SMARANDACHE SOFT NEUTROSOPHIC NEAR-RING AND SOFT NEUTROSOPHIC IDEAL

N. KANNAPPA [1], B. FAIROSEKANI [2]

[1,2] Department of Mathematics, T.B.M.L. College, Porayar-609307, Tamil Nadu, India
Email: sivaguru91@yahoo.com, fairosemaths@gmail.com

Abstract

In this paper, we introduce new algebraic structures of Soft Neutrosophic Near-Ring, namely Smarandache Soft Neutrosophic Near-Ring, Smarandache Soft Neutrosophic Ideal and Smarandache Soft Neutrosophic Homomorphism. We define Smarandache Soft Neutrosophic Near-Ring and obtain some characterizations through the concept of Soft Neutrosophic Ideals. For the core concept of Near-Ring, we refer to G. Pilz [4], for the concept of Near-Field we refer to P. Dheena [1] and for the concept of Soft Neutrosophic Algebraic Structures we refer to Muhammed Shabir, Mumtaz Ali, Munazza Naz, and Florentin Smarandache [5, 6].

Keywords

Soft Neutrosophic Near-Ring, Soft Neutrosophic Near-Field, Soft Neutrosophic Ideal, Smarandache Soft Neutrosophic Near-Ring, Smarandache Soft Neutrosophic Ideal, Smarandache Soft Neutrosophic Homomorphism.

1. Introduction

To better study the congruence in number theory, Florentin Smarandache introduces new notions in algebra [2]. By <proper subset> of a set A he considers a set P included in A, but different from A, also different from the empty set, and from the unit element in A, ranking the algebraic structures using an order relationship.

We have an algebraic structure $S_1 << S_2$ if: both are defined on the same set; all S_1 laws are also S_2 laws; all axioms of a S_1 law are accomplished by the corresponding S_2 law; S_2 laws accomplish more axioms that S_1 laws, or S_2 has more laws than S_1.

For example: *semigroup* << *monoid* << *group* << *ring* << *field*, or *semigroup* << *commutative semigroup*, *ring* << *unitary ring* etc. The author defines a *general special structure* to be a structure SM on a set A, different from a structure SN, such that a proper subset of A is a structure, where SM << SN.

In addition, see our papers [8, 9].

2. Preliminaries

Definition 1. Let $\langle N\, UI\rangle$ be a neutrosophic near-ring and (F, A) be a soft set over $\langle N\, UI\rangle$. Then (F, A) is called a soft neutrosophic near-ring if and only if $F(a)$ is a neutrosophic sub near-ring of $\langle N\, UI\rangle$ for all $a \in A$.

Definition 2. Let $K(I) = \langle KUI\rangle$ be a neutrosophic near-field and let (F, A) be a soft set over $K(I)$. Then (F, A) is said to be a soft neutrosophic near-field if and only if $F(a)$ is a neutrosophic sub near-field of $K(I)$ for all $a \in A$.

Definition 3. Let (F, A) be a soft neutrosophic near-ring over $\langle N \cup I\rangle$ with more than one element. Then, the non-zero elements of (F, A) form a group under multiplication if and only if for every $F(a) \neq 0$ in (F, A) there exists a unique $F(b)$ in (F, A) such that $F(a)F(b)F(a) = F(a)$.

Definition 4. Let (F, A) be a soft neutrosophic zero symmetric near-ring over $\langle N \cup I\rangle$, which contains a distributive element $F(a1) \neq 0$. Then (F, A) is a near-field if and only if for each $F(a) \neq 0$ in $(F, A), (F, A)F(a) = (F, A)$.

Definition 5. Let (F, A) be a finite soft neutrosophic zero symmetric near-ring that contains a distributive element $F(w) \neq 0$, and for each $F(x) \neq 0$ in (F, A) there exists $F(y)$ in (F, A) such that $F(y)F(x) \neq 0$. Then (F, A) is a soft neutrosophic near-field if and only if (F, A) has no proper left ideal.

