proposition3.** Every fuzzy strong ideal of a Smarandache BH-algebra X is a Q-Smarandache fuzzy strong ideal of X.

**Proof :**Let$\mu$ be a fuzzy strong ideal of X .
i. Let x ∈ X ⇒ $\mu$(0) ≥$\mu$(x) .
[By definition 13(i)]
ii. let x, z ∈X and y ∈ X ⇒x,z∈ Q
⇒$\mu$(x*z) ≥ min{$\mu$((x*y)*z) , $\mu$(y)}
[By definition13(ii)]
⇒$\mu$ is a Q-Smarandache fuzzy strong ideal of X. ■

**Theorem 1**. Let X be SmarandacheBH-algebra and let $\mu$ be a fuzzy set.Then $\mu$ is a Q-Smarandache fuzzy strong ideal if and only if $v$(x) = $\mu$ (x) / $\mu$ (0) is a Q-Smarandache fuzzy strong ideal.

**Proof:** Let $\mu$ be a Q-Smarandache fuzzy strong ideal,
1) $v$(0)= $\mu$ (0) / $\mu$ (0),⇒ $v$(0)=1
⇒ $v$(0) ≥ $v$(x) ∀x∈X

2) $v(x*z)= \mu(x*z)/\mu(0)$
$\geq \min\{\mu((x*y)*z), \mu(y)\}/\mu(0)\}$
[Since $\mu$ is a Q-Smarandache fuzzy strong ideal. .By definition1(ii)]
$\geq \min\{\mu((x*y)*z) / \mu(0), \mu(y) / \mu(0)\}$
$\geq \min\{v((x*y)*z), v(y)\}$
$\Rightarrow v(x*z) \geq \min\{v((x*y)*z), v(y)\}$
$\Rightarrow v$ is a Q-Smarandache fuzzy strong ideal.**Conversely.**Let $v$ be a Q-Smarandache fuzzy strong ideal.
i.$\mu(0)=v(0). \mu(0), \Rightarrow \mu(0) \geq v(x).\mu(0)$
$\Rightarrow \mu(0) \geq \mu(x)\ \forall x \in X$
ii.$\mu(x*z)= v(x*z).\mu(0) \geq \min\{v(x*(y*z)), v(y)\}.\mu(0)$
[Since $\mu'$ is a Q-Smarandache fuzzy ideal.By definition1(i)]
$\geq \min\{v((x*y)*z).\mu(0), v(y).\mu(0)\}$
$\geq \min\{\mu((x*y)*z), \mu(y)\}$
$\Rightarrow \mu(x) \geq \min\{\mu((x*y)*z), \mu(y)\}$
$\Rightarrow \mu$ is a Q-Smarandache fuzzy strong ideal.■

**Proposition 4.**Let X be a BH*-algebra ,and $\mu$ be a Q-Smarandache fuzzy strong ideal. Then
i.$\mu$ is a Q-Smarandache fuzzy closed idealof X.
ii. $\mu$ is a Q-Smarandache fuzzy completely closed ideal of X,if $X*X/\{0\} \subseteq Q$.

**Proof :**Let $\mu$ be a fuzzy strong ideal of X.
[ByProposition1]
$\Rightarrow \mu$ is a Q-Smarandache fuzzy ideal of X.
i. $\Rightarrow \mu$ is a Q-Smarandache fuzzy closed ideal of X. [By proposition 1]
ii. Let $x,y \in X \Rightarrow \mu$ is a Q-Smarandache fuzzy completely closed ideal of X.
[By proposition 1].■

**Theorem 2.**Let A be a non-empty subset of a Q-Smarandache BH-algebra X and let μ be a fuzzy set in X defined by:

$$\mu(x)=\begin{cases}\alpha_1 & x\in Q\\ \alpha_2 & \text{otherwise}\end{cases}$$

where $\alpha_1 > \alpha_2$ in [0, 1].Then μ is a Q-Smarandache fuzzy strong ideal X

**Proof :** Let μ be a fuzzy set of X.
i. $0 \in Q \Rightarrow \mu(0) = \alpha_1. \Rightarrow \mu(0) \geq \mu(x)$ [Since$\alpha_1 > \alpha_2$ ]
ii. let $x,z \in Q$ and $y \in X \Rightarrow x*z \in Q$
$\Rightarrow \mu(x*z) = \alpha_1$
Then we have for cases.

