

## Smarandache's Ratio Theorem

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### Abstract.

In this paper we present the Smarandache's Ratio Theorem in the geometry of the triangle.

### Smarandache's Ratio Theorem.

If the points  $A_1, B_1, C_1$  divide the sides  $\|BC\| = a, \|CA\| = b$ , respectively  $\|AB\| = c$  of a triangle  $\Delta ABC$  in the same ratio  $k > 0$ , then

$$\|AA_1\|^2 + \|BB_1\|^2 + \|CC_1\|^2 \geq \frac{3}{4}(a^2 + b^2 + c^2).$$

### Proof.

Suppose  $k > 0$  because we work with distances.

$$\|BA_1\| = k \|BC\|, \|CB_1\| = k \|CA\|, \|AC_1\| = k \|AB\|$$

We'll apply three times Stewart's theorem in the triangle  $\Delta ABC$ , with the segments  $AA_1, BB_1$ , respectively  $CC_1$ :

$$\|AB\|^2 \cdot \|BC\|(1-k) + \|AC\|^2 \cdot \|BC\|k - \|AA_1\|^2 \cdot \|BC\| = \|BC\|^3(1-k)k$$

where

$$\|AA_1\|^2 = (1-k)\|AB\|^2 + k\|AC\|^2 - (1-k)k\|BC\|^2$$

similarly,

$$\|BB_1\|^2 = (1-k)\|BC\|^2 + k\|BA\|^2 - (1-k)k\|AC\|^2$$

$$\|CC_1\|^2 = (1-k)\|CA\|^2 + k\|CB\|^2 - (1-k)k\|AB\|^2$$

By adding these three equalities we obtain:

$$\|AA_1\|^2 + \|BB_1\|^2 + \|CC_1\|^2 = (k^2 - k + 1)(\|AB\|^2 + \|BC\|^2 + \|CA\|^2),$$

which takes the minimum value when  $k = \frac{1}{2}$ , which is the case when the three lines from the enunciation are the medians of the triangle.

The minimum is  $\frac{3}{4}(\|AB\|^2 + \|BC\|^2 + \|CA\|^2)$ .

**Open Problems on Smarandache's Ratio Theorem.**

1. If the points  $A_1', A_2', \dots, A_n'$  divide the sides  $A_1A_2, A_2A_3, \dots, A_nA_1$  of a polygon in a ratio  $k > 0$ , determine the minimum of the expression:

$$\|A_1A_1'\|^2 + \|A_2A_2'\|^2 + \dots + \|A_nA_n'\|^2.$$

2. Similarly question if the points  $A_1', A_2', \dots, A_n'$  divide the sides  $A_1A_2, A_2A_3, \dots, A_nA_1$  in the positive ratios  $k_1, k_2, \dots, k_n$  respectively.

3. Generalize this problem for polyhedrons.

### References:

[1] F. Smarandache, *Problèmes avec et sans... problèmes!*, Problem 5.38, p. 56, Somipress, Fés, Morocco, 1983.

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