Definition 6. Let (F, A) be a soft neutrosophic near-ring over $\langle N \cup I\rangle$. Then (F, A) is called soft neutrosophic zero symmetric near-ring over $\langle N \cup I\rangle$, if $F(n)0 = 0$ for all $F(n)$ in (F, A).

Definition 7. Let (F, A) be a soft neutrosophic near-ring over $\langle N \cup I\rangle$. An element $F(e)$ in a soft neutrosophic near-ring (F, A) over $\langle N \cup I\rangle$ is called idempotent if $F(e^2) = F(e)$.

Definition 8. Let (F, A) be a soft neutrosophic near-ring over $\langle N \cup I\rangle$. An element $F(b)$ in (F, A) is called distributive if $F(b)(F(a_1) + F(a_2)) = F(b)F(a_1) + F(b)F(a_2)$ for all $F(a_1), F(a_2)$ in (F, A).

Definition 9. Let (F,A) be a soft neutrosophic near-ring over $\langle N \cup I \rangle$. A soft neutrosophic subgroup (H,A) of (F,A) is called (F,A) subgroup if $(F,A)(H,A) \subset (H,A)$.

Definition 10. Let (F,A) be a soft neutrosophic near-ring over $\langle N \cup I \rangle$; it is called regular if for each $F(a)$ in (F,A) there exists $F(x)$ in (F,A) such that $F(a)F(x)F(a) = F(a)$.

Definition 11. Let (F,A) be a soft set over a neutrosophic near-ring over $\langle N \cup I \rangle$. Then (F,A) is called soft neutrosophic ideal over $\langle N \cup I \rangle$ if and only if $F(a)$ is a neutrosophic ideal over $\langle N \cup I \rangle$.

Definition 12. Let (F,A) and (K,B) be two soft neutrosophic near-ring over $\langle N \cup I \rangle$. Then (K,A) is called soft neutrosophic ideal of (F,A) if $B \subseteq A$, and $k(a)$ is a neutrosophic ideal of $F(a)$ for all $a \in A$.

Definition 13. Let (F,A) be a Smarandache soft neutrosophic near-ring over $\langle N \cup I \rangle$. A normal subgroup (L,A) of (F,A) is called a Smarandache soft neutrosophic ideal of (F,A) over $\langle N \cup I \rangle$ related to (G,A) over $\langle N \cup I \rangle$ if $(L,A)(G,A) \subseteq (L,A)$, an for all $G(a), G(b)$ in (G,A), and for all $L(a)$ in (L,A), $G(a)(G(b)+L(a)) - G(a)G(b)$ in (L,A), where (G,A) is the soft neutrosophic near-field contained in (H,A).

Definition 14. The extended union of two Smarandache soft neutrosophic ideals (L_1,A) and (L_2,B) over a Smarandache soft neutrosophic near-ring $\langle N \cup I \rangle$ is the soft neutrosophic ideal (L_3,C), where $C = A \cup B$ and for all $c \in C, L_3(c)$ is defined as

$$L_3(c) = \begin{cases} L_1(c) & \text{if c} \in \text{A} - \text{B} \\ L_2(c) & \text{if c} \in \text{B} - \text{A} \\ L_1(c) \cup L_2(c) & \text{if c} \in \text{A} \cap \text{B} \end{cases}$$

We write $(L_1,A) \cup_E (L_2,B) = (L_3,C)$.

Definition 15. The restricted union of two Smarandache soft neutrosophic ideals (L_1,A) and (L_1,B) over a Smarandache soft neutrosophic near-ring $\langle N \cup I \rangle$ is a soft neutrosophic ideal (L_1,C), where $C = A \cup B$ and for all $c \in C$, (L_3,C) is defined as $(L_3,C) = (L_1,A) \cup_R (L_2,B)$ where $C = A \cap B$ and $L_3(c)$ = L_1, (c) ∪ L_2 (c) for all $c \in C$.