<u>Case1</u> If $((x*y)*z)= \alpha_1$and $\mu(y) =\alpha_1$
$\Rightarrow \min\{\mu((x*y)*z), \mu(y)\} =\alpha_1$
$\Rightarrow \mu(x*z) \geq \min\{\mu((x*y)*z), \mu(y)\}$

<u>Case2</u> If $((x*y)*z)= \alpha_2$and $\mu(y) =\alpha_1$
$\Rightarrow \min\{\mu((x*y)*z), \mu(y)\} =\alpha_2$
$\Rightarrow \mu(x*y) = \min\{\mu(x*y), \mu(y)\}$

<u>Case3</u> If $((x*y)*z)= \alpha_1$and $\mu(y) =\alpha_2$
$\Rightarrow \min\{\mu((x*y)*z), \mu(y)\} =\alpha_2$
$\Rightarrow \mu(x*y) = \min\{\mu(x*y), \mu(y)\}$

<u>Case3</u> If $((x*y)*z)= \alpha_2$and $\mu(y) =\alpha_2$
$\Rightarrow \min\{\mu((x*y)*z), \mu(y)\} =\alpha_2$
$\Rightarrow \mu(x*y) = \min\{\mu(x*y), \mu(y)\}$
$\Rightarrow$μ is a Q-Smarandache fuzzy strong ideal of X. ■

**Theorem 3.** Let X be BH-algebra and let $\mu$ be a fuzzy set.Then$\mu$ is a Q-Smarandache fuzzy strong ideal if and only if $\mu'(x)= \mu(x)+1- \mu(0)$ is a Q-Smarandache fuzzy strong ideal.

**Proof:** Let $\mu$ be a Q-Smarandache fuzzy strong ideal,
i. $\mu'(0)= \mu(0)+1- \mu(0), \Rightarrow \mu'(0)=1$
$\Rightarrow \mu'(0) \geq \mu'(x)\ \forall x \in X$
ii. $\mu'(x*z)= \mu(x*z)+1- \mu(0)$
$\geq \min\{\mu((x*y)*z)), \mu(y)\}+1- \mu(0)$[Since $\mu$ is a Q-Smarandache fuzzy strong ideal. .By definition1]
$\geq \min\{\mu((x*y)*z)) +1- \mu(0), \mu(y) +1- \mu(0)\} \geq \min\{\mu'((x*y)*z)), \mu'(y)\}$
$\Rightarrow \mu'(x*z) \geq \min\{\mu'((x*y)*z)), \mu'(y)\}$
$\Rightarrow \mu'$ is a Q-Smarandache fuzzy strong ideal.

<u>Conversely</u>Let $\mu'$ be a Q-Smarandache fuzzy strong ideal.
1) $\mu(0)= \mu'(0)-1+ \mu(0), \Rightarrow \mu(0) \geq \mu'(x)-1+ \mu(0) \Rightarrow \mu(0) \geq \mu(x)\ \forall x \in X$
2) $\mu(x*z)= \mu'(x*z)-1+ \mu(0)$
$\geq \min\{\mu'((x*y)*z)), \mu'(y)\}-1+ \mu(0)$ [Since $\mu'$ is a Q-Smarandache fuzzy strong ideal. By definition1]
$\geq \min\{\mu'((x*y)*z)) -1+ \mu(0), \mu'(y) -1+ \mu(0)\}$
$\geq \min\{\mu((x*y)*z)), \mu(y)\}$
$\Rightarrow \mu(x) \geq \min\{\mu((x*y)*z)), \mu(y)\}$
$\Rightarrow \mu$ is a Q-Smarandache fuzzy strong ideal.■

**Theorem4.** Let X be a Smarandache BH-algebra such that $X = X_+$ and $\mu$ be a Q-Smarandachestrong ideal of X. Then $\mu$ is a Q-Smarandache closed ideal of X .

