SMARANDACHE PASCAL DERIVED SEQUENCES

(Amarnath Murthy, S.E.(E&T), WLS, Oil and Natural Gas Corporation Ltd., Sabarmati, Ahmedabad,-380005 INDIA.)

Given a sequence say S_b. We call it the base sequence. We define a **Smarandache Pascal derived sequence S_d** as follows:

$$T_{n+1} = \sum_{k=0}^{n} {}^{r}C_k \, .t_{k+1}$$, where t_k is the k^{th} term of the base sequence.

Let the terms of the the base sequence S_b be

$b_1, b_2, b_3, b_4, \ldots$

Then the Smarandache Pascal derived Sequence S_d

$d_1, d_2, d_3, d_4, \ldots$ is defined as follows:

$d_1 = b_1$

$d_2 = b_1 + b_2$

$d_3 = b_1 + 2b_2 + b_3$

$d_4 = b_1 + 3b_2 + 3b_3 + b_4$

...

$$d_{n+1} = \sum_{k=0}^{n} {}^{n}C_k \, .b_{k+1}$$

These derived sequences exhibit interesting properties for some base sequences.

Examples:

{1} $S_b \to 1, 2, 3, 4, \ldots$ (natural numbers)

$S_d \to 1, 3, 8, 20, 48, 112, 256, \ldots$ (Smarandache Pascal derived natural number sequence)

The same can be rewritten as

$2x2^{-1}, 3x2^0, 4x2^1, 5x2^2, 6x2^3, \ldots$

It can be verified and then proved easily that $\mathbf{T_n = 4(T_{n-1} - T_{n-2})}$ for $n > 2$.

And also that $\mathbf{T_n = (n+1) .2^{n-2}}$

{2} $S_b \to 1, 3, 5, 7, \ldots$ (odd numbers)

$S_d \to 1, 4, 12, 32, 80, \ldots$

The first difference $1, 3, 8, 20, 48, \ldots$ is the same as the S_d for natural numbers.

The sequence S_d can be rewritten as

$1.2^0, 2.2^1, 3.2^2, 4.2^3, 5.2^4, \ldots$

Again we have $\mathbf{T_n = 4(T_{n-1} - T_{n-2})}$ for $n > 2$.Also $\mathbf{T_n = n.2^{n-1}}$.

{3} Smarandache Pascal Derived Bell Sequence:

Consider the Smarandache Factor Partitions (SFP) sequence for the square free numbers:

(The same as the **Bell number** sequence.)

$S_b \to \mathbf{1}, 1, 2, 5, 15, 52, 203, 877, 4140, \ldots$

We get the derived sequence as follows

$S_d \to 1, 2, 5, 15, 52, 203, 877, 4140, \ldots$

The **Smarandache Pascal Derived Bell Sequence** comes out to be the same. We

call it **Pascal Self Derived Sequence.** This has been established in **ref. [1]**
In what follows, we shall see that this Transformation applied to Fibonacci Numbers gives beautiful results.

****{4} Smarandache Pascal derived Fibonacci Sequence:**

Consider the Fibonacci Sequence as the Base Sequence:

$S_b \to$ **1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 114, 233, . . .**

We get the following derived sequence

$S_d \to 1, 2, 5, 13, 34, 89, 233, \ldots$ **(A)**

It can be noticed that the above sequence is made of the alternate (even numbered terms of the sequence) Fibonacci numbers.

This gives us the following result on the Fibonacci numbers.

$$F_{2n} = \sum_{k=0}^{n} {}^{n}C_k . F_k$$ **, where F_k is the k^{th} term of the base Fibonacci sequence.**

Some more interesting properties are given below.

If we take **(A)** as the base sequence we get the following derived sequence S_{dd}

$S_{dd} \to 1, 3, 10, 35, 125, 450, 1625, 5875, 21250, \ldots$

An interesting observation is ,the first two terms are divisible by 5^0, the next two terms by 5^1 , the next two by 5^2 , the next two by 5^3 and so on.

$$T_{2n} \equiv T_{2n-1} \equiv 0 \pmod{5^n}$$

On carrying out this division we get the following sequence i.e.

1, 3, 2, 7, 5, 18, 13, 47, 34, 123, 89, . . . **(B)**

The sequence formed by the odd numbered terms is

1, 2, 5, 13, 34, 89, . . .

which is again nothing but S_d (the base sequence itself.).

Another interesting observation is every even numbered term of **(B)** is the sum of the two adjacent odd numbered terms. (3 = 1+2, 7 = 2 +5, 18 = 5 + 13 etc.)

CONJECTURE: Thus we have the possibility of another beautiful result on the Fibonacci numbers which of-course is yet to be established.

$$F_{2m+1} = (1/5^m) \sum_{r=0}^{2m+1} \{ {}^{2m+1}C_r (\sum_{k=0}^{r} {}^{r}C_k F_k) \}$$

Note: It can be verified that all the above properties hold good for the **Lucas sequence (1 , 3, 4, 7, 11, . . .)** as well.