Definition 16. The extended intersection of two Smarandache soft neutrosophic ideals (L_1,A) and (L_2,B) over a Smarandache soft neutrosophic near-ring $\langle N \cup I \rangle$ is the soft neutrosophic ideal (L_3, C) where $C = A \cup B$ and for all $c \in C$, (L_3, c) is defined as

$$L_3(c) = \begin{cases} L_1(c) & \text{if } c \in A - B \\ L_2(c) & \text{if } c \in B - A \\ L_1(c) \cap L_2(c) & \text{if } c \in A \cap B \end{cases}$$

We write $(L_1, A) \cap_E (L_2, B) = (L_3, C)$.

Definition 17. The restricted intersection of two Smarandache soft neutrosophic ideals (L_1, A) and (L_2, B) over a Smarandache soft neutrosophic near-ring $\langle N \cup I \rangle$ is the soft Neutrosophic ideal (L_3, C), such that $A \cap B = \phi$. T (L_3, C) is defined as (L_3, C)=$(L_1, A) \cup_R (L_2, B)$, where $C = A \cap B$ and $L_3(c)$=$L_1(c) \cup L_2(c)$ for all $c \in C$.

Definition 18. Let (F, A) and (G, B) be two Smarandache soft neutrosophic near-ring over $\langle N \cup I \rangle$ and $\langle N^{|} \cup I \rangle$ respectively. Let $f: \langle N \cup I \rangle \to \langle N^{|} \cup I \rangle$ and $g: A \to B$ be two mappings. Then $(f, g): (F, A) \to (G, B)$ is called Smarandache soft neutrosophic near-ring homomorphism, if f is a neutrosophic near-ring homomorphism from $\langle N \cup I \rangle$ onto $\langle N^{|} \cup I \rangle$; g is a mapping from A onto B; $f(F(a)) = G(g(a))$ for all $a \in A$.

If f is a neutrosophic isomorphism from $\langle N \cup I \rangle$ to $\langle N^{|} \cup I \rangle$ and g is one to one mapping from A onto B, then (f, g) is called a Smarandache soft neutrosophic near-ring isomorphism from (F, A) to (G, B), where $F(a)$ & $g(a)$ is a proper subset of (F, A) which is a soft neutrosophic near-field.

Now we introduce the core concept, called SMARANDACHE –SOFT NEUTROSOPHIC–NEAR-RING.

Definition 19. A soft neutrosophic near-ring is said to be Smarandache soft neutrosophic near-ring if a proper subset of it is a soft neutrosophic near-field with respect to the same operations.

3. Results

Theorem 1. Let (F, A) be a soft neutrosophic near-ring over $\langle N \cup I \rangle$ in which idempotents commute and suppose that for each $H(x)$ in (H, A) there exists $H(y)$ in (H, A) such that $H(y)H(x) \neq 0$. Then (F, A) is a Smarandache soft neutrosophic near-ring if and only if (H, A) has no proper left ideal, and $(H, A)0 \neq (H, A)$, where (H, A) is a soft neutrosophic near-ring, which is a proper subset of (F, A).

Proof. (I) We assume that (H, A) has no proper left ideal.

Now we claim that (H, A) is a soft neutrosophic near-field.

Let $H(x) \neq 0$ in (H, A) and let $K(H(x) = \{H(n)$ in (H, A): $H(n)H(x) = 0\}$. Clearly $K(H(x))$ is a left ideal, since there exists $H(y)$ in (H, A) such that $H(y)H(x) \neq 0$. We have $H(y)$ that is not in $K(H(x))$, so $K(H(x)) = 0$.

Let ϕ:$((H, A), +) \rightarrow ((H, A)H(x), +)$ given by $\phi\big(H(n)\big) = H(n)H(x)$; then ϕ is an isomorphism; since (H, A) is finite, $(H, A)H(x) = (H, A)$.