**Proof :**Let $\mu$ be a Q-Smarandache strong ideal $\Rightarrow \mu I$ is a Q-Smarandache ideal of X. [Byproposition1]
Now, let $x \in I$
$\Rightarrow (0*x) = \mu(0) \geq \mu(x)$ [By definition 6]
$\Rightarrow \mu$ is a Q-Smarandache closed ideal of X . ■

**Proposition5.** Let X be a Smarandache normal BH-algebra such that $X=X_+$ and let $\mu$ be a Q-Smarandache fuzzy strong ideal such that $x*y \in Q, \forall x,y \in X$ and $y \neq 0$.Then $\mu$ is a Q-Smarandache fuzzy completely closed ideal of X.

**Proof:** Let $\mu$ I be a Q-Smarandache fuzzy strong ideal of X.
$\Rightarrow \mu$ is a Q-Smarandache fuzzy ideal of X. [By remark3 ]
Now, let $x,y \in X \Rightarrow x*y \in Q$
[Since $x*y \in Q, \forall x,y \in X$ ]
We have
$\mu(x*y) = \mu((x*y)*0) \geq \min\{\mu(((x*y)*x)*0)), \mu(x)\}$ [By definition 3(ii)]
$= \min\{\mu((x*y)*x), \mu(x)\}$[By definition 3(ii)] $= \min\{\mu(0*y), \mu(x)\}$ [By definition 5(ii)]
$= \min\{\mu(0), \mu(x)\} = \mu(x)$
$\Rightarrow \mu(x*y) \geq \min\{\mu(y), \mu(x)\}$
$\Rightarrow \mu$ is a Q-Smarandache fuzzy completely closed ideal. ■

**Proposition 6.**Let X be a normal BH-algebra such that $X*X/\{0\} \subseteq Q$ .Then every Q-Smarandache fuzzy strong ideal and closed of X is a Q-Smarandache fuzzy completely closed ideal of X.

**Proof :**Let $\mu$ be a Q-Smarandache fuzzy strong ideal of X.

$\Rightarrow\mu$ is a Q-Smarandache fuzzy ideal of X. [By proposition1]
Now, let x, y ∈X. ⇒ x*y ∈Q [Since X*X /{0} ⊆Q]
$\Rightarrow\mu(x*y) \geq \min\{\mu((x*y)*x)*0)\ ,\ \mu(x)\}$ [By definition1 ]
$= \min\{\mu(0*y)\ ,\ \mu(x)\}$[By definition 5(ii)]
$\geq \min\{\mu(y)\ ,\ \mu(x)\}$[By definition14]
$\Rightarrow\mu$ is a Q-Smarandache fuzzy completely closed ideal of X. ■

**Remark 1.** Let μ be a fuzzy set of a Smarandache BH-algebra X and w ∈ X .The set {x∈X: (w) ≤$\mu$(x)} is denoted by ↑$\mu$(w) .

**Theorem 5.**Let X be a Smarandache BH-algebra, w∈X and $\mu$ is a Q-Smarandache fuzzy strong ideal of X. Then↑(w) is a Q-Smarandachestrong ideal of X.