Pascalisation of Fibonacci sequence with index in arithmetic progression:

Consider the following sequence formed by the Fibonacci numbers whose indexes are in A. P.

$F_1, F_{d+1}, F_{2d+1}, F_{3d+1}, \ldots$ on pascalisation gives the following sequence

$1, d.F_2, d^2.F_4, d^3.F_6, d^4.F_8, \ldots, d^n.F_{2n}, \ldots$

for d = 2 and d = 3.

For d = 5 we get the following

Base sequence : $F_1, F_6, F_{11}, F_{16}, \ldots$

1, 13, 233, 4181, 46368, . . .

Derived sequence: 1, 14, 260, 4920, 93200, . . .in which we notice that
260= 20.(14- 1), 4920 = 20.(260 - 14) , 93200 = 4920 - 260) etc . which suggests the possibility of

Conjecture: The terms of the pascal derived sequence for d = 5 are given by $T_n = 20.(T_{n-1} - T_{n-2})$ (n > 2)

For d = 8 we get

Base sequence : F_1, F_9, F_{17}, F_{25} , . . .

$S_b \longrightarrow$ 1, 34, 1597, 75025, . . .

$S_d \longrightarrow$ 1, 35, 1666, 79919, . . .

= 1, 35, (35-1). 7^2 , (1666 - 35). 7^2 , . . . etc. which suggests the possibility of

Conjecture: The terms of the pascal derived sequence for d = 8 are given by $T_n = 49.(T_{n-1} - T_{n-2})$, (n > 2)

Similarly we have Conjectures:

For d = 10 , **$T_n = 90.(T_{n-1} - T_{n-2})$, (n > 2)**

For d = 12 , **$T_n = 18^2.(T_{n-1} - T_{n-2})$, (n > 2)**

Note: There seems to be a direct relation between d and the coefficient of ($T_{n-1} - T_{n-2}$) (or the common factor) of each term which is to be explored.

{5} Smarandache Pascal derived square sequence:

$S_b \rightarrow$ 1, 4, 9, 16, 25, . . .

$S_d \rightarrow$ 1, 5, 18, 56, 160, 432, . . .

Or 1, 5x1, 6x 3, 7x 8, 8x20, 9x48 , . . . , (**$T_n = (n+3)t_{n-1}$**) , where t_r is the r^{th} term of Pascal derived natural number sequence.

Also one can derive **$T_n = 2^{n-2} . (n+3)(n)/ 2$.**

{6} Smarandache Pascal derived cube sequence:

$S_b \rightarrow$ 1, 8, 27, 64, 125

$S_d \rightarrow$ 1, 9, 44, 170, 576, 1792, . . .

We have $T_n \equiv 0 \pmod{(n+1)}$.

Similarly we have derived sequences for higher powers which can be analyzed for patterns.

{7} Smarandache Pascal derived Triangular number sequence:

$S_b \rightarrow$ 1, 3, 6, 10, 15, 21, . . .

$S_d \rightarrow$ 1, 4, 13, 38, 104, 272, . . .

{8} Smarandache Pascal derived Factorial sequence:

$S_b \rightarrow$ 1, 2, 6, 24, 120, 720, 5040, . . .

$S_d \rightarrow$ 1, 3, 11, 49, 261, 1631, . . .

We can verify that **$T_n = n.T_{n-1} + \Sigma T_{n-2} + 1$.**

Problem: Are there infinitely many primes in the above sequence?

Smarandache Pascal derived sequence of the k^{th} order.

Consider the natural number sequence again:

$S_b \rightarrow$ 1, 2, 3, 4, 5, . . . The corresponding derived sequence is

$S_d \rightarrow 2x2^{-1}$, $3x2^0$, $4x2^1$, $5x2^2$, $6x2^3$, . . . With this as the base sequence we get the derived sequence denoted by S_{d2} as

S_{dd} or $S_{d2} \rightarrow$ 1, 4, 15, 54, 189, 648, . . . which can be rewritten as

1, $4x3^0$, $5x3^1$, $6x3^2$, $7x3^3$ . . .

similarly we get S_{d3} as 1, $5x4^0$, $6x4^1$, $7x4^2$, $8x4^3$, . . . which suggests the possibility of the terms of S_{dk} , the k^{th} order Smarandache Pascal derived natural

number sequence being given by
1, (k+2) .(k+1)^0, (k+3).(k+1)^1 , (k+4).(k+1)^2, . . ., (k+r).(k+1)$^{r-2}$ etc. This can be proved by induction.

We can take an arithmetic progression with the first term 'a' and the common difference 'b' as the base sequence and get the derived kth order sequences to generalize the above results.

Reference: [1] Amarnath Murthy, ' Generalization of Partition Function,. Introducing Smarandache Factor Partitions' SNJ, Vol. 11, No. 1-2-3,2000.