Hence, by theorem, we have "Let (F, A) be a soft neutrosophic near-ring over $\langle N \cup I \rangle$. Then (F, A) is a Smarandache soft neutrosophic near-ring if and only if for each $H(a) \neq 0$ in $(H, A), (H, A)H(a) = (H, A)$ and $(H, A)0 \neq (H, A)$, where (H, A) is a soft neutrosophic near-ring, which is a proper subset of (F, A), in which idempotents commute".

Therefore (H, A) is a soft neutrosophic near-field. Then, by definition, (F, A) is a Smarandache soft neutrosophic near-ring.

(II) Conversely, we assume that (F, A) is a Smarandache soft neutrosophic near-ring. Then, by definition, there exists a proper subset $(H, A) \neq 0$ of (F, A) which is a soft neutrosophic near-field. Now, we prove that (H, A) has no proper left ideal: since (H, A) is a soft neutrosophic near-field if and only if [0] and itself are the ideals of (H, A), therefore (H, A) has no proper left ideal.

Theorem 2. Let (F, A) be a soft neutrosophic near-ring over $\langle N \cup I \rangle$ that contains a distributive element $H(r) \neq 0$, and suppose that for each $H(x) \neq 0$ in (H, A), there exists $H(y)$ in (H, A) such that $H(y)H(x) \neq 0$. Then (F, A) is a Smarandache soft neutrosophic near-ring if and only if (H, A) is regular and (H, A) has no proper left ideal, where (H, A) is a soft neutrosophic near-ring, which is a proper subset of (F, A).

Proof. (I) We assume that (H, A) is regular, with no proper left ideal.

Now we claim that (H, A) is a soft neutrosophic near-field.

Let $H(b) \neq 0$ in (H, A) and let $K(H(b)) = \{H(n)$ in (H, A): $H(n)H(b) = 0\}$; then $K(H(b))$ is a left ideal; since there exist some $H(t)$ in (H, A), such that $H(t)H(b) \neq 0$, we have $K\big(H(b)\big) = 0$; so if $H(a) \neq 0$ in (H, A) and $H(b) \neq 0$ in (H, A); then $H(a)H(b) \neq 0$. Hence (H, A) is without zero divisors. Since (H, A) is regular, there exists $H(x)$ in (H, A) such that $H(r)H(x)H(r) = H(r)$.

Let $H(r)H(x) = H(e)$, so $H(e)H(r) = H(r)$ and $H(e) \neq 0$ and $H(e)^2 = H(e)$. So $(H(r)H(e) - H(r))H(e) = 0$, hence $H(r) = H(r)H(e) = H(e)H(r)$.

Let $H(z)$ in (H, A), then $(H(z)H(e) - H(z))H(r) = 0$, so $H(z)H(e) = H(z)$.

Moreover, $H(r)(H(e)H(z) - H(z)) = 0$, hence $H(z) = H(e)H(z) = H(z)H(e)$, so $H(e)$ is an identity in (H, A).

If $K(H(0)) = \{H(n) \text{ in } (H, A): H(n)H(0) = 0\}$, then $K(H(0))$ is a left ideal. Since $H(e)$ in $K(H(0))$, $K(H(0)) \neq 0$. So $K(H(0)) = (H, A)$ and (H, A) is zero symmetric.

Therefore (H, A) is a soft neutrosophic near-field. By definition, (F, A) is a Smarandache soft neutrosophic near-ring.

(II) Conversely, we assume that (F, A) is a Smarandache soft neutrosophic near-ring. Then, by definition, there exists a proper subset $(H, A) \neq 0$ of (F, A) which is a soft neutrosophic near-field. Now to prove that (H, A) has no proper left ideal. Since (H, A) is a soft neutrosophic near-field if and only if [0] and itself are the ideals of (H, A), therefore (H, A) has no proper left ideal.