**Proof :**Let$\mu$ be a Q-Smarandache fuzzy strong ideal of X.
To prove that ↑(*w*) is a Q-Smarandache strong ideal of X.
1) Let x ∈↑(w) ⇒$\mu$(0) ≥$\mu$(x) [By definition 1(i)]
⇒(0) ≥$\mu$(w) ⇒ 0 ∈↑$\mu$(w)
2) Let x,z∈ Q, y ∈↑$\mu$(w) and (x*y)*z∈↑$\mu$(w).
⇒(w) ≤$\mu$(y) and $\mu$(w) ≤$\mu$((x*y)*z)
⇒(w) ≤ min{ $\mu$(y) , $\mu$((x*y)*z)}
But μ(x*z) ≥ min{μ((x *y)*z), μ(y)} [By definition 1(ii)]
⇒(w) ≤$\mu$(x*z) ⇒ x*z∈↑$\mu$(w)
⇒↑(*w*) is a Q-Smarandache strong ideal of X. ■

**Corollary1.**Let X be a Smarandache BH-algebra . Then $\mu$ is a Q-Smarandache fuzzy strong ideal of X if and only if $\mu_t$ is a Q-Smarandache strong ideal of X, for all t ∈ $[0,\sup_{x\in X}\mu(x)]$

**Proof** *:*Let t ∈ $[0,\sup_{x\in X}\mu(x)]$. To prove that $\mu_t$ is a Q-Smarandache strong ideal of X.Since $\mu$ is a Q-Smarandache fuzzy strong ideal of X.
Now, let y ∈ $\mu_t$ and x*(y*z) ∈ $\mu_t$⇒μ(y) ≥ t and μ((x*y)*z)) ≥ t .
To prove that x*z ∈ $\mu_t$
We have $\mu$(x*z) ≥min{$\mu$((x*y)*z)) ,$\mu$(y)} [By definition 1]
Since ((x*y)*z)) ≥t and $\mu$(y) ≥t ⇒min{$\mu$((x*y)*z)) ,$\mu$(y)}≥ t
⇒$\mu$(x*z) ≥t ⇒ x*z ∈ $\mu_t$
⇒$\mu_t$ is a Q-Smarandache strong ideal of X.
***Conversely,***
To prove that $\mu$ is a Q-Smarandache fuzzy strong ideal of X.
Since $\mu_t$ is a Q-Smarandache strong ideal of X.
Let $t = \sup_{x\in X}\mu(x)$ ,x,z∈Q and(x*y)*z, y ∈ $\mu_t$
⇒ x*z ∈ $\mu_t$[By definition9]
⇒$\mu$(x*z) ≥t ⇒$\mu$(x*z) = t [Since $t = \sup_{x\in X}\mu(x)$]
Similarly,((x*y)*z)=t and $\mu$(y)=t
⇒t = min {((x*y)*z),$\mu$(y)}
⇒$\mu$(x*z) ≥ min {((x*y)*z),$\mu$(y)}
⇒$\mu$ is a Q-Smarandache fuzzy strong ideal of X.■

**Proposition7** .Let $f$ : (X, *, 0) →(Y, $*'$ ,$0'$) be a Smarandache BH-epimorphism. If $\mu$ is a Q-Smarandache fuzzy strong ideal of X, then $f$ (μ) is a $f$ (Q)-Smarandache fuzzy strong ideal of Y .

