



SMARANDACHE-R-MODULES AND ALGORITHMS

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ABSTRACT

In this paper we introduced Smarandache-2-algebraic structure of R-modules namely Smarandache- R-modules. A Smarandache-2-algebraic structure on a set N means a weak algebraic structure A_0 on N such that there exists a proper subset M of N, which is embedded with a stronger algebraic structure A_1 , stronger algebraic structure means satisfying more axioms, by proper subset one understands a subset different from the empty set, form the unit element if any, from the whole set. We define Smarandache-R-modules and obtain some of its algorithms through on CS-Algebras, on BF-Algebras, and on BRK-Algebras. We refer to Raul Padilla[10].

Keywords : *R-modules, Smarandache-R-modules, CS-Algebras, BF-Algebras, and BRK-Algebras*

INTRODUCTION

In order that new notions are introduced in algebra to better study the congruence in number theory by Florentin Smarandache [1]. By <proper subset> of a set A we consider a set P included in A, and different from A, different from the empty set, and from the unit element in A-if any they rank the algebraic structures using an order relationship:

They say that the algebraic structures $S_1 \ll S_2$ if: both are defined on the same set; all S_1 laws are also S_2 laws; all axioms of an S_1 law are accomplished by the corresponding S_2 law; S_2 law accomplish strictly more axioms than S_1 laws, or S_2 has more laws than S_1 .

For example: Semi group \ll Monoid \ll group \ll ring \ll field, or Semi group \ll commutative semi group, ring \ll unitary, ring etc. They define a General special structure to be a structure SM on a set A, different from a structure SN, such that a proper subset of A is on structure, where $SM \ll SN \ll$.

PRELIMINARIES:

DEFINITION:1.1

A left R- modules A is a system with two binary operations, addition and multiplication, such that

- (i) the elements of A form a group $(A,+)$ under addition,
- (ii) the elements of A form a multiplicative semi-group,
- (iii) $x(y+z) = xy + xz$, for all $x,y,z \in A$
In particular, if A contains a multiplicative semi-group S whose elements generate $(A,+)$ and satisfy
- (iv) $(x+y)s = xs + ys$, for all $x,y \in A$ and $s \in S$, then we say that A is a distributively generated R-modules.

DEFINITION :1.2

A R – Modules $(B,+,\cdot)$ is said to be Smarandache- R-modules whose proper subset A is a S - algebra with respect to same induced operation of B.

DEFINITION :1.3 (Alternative definition for S-R-modules)

If there exists a non-empty set A which is a R-modules such that it superset B of A is a S-algebra with respect to the same induced operation, then B is called Smarandache- R-modules It can also written as S-R-modules.

ALGORITHMS

BE algebras: Ahn.S.S and So.K.S has introduced BE algebras and satisfies the following conditions for all x, y and z in A

- 1) $x * x = 1$
- 2) $x * 1 = 1$
- 3) $1 * x = x$
- 4) $x * (y * z) = y * (x * z)$.

According to Raul Padilla (4,Thm.1.60d) R is a module, Now by definition, R is a Smarandache-R-modules

ALGORITHMS:I

- Step 1: Consider a R-module R
- Step 2: Let x, y and z in A
- Step 3: Choose $x * y \leq y * x$
- Step 4: Choose $x \leq y$ implies $y * x \leq x * y$
- Step 5: Verify $x * (y * z) = (x * y) * (x * z)$
- Step 6: If step 5 is true then R is a Smarandache-R-module

ALGORITHM:II

- Step1: Consider a R-module R
- Step 2: Let x, y and z in A
- Step 3: Choose $(y * x) * y \leq x * y$.
- Step 4: Choose $x * (x * y) = x * y$.
- tep 5: Verify $x * (y * z) = (x * y) * (x * z)$,

Step 6: If step 5 is true then by definition, we write R is a Smarandache-R-module

BRK ALGEBRA

BRK algebras: Imai and Iseki has introduced BRK algebras and satisfies the following conditions

- 1) $(x * y) * (x * z) \leq (z * y)$
- 2) $x * (x * y) \leq y$
- 3) $x \leq x$
- 4) $x \leq y$ and $y \leq x$ imply $x = y$
- 5) $x \leq 0$ implies $x = 0$, where $x \leq y$ is defined by $x * y = 0$, for all $x, y, z \in X$.

According to Raul Padilla (4,Thm.1.60d) R is a module, now by definition, R is a Smarandache-R-modules.

ALGORITHMS III

- Step1: Consider a R-module R
 Step 2: Let x, y in A
 Step 3: Let $x * x = 0$
 Step 4: Choose $x * y = 0$
 Step 5: Choose $y * x = 0$
 Step 6: Verify that $0 * x = 0 * y$.
 Step 7: If step 6 is true then we write R is a Smarandache-R-module.

ALGORITHM:IV

- Step1: Consider a R-module R
 Step 2: Let a, b and c in A
 Step 3: Choose $a * b$
 Step 4: Choose $a * c$
 Step 5: Verify $a * b = a * c$ then $0 * b = 0 * c$
 Step 6: If step 5 is true then R is a Smarandache-R-module

BF-ALGEBRA

According to Andrzej Walendziak has introduced on BF algebras for the following conditions

- a) $0 * (x * y) = y * x$.
 b) $0 * (0 * x) = x$
 c) if $0 * x = 0 * y$, then $x = y$
 d) if $x * y = 0$, then $y * x = 0$
 for any $x, y \in A$;

According to Raul Padilla (4,Thm.1.60d) R is a module, now by definition, R is a Smarandache-R-modules
 Given R be a Smarandache-R-module, if there exists a proper subset A of R in which satisfies the following statements

- (a) A is a BF-algebra;
 (b) $x = [x * (0 * y)] * y$ for all $x, y \in A$;
 (c) $x = y * [(0 * x) * (0 * y)]$ for all $x, y \in A$.

According to Raul Padilla (4,Thm.1.60d) R is a module, now by definition, R is a Smarandache-R-modules

ALGORITHM:V

- Step1: Consider a R-module R
 Step 2: Let x, y in A
 Step 3: Choose $x * y = 0$
 Step 4: Choose $y * x = 0$
 Step 5: Check $x = 0 * (0 * x) = x * 0$
 Step 6: Verify that $x = y$
 Step 7: If step 6 is true then we write R is a Smarandache-R-module

ALGORITHM:VI

Step1: Consider a R-module R

Step 2: Let x, y in A

Step 3: Choose $x * y = 0$

Step 4: Choose $y * x = 0$

Step 5: Check $x = y * (0 * x) * (0 * y)$

Step 6: Verify that $x = y$

Step 7: If step 6 is true then we write R is a Smarandache-R-module

Given R be a smarandache-R-module, if there exists a proper subset A of R in which *BG-algebra* satisfies the following statements

(a) A is a *BG-algebra*;

(b) For $x, y \in A$, $x * y = 0$ implies $x = y$;

(c) The right cancellation law holds in A. i.e., If $x * y = z * y$, then $x = z$ for any $x, y, z \in A$;

(d) The left cancellation law holds in A. i.e., if $y * x = y * z$, then $x = z$ for any $x, y, z \in A$.

According to Raul Padilla (4,Thm.1.60d) R is a module, now by definition, R is a Smarandache-R – Modules

ALGORITHM:VII

Step1: Consider a R-module R

Step 2: Let x, y in A

Step 3: Choose $x * y = 0$

Step 4: Choose $y * x = 0$

Step 5: Check $x = (x * y) * (0 * y) = 0 * (0 * y)$

Step 6: Verify that $x = y$

Step 7: If step 6 is true then we write R is a Smarandache-R-module

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