Theorem 3. Let (F, A) be a soft neutrosophic near-ring over $\langle N \cup I \rangle$ in which idempotents commute. Then (F, A) is a Smarandache soft neutrosophic near-ring if and only if (H, A) is regular and (H, A) has no proper left ideal, where (H, A) is a soft neutrosophic near-ring, which is a proper subset of (F, A).

Proof. (I) We assume that (H, A) is regular with no proper left ideal.

Now we claim that (H, A) is a soft neutrosophic near-field.

For any $H(n)$ in (H, A), $(H(n)0)^2 = H(n)0(H(n)0) = H(n)0$. Since 0 is also idempotent, we have $0 = 0(H(n)0) = (H(n)0)0 = H(n)0$, hence (H, A) is zero symmetric, since (H, A) is regular. Being given $H(a)$ in (H, A), there exists $H(x)$ in (H, A) such that $H(a)H(x)H(a) = H(a)$.

Let $H(b) = H(x)H(a)H(x)$, then $H(a)H(b)H(a) = H(a)$, and $H(b)H(a)H(b) = H(b)$, so $H(a)H(b)$, and $H(b)H(a)$ are idempotent.

For any $H(a)$ in (H, A), if $H(a)^2 = 0$, there exists $H(b)$ in (H, A) such that $H(a)H(b)H(a) = H(a)$ and $H(b)H(a)H(b) = H(b)$.

So: $H(b)^2 = \big(H(b)H(a)H(b)\big)\big(H(b)H(a)H(b)\big) =$
$H(b)\big(H(a)H(b)\big)\big(H(b)H(a)\big)H(b) = H(b)\,(H(b)H(a))(H(a)H(b))\,H(b) =$
$H(b)^2\, H(a)^2\, H(b)^2 = 0$.

$$[H(a)(H(b)H(a) + H(b))]^2 = H(a)(H(b)H(a) + H(b))\,H(a)\,(H(b)H(a) + H(b)) = [H(a)\,(H(b)H(a) + H(b))\,H(a)](H(b)H(a) + H(b)) = H(a)H(b)H(a)(H(b)\,H(a) + H(b) = H(a)(H(b)H(a) + H(b)).$$

Hence:

$$0 = H(b)H(a)^2\big(H(b)H(a) + H(b)\big) = \big(H(b)H(a)\big)$$
$$\big[\,H(a)\big(H(b)H(a) + \ H(b)\big)\big] =$$
$$= H(a)(H(b)H(a) + H(b))H(b)H(a) = H(a)H(b)H(a)H(b)H(a)$$
$$= H(a)H(b)H(a) = H(a).$$

Hence $if\ H(a) \neq 0$ in (H, A), then $H(a)^2 = 0$. Let $H(b) \neq 0$ in (H, A).

Let $K(H(b)) = \{H(n)$ in $(H, A): H(n)H(b) = 0\}$. Since $H(b)$ is not in $K(H(b))$, we have $K(H(b)) = 0$. So if $H(a) \neq 0$ and $H(b) \neq 0$ in (H, A), then $H(a)$ $H(b) \neq 0$ and (H, A) is without zero divisors. Let $H(e)$ be any non-zero idempotent. Then for any $H(n)$ in (H, A). We have $(H(n)H(e) - H(n))H(e) = 0$ and hence $H(n)H(e) = H(n)$. Hence, every non-zero idempotent is a right identity. Let $H(e), H(f)$ be any two non-zero idempotents in (H, A), then $H(e) = H(e)H(f) = H(f)H(e) = H(f)$. So (H, A) contains a unique non-zero idempotent, namely $H(e)$.