**Proof :**Let $\mu$ be a Q-Smarandache fuzzy strong ideal of X.
i. Let y ∈$f$ (μ) such thaty = $f$ (x) .
( $f$ (μ))( $0'$ )=sup {$\mu$(x) | x ∈ $f^{-1}$( $0'$ )}
=(0) ≥ $\mu$(x) [By definition 1(i)]
=( $f$ (μ))( $f$ (x))
= ( $f$ (μ))(y)⇒( $f$ (μ))( $0'$ )≥( $f$ (μ))(y)
ii. Let $y_1$,$y_3$ ∈$f$ (Q), $y_2$ ∈ Y, there exists $x_1, x_3$ ∈Q and $x_2$ ∈X such that $y_1$= $f$ ($x_1$), $y_3$= $f$ ($x_3$) and$y_2$ =$f$ ($x_2$)⇒ ( $f$ (μ))($y_1 * y_3$ ) = sup{$\mu$($x_1$*$x_3$) | x ∈ $f^{-1}$($y_1 * y_3$)}
( $f$ (μ))($y_1$* $y_1$) ≥($x_1 * x_3$)≥ min{ $\mu$(($x_1 * x_2$)*$x_3$),$\mu$($x_2$)} [By definition 1(ii)]
= min {( $f$ (μ))($f$ (($x_1$*$x_2$)*$x_3$)) , ( $f$ (μ))($x_2$)} =min{($f$ (μ))(($f$ ($x_1$) $*'$ $f$ ($x_2$)) $*'$ $f$($x_3$)),($f$ (μ))($f$ ($x_2$)}= min{($f$ (μ))($y_1$ $*'$ $y_2$) $*'$ $y_3$),( $f$ (μ))($y_2$)}
⇒ ( $f$ (μ))($y_1$ ) ≥ min { ( $f$ (μ))(($y_1$ $*'$ $y_2$) $*'$ $y_2$),( $f$ (μ))($y_2$ )}
⇒ $f$ ($\mu$) is a $f$ (Q)-Smarandache fuzzy strong ideal of Y. ■

**Theorem 6.**Let $f$ :(X,*,0)→(Y, $*'$ , $0'$ )be a Smarandache BH-epimorphism.If$\nu$is a Q -Smarandache fuzzy strong ideal of Y, then $f^{-1}(\nu)$ is a$f^{-1}$(Q)-Smarandache fuzzy strong ideal of X

**Proof** *:*i. Let x ∈X. Since $f$ (x) ∈Y and $\nu$ is a Q-Smarandachestrong fuzzy ideal of Y .
($f^{-1}(\nu)$)(0)= ν( $f$ (0))= ν( $0'$ ) ≥ ν( $f$ (x)) =($f^{-1}(\nu)$)(x)
ii. Let x ∈ $f^{-1}$(Q), y ∈X .
$f^{-1}(\nu)$(x*z)=( $f$ (x*z)) [By definition 12]
≥ min{$\nu$( $f$ (x) $*'$ $f$ (y)) $*'$ $f$ (z)), $\nu$( $f$ (y))} [By remark1 ]
=min{$\nu$( $f$ ((x * y)*z), $\nu$( $f$ (y))}
⇒$f^{-1}(\nu)$(x)≥min{$f^{-1}(\nu)$((x*y)*z),$f^{-1}(\nu)$(y)}[By definition1]
⇒$f^{-1}(\nu)$ is a Q-Smarandache fuzzy strong ideal of X . ■

**Definition 2**. Let X be a Smarandache BH-algebra and n be apositive integer**.** A nonempty subset I of X is called a ***Smarandachen-fold strong ideal of X related toQ*** ( or briefly**,** ***Q- Smarandachen-fold strong ideal*** of X) if it satisfies: i. 0 ∈ I , ii. ∀y ∈I and (x* y)*$z^n$∈I ⇒x*z ∈I, ∀ x ,z ∈Q.

**Example 2.**Consider the set I={0,3} in example 1 is a Q-Smarandachen-fold strongideal of X.But the set I={0,2,3} is not a Q-Smarandachen-fold strongideal since(1*3)*$0^n$= 2∈I,but 1*$0^n$=1∉ I.

**Remark 2.** Every n-fold strong ideal of a Smarandache BH-algebra X is a Q-Smarandachen-fold strong ideal of X.

**Remark3.**Every Q-Smarandachen-fold strong ideal of a Smarandache BH-algebra X is a Q-Smarandachestrong ideal of X.**Remark4.**Every Q-Smarandachen-fold strong ideal of a Smarandache BH-algebra X is a Q-Smarandache ideal of X.

**Proposition8 .** Let X be a Smarandache BH-algebra . Then every Q-Smarandachen-foldstrong ideal which is contained in Q is a Q-Smarandache completely closed ideal of X.

