Topics in Recreational Mathematics 3/2015

Charles Ashbacher

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SMARANDACHE'S ORTHIC THEOREM

Edited by Prof. Ion Patrascu

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Abstract

We present the Smarandache's Orthic Theorem in the geometry of the triangle.

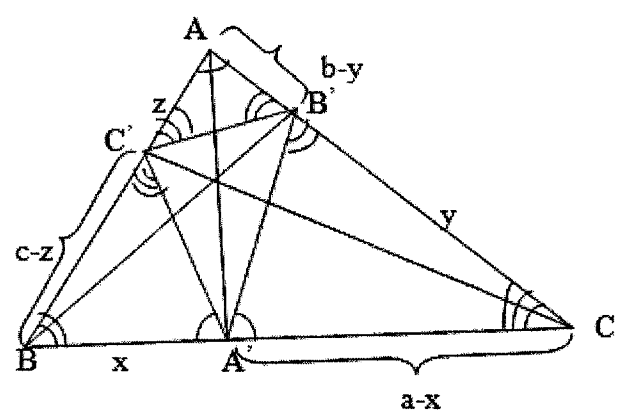
Smarandache's Orthic Theorem

Given a triangle ABC whose angles are all acute (acute triangle), we consider $A'B'C'$, the triangle formed by the legs of its altitudes.

What are the conditions where the expression

$$\|A'B'\|\cdot\|B'C'\|+\|B'C'\|\cdot\|C'A'\|+\|C'A'\|\cdot\|A'B'\|$$

is maximum?



Solution:

We have

$$\Delta ABC \sim \Delta A'B'C' \sim \Delta AB'C \sim \Delta A'BC' \qquad (1)$$

We note

$$\|BA'\| = x,\ \|CB'\| = y,\ \|AC'\| = z.$$

It follows that

$$\|A'C\| = a - x,\ \|B'A\| = b - y,\ \|C'B\| = c - z$$

$$\widehat{BAC} = B'\widehat{A'}C = B\widehat{A'}C';\ \ \widehat{ABC} = A\widehat{B'}C' = A'\widehat{B'}C;\ \ B\widehat{CA} = B\widehat{C'}A' = B'\widehat{C'}A$$

From these equalities we have the relation (1)

$$\Delta A'BC' \sim \Delta A'B'C \Rightarrow \frac{\|A'C'\|}{a-x} = \frac{x}{\|A'B'\|} \qquad (2)$$

$$\Delta A'B'C \sim \Delta AB'C' \Rightarrow \frac{\|A'C'\|}{z} = \frac{c-z}{\|B'C'\|} \qquad (3)$$

$$\Delta AB'C' \sim \Delta A'B'C \Rightarrow \frac{\|B'C'\|}{y} = \frac{b-y}{\|A'B'\|} \qquad (4)$$

From (2), (3) and (4) we observe that the sum of the products from the problem is equal to:

$$x(a-x) + y(b-y) + z(c-z) = \frac{1}{4}(a^2+b^2+c^2) - \left(x - \frac{a}{2}\right)^2 - \left(y - \frac{b}{2}\right)^2 - \left(z - \frac{c}{2}\right)^2$$

which will reach its maximum when $x=\frac{a}{2}$, $y=\frac{b}{2}$, $z=\frac{c}{2}$, that is when the altitudes' legs are in the middle of the sides, therefore when ΔABC is equilateral. The maximum of the expression is

$$\frac{1}{4}\left(a^2+b^2+c^2\right).$$

Conclusion (Smarandache's Orthic Theorem)

If we note the lengths of the sides of the triangle ΔABC by $\|AB\| = c$, $\|BC\| = a$, $\|CA\| = b$, and the lengths of the sides of its orthic triangle $\Delta A`B`C`$ by $\|A`B`\| = c`$, $\|B`C`\| = a`$, $\|C`A`\| = b`$, then we have proved that:

$$4(a`b` + b`c` + c`a`) \le a^2 + b^2 + c^2.$$

Open Problems related to Smarandache's Orthic Theorem:

1. Generalize this problem to polygons. Let $A_1A_2\ldots A_m$ be a polygon and P a point inside it. From P we draw perpendiculars on each side A_iA_{i+1} of the polygon and we note by Ai' the intersection between the perpendicular and the side A_iA_{i+1}. A pedal polygon $A_1'A_2'\ldots A_m'$ is formed. What properties does this pedal polygon have?
2. Generalize this problem to polyhedrons. Let $A_1A_2\ldots A_n$ be a poliyhedron and P a point inside it. From P we draw perpendiculars on each polyhedron face F_i and we note by Ai' the intersection between the perpendicular and the side F_i. A pedal polyhedron $A_1'A_2'\ldots A_p'$ is formed, where p is the number of polyhedron's faces. What properties does this pedal polyhedron have?

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