Let $H(a)$ be any element in $(H, A)^*$, then there exists $H(b)$ in (H, A) such that $H(a)H(b)H(a) = H(a)$. Since $H(b)H(a)$ is a non-zero idempotent, $H(b)H(a) = H(e)$ and $H(a)H(e) = H(a)$. Hence $H(e)$ is a right identity for $(H, A)^*$. Since $H(a)H(b)$ is non-zero idempotent $H(a)H(b) = H(e)$. So $H(b)$ is the right inverse of $H(a)$. Hence $((H, A)^*, \cdot)$ is a group. So (H, A) is a near-field.

Therefore, (H, A) is a soft neutrosophic near-field. Then, by definition, (F, A) is a Smarandache soft neutrosophic near-ring.

(II) Conversely, we assume that (F, A) is a Smarandache soft neutrosophic near-ring. Then, by definition, there exists a proper subset $(H, A) \neq 0$ of (F, A) is a soft neutrosophic near-field. Now we prove that (H, A) has no proper left ideal: since in (H, A), the conditions are trivially hold.

Theorem 4. Every Smarandache soft neutrosophic ideals (L, A) over a neutrosophic near-ring $\langle N \cup I \rangle$ is trivially a Smarandache soft neutrosophic near-ring.

Proof. Let (L, A) be a a Smarandache soft neutrosophic ideal over a neutrosophic near-ring $\langle N \cup I \rangle$. Then, by definition, $F(a)$ is a neutrosophic ideal for all $a \in A$. Since we know that every neutrosophic ideal is a neutrosophic sub near-ring of $\langle N \cup I \rangle$, it follows that $F(a)$ is a neutrosophic sub near-ring of $\langle N \cup I \rangle$. Thus, by definition of Smarandache soft neutrosophic near-ring, we get that (F, A) is a Smarandache soft neutrosophic near-ring.

Acknowledgement

The authors acknowledge their thanks to Professor Florentin Smarandache for valuable suggestions.

References

1. Dheen P., "On Near-Fields", in *Indian J. Pure Appl. Maths.*, 17(3), pp. 322-326, March 1986.
2. Smarandache F., "Special Algebraic Structures", University of New Mexico, http://arxiv.org/ftp/math/papers/0010/0010100.pdf.
3. Smarandache F., Mumtaz A., Munazza N., Shabir M., "Soft Neutrosophic Left Almost Semigroup", SISOM & ACOUSTICS, Bucharest, 22-23 May 2014.
4. Pilz. G., "Near-rings", North Holland, American Research Press, Amsterdam, 1977.
5. Shabir M., Mumtaz A., Munazza N., Smarandache F., in *Neutrosophic Sets and System*, Vol. 1, 2013.
6. Mumtaz A., Smarandache F., Shabir M., Munazza N., in *Neutrosophic Sets and System*, Vol. 3, 2014.
7. Mumtaz A., Smarandache F., Shabir M., Vladareanu L., in *Neutrosophic Sets and System*, Vol. 5, 2014.
8. Kannappa N., Fairosekani B., "On Some Characterization of Smarandache Soft Neutrosophic Near-ring", in *Jamal Academic Research Journal*, Trichirapalli, India, 2015.
9. Kannappa N., Fairosekani B., "Some Equivalent Conditions of Smarandache Soft Neutrosophic Near-ring", accepted for publication in *Neutrosophic Sets and System*, Vol. 8, 2015.
10. Ramaraj T., Kannappa N., "On Bi-ideals of Smarandache Near-rings", in *Acta Sciencia Indica*, Meerut, India, Vol. XXXIM, No. 3, pp. 731-733, 2005.
11. Ramaraj T., Kannappa N., "On Finite Smarandache Near-rings", in *Scientia Magna*, ISSN 1556-6706, Department of Mathematics, North West University, Xi'an, Shaanxi, P.R. China, Vol. I, No. 2, pp. 49-51, 2005.
12. Ramaraj T., Kannappa N., "On Six Equivalent Conditions of Smarandache Near-rings", in *Pure and Applied Mathematic Science*, ISSN 0379-3168, Saharanpur, India, Vol. LXVI, No. 1-2, pp. 87-91, September 2007.