**Proof :**Let I be a Q-Smarandachen-foldstrong ideal of X⇒ I is a Q-Smarandache ideal of X. [By remark4]
Now, let x,y∈I ⇒x,y∈Q ⇒ x*y∈Q [Since I⊆ Q ]

Where ((x*y)*x)*0 =((x*x)*y)*0 [Since (x*y)*z =(x*z)*y. By definition 1(iii)]
=(0*y)*0 [By definition 3(i)]
=0*y [By definition 3(ii)]
=0∈I [Since 0*x=0.By definition 2(v)]
$\Rightarrow$((x*y)*x)*$0^n$∈I and x∈I.$\Rightarrow$ (x*y)*$0^n$∈ I $\Rightarrow$ x*y∈ I[By definition 3(ii)]
$\Rightarrow$ I is a Q-Smarandache completely closed ideal of X. ■

**Definition 3.**A fuzzy subset of a Smarandache BH-algebra Xand n be a positive integer is called a Q-***Smarandache* fuzzy *n-fold strong* ideal** ,iff i. μ (0) ≥μ (x) ) ∀x∈X. ii. μ (x*$z^n$) ≥ min{ μ ((x*y)*$z^n$), μ (y)},∀x,z∈Q.

**Example3.**Consider thefuzzy set $\mu$ which is defined by:

$$\mu(x) = \begin{cases} 0.5 & x = 0,2 \\ 0.4 & x = 1,3 \end{cases}$$

is a Q-Smarandache fuzzy n-foldstrong ideal , since: i.$\mu$(0) = 0.5 ≥$\mu$(x) ∀x∈X, ii. $\mu$ (x*$z^1$) ≥ min{ $\mu$((x*y)*$z^1$), $\mu$(y)},∀x, z∈Q.

But the fuzzy set$\mu(x) = \begin{cases} 0.5 & x = 0,2,3 \\ 0.4 & x = 1 \end{cases}$

is not a Q-Smarandache fuzzy n-foldstrong ideal since$\mu(1*0^3)=\mu(1)=0.4 < \min\{\mu(1*3)*0^3), \mu(3)\}=0.5$

**Remark5.** Every fuzzy n-fold strong ideal of a Smarandache BH-algebra X is a Q-Smarandache fuzzy n-fold strong ideal of X.

**Proposition 9 .**Every Q-Smarandache fuzzy n-fold strong ideal of a Smarandache BH-algebra X is a Q-Smarandache fuzzy strong ideal of X.

**Proof :** Let $\mu$ be a Q-Smarandache fuzzy n-fold strong ideal of X .**i.** Let x ∈ X $\Rightarrow$ (0) ≥$\mu$(x) .[By definition 3(i)]
ii. let x,z∈Q and y ∈ X $\Rightarrow$$\mu$(x*$z^n$) ≥ min{$\mu$((x*y)*$z^n$) , $\mu$(y)}[By definition 3(ii)]
When n=1$\Rightarrow$$\mu$(x*z) ≥ min{$\mu$((x*y)*z) , $\mu$(y)}
$\Rightarrow$μ is a Q-Smarandache fuzzy strong ideal of X. ■

**Theorem 7.** Let $f$ :(X,*,0)→(Y, $*'$ , $0'$ )be a Smarandache BH-epimorphism. If $\nu$is a Q-Smarandache fuzzy n-fold strongideal of Y, then $f^{-1}(\nu)$ is a$f^{-1}$(Q)-Smarandache fuzzy n-fold strongideal of X.

**Proof :**i.Let x ∈X. Since $f$ (x) ∈Y and $\nu$ is a Q-Smarandache fuzzy n-fold strongideal of Y.
$(f^{-1}(\nu))(0)= \nu(f(0))= \nu(0') \geq \nu(f(x)) =(f^{-1}(\nu))(x)$
ii. Let x,z∈ $f^{-1}$(Q), y ∈X .
$f^{-1}(\nu)(x*z^n)=(f(x*z^n))$ [By definition11]
$\geq \min\{\nu(f(x) *' f(y)) *' f(z^n)), \nu(f(y))\}$
[By remark1]
$=\min\{\nu(f((x*y)*z^n), \nu(f(y))\}$
$\Rightarrow f^{-1}(\nu)(x*z^n)\geq\min\{f^{-1}(\nu)((x*y)*z^n), f^{-1}(\nu)(y)\}$
$\Rightarrow f^{-1}(\nu)$ is a Q-Smarandache fuzzyn-foldstrong ideal of X . ■

**Theorem 8.**Let X be Smarandache BH-algebra. If $\mu$ is a fuzzy set such that Q= $X_\mu$={x∈X: $\mu$(x)=$\mu$(0)}and $\mu$(0)≥ $\mu$(x) ∀x∈X, then $\mu$ is a Q-Smarandache fuzzy n-fold strong ideal of X.

***Proof*** **:**Let $\mu$ be a fuzzy set of X, such that Q =$X_\mu$ and $\mu$(0)≥ $\mu$(x) ∀x∈X.
i. (0)≥$\mu$(x) ∀x∈X.
ii. Let x,z∈Q and y∈X .
(0)≥$\mu$(y) and $\mu(0) \geq \mu((x*y)*z^n)$[Since$\mu$(0)≥$\mu$(x) ∀x∈X]
$\Rightarrow$(0) ≥ min{$\mu$((x*y)*$z^n$),$\mu$(y)}
But $\mu$(x*$z^n$)=$\mu$(0)[Since Q= $X_\mu$]
$\Rightarrow$$\mu$(x*$z^n$) ≥ min{$\mu$((x*y)*$z^n$),$\mu$(y)}$\Rightarrow$$\mu$ is a Q-Smarandache fuzzy n- fold strong ideal of X.■

**Proposition10.**Let {$\mu_\alpha$:$\alpha\in\lambda$} be a family of Q-Smarandache fuzzy n- fold strongideals of a Smarandache BH-algebra X. Then $\bigcap_{\alpha\in\lambda}\mu_\alpha$ is a fuzzy n-fold strongideal of X.

**Proof :**Let {$\mu_\alpha$:$\alpha\in\lambda$}be a family of Q-Smarandache fuzzy n-foldstrong ideals of X.

i. Let x∈X. $\bigcap_{\alpha\in\lambda}\mu_\alpha(0) = \inf\{\mu_\alpha(0), \alpha\in\lambda\}\geq\inf\{\mu_\alpha(x), \alpha\in\lambda\}$

[Since $\mu_\alpha$ is a Q-Smarandache fuzzy n-foldideal, ∀$\alpha\in\lambda$. By definition 3(i)]

$= \bigcap_{\alpha\in\lambda}\mu_\alpha(x) \Rightarrow \bigcap_{\alpha\in\lambda}\mu_\alpha(0) \geq \bigcap_{\alpha\in\lambda}\mu_\alpha(x)$

ii. Let x,z∈ Q and y∈X

$\bigcap_{\alpha\in\lambda}\mu_\alpha(x*z^n) = \inf\{\mu_\alpha(x*z^n), \alpha\in\lambda\} \geq \inf\{\min\{\mu_\alpha((x*y)*z^n), \mu_\alpha(y)\}, \alpha\in\lambda\}$[Since $\mu_\alpha$ is a Q-Smarandache fuzzy n-foldstrong ideal, ∀$\alpha\in\lambda$. By definition3(ii)]= min{ inf{ $\mu_\alpha$((x*y)*$z^n$), $\alpha\in\lambda$ }, inf{ $\mu_\alpha$(y), $\alpha\in\lambda$ } }

$= \min\{\bigcap_{\alpha\in\lambda}\mu_\alpha((x*y)*z^n), \bigcap_{\alpha\in\lambda}\mu_\alpha(y)$

$\Rightarrow \bigcap_{\alpha\in\lambda}\mu_\alpha(x*z^n)\geq\min\{\bigcap_{\alpha\in\lambda}\mu_\alpha((x*y)*z^n), \bigcap_{\alpha\in\lambda}\mu_\alpha(y)\} \Rightarrow$

$\bigcap_{\alpha\in\lambda}\mu_\alpha$ is a Q-Smarandache fuzzy n-foldstrong ideal of X. ■

**Proposition 11.**Let {$\mu_\alpha$: $\alpha\in\lambda$} be a chain of Q-Smarandache fuzzy n-fold strongideals of a Smarandache BH-algebra X. Then $\bigcup_{\alpha\in\lambda}\mu_\alpha$ is a Q-Smarandache fuzzy n-foldstrong ideal of X.

**Proof :**Let {$\mu_\alpha$: $\alpha\in\lambda$} be a chain of Q-Smarandache fuzzy n-fold strongideals of X.

i. Let x∈X. $\bigcup_{\alpha\in\lambda}\mu_\alpha(0)=\sup\{\mu_\alpha(0),\alpha\in\lambda\} \geq \sup\{\mu_\alpha(x),\alpha\in\lambda\}$
[Since $\mu_\alpha$ is a Q-Smarandache fuzzy n-foldstrong ideal,∀$\alpha\in\lambda$.Bydefinition 3(i)]

$= \bigcup_{\alpha\in\lambda} \mu_\alpha(x) \Rightarrow \bigcup_{\alpha\in\lambda} \mu_\alpha(0) \geq \bigcup_{\alpha\in\lambda} \mu_\alpha(x)\ \forall\ x\in X$ .

ii. Let x,z∈Q,y∈X. $\bigcup_{\alpha\in\lambda} \mu_\alpha(x*z^n) = \sup\{\ \mu_\alpha\ (x),\alpha\in\lambda\} \geq$ $\sup\{\min\{\mu_\alpha((x*y)*z^n),\mu_\alpha(y)\}, \alpha\in\lambda\}$

[Since $\mu_\alpha$ is a Q-Smarandache fuzzy n-foldstrong ideal, $\forall\alpha\in\lambda$. By definition 3(ii)]

But $\{\ _\alpha\ ,\alpha\in\lambda\}$ is a chain ⇒ there exist, $j\in\lambda$ such that

$\sup\{\ \min\{\mu_\alpha((x*y)*z^n),\ \mu_\alpha(y)\},\ \alpha\in\lambda\ \} = \min\{\mu_j((x*y)*z^n), \mu_j(y)\}$

$=\min\{\sup\{\mu_\alpha((x*y)*z^n),\alpha\in\lambda\}, \sup\{\mu_\alpha(y), \alpha\in\lambda\}\}$

$\Rightarrow \bigcup_{\alpha\in\lambda} \mu_\alpha(x*z^n) \geq \min\{\mu_j((x*y)*z^n)\ , \mu_j(y)\}$

$\geq \min\{\sup\{\mu_\alpha((x*y)*z^n),\alpha\in\lambda\},\ \sup\{\mu_\alpha(y),\ \alpha\in\lambda\}= \min\{ \bigcup_{\alpha\in\lambda} \mu_\alpha((x*y)*z^n), \bigcup_{\alpha\in\lambda} \mu_\alpha(y)\ \}$

$\Rightarrow \bigcup_{\alpha\in\lambda} \mu_\alpha(x*z^n) \geq \min\{ \bigcup_{\alpha\in\lambda} \mu_\alpha((x*y)*z^n), \bigcup_{\alpha\in\lambda} \mu_\alpha(y)\} \Rightarrow$

$\bigcup_{\alpha\in\lambda} \mu_\alpha$ is a Q-Smarandache fuzzy n-fold strong ideal of X.

■

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