SOME NOTIONS ON LEAST COMMON MULTIPLES

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In [1] Smarandache LCM Sequence has been defined as T_n = LCM (1 to n) = LCM of all the natural numbers up to n.
The SLS is
1, 2, 6, 60, 60, 420, 840, 2520, 2520, . . .
We denote the LCM of a set of numbers a, b, c, d, etc. as LCM(a,b,c,d)
We have the well known result that n! divides the product of any set of n consecutive numbers. Using this idea we define **Smarandache LCM Ratio Sequence** of the r^{th} kind as **SLRS(r)**

The n^{th} term ${}_rT_n$ =LCM (n , n+1, n+2, . . .n+r-1) /LCM (1, 2, 3, 4, . . . r)
As per our definition we get SLRS(1) as
1 , 2, 3, 4, 5, . . . ${}_1T_n$ (= n.)
we get SLRS(2) as
1, 3, 6, 10, . . . ${}_2T_n$ = n(n+1)/2 (triangular numbers).
we get SLRS(3) as
LCM (1, 2, 3)/ LCM (1, 2, 3), LCM (2, 3, 4)/ LCM (1, 2, 3) , LCM (3, 4, 5,)/ LCM (1, 2, 3)
LCM (4, 5, 6)/ LCM (1, 2, 3) LCM (5, 6, 7)/ LCM (1, 2, 3)

═ 1 , 2 , 10 , 10 , 35 . . . similarly we have
SLRS(4) ═ 1, 5 , 5, 35, 70, 42, 210 , . . .

It can be noticed that for r > 2 the terms do not follow any visible patterns.
OPEN PROBLEM : To explore for patterns/ find reduction formullae for ${}_rT_n$.

Definition: Like ${}^nC_r$, the combination of r out of n given objects , We define a new term ${}^nL_r$
As
${}^nL_r$ = LCM (n, n-1, n-2, . . . n-r+1) / LCM (1, 2 , 3 , . . .r)
(Numeretor is the LCM of n , n-1 , n-2, . . .n-r+1 and the denominator is the LCM of first natural numbers.)
we get ${}^1L_0$ =1, ${}^1L_1$ =1, ${}^2L_0$ =1, ${}^2L_1$ =2, ${}^2L_2$ =2 etc. define ${}^0L_0$ =1
we get the following triangle:

1

1 ,1

1 ,2, 1

1 ,3 ,3, 1

1, 4 ,6, 2, 1

1, 5, 10,, 10 5, 1

1, 6, 15, 10, 5, 1, 1

1, 7, 21, 35, 35, 7,7, 1

1, 8, 28, 28, 70, 14, 14, 2, 1

1, 9, 36, 84, 42, 42, 42, 6, 3, 1

1, 10, 45, 60, 210, 42, 42, 6, 3,1, 1

Let this traingle be called **Smarandache AMAR LCM Triangle**

Note: As r! divides the product of r consecutive integers so does the LCM (1, 2, 3, ... r) divide the LCM of any r consecutive numbers Hence we get only integers as the members of the above triangle.

Following properties of **Smarandache AMAR LCM Triangle** are noticable.

1. The first column and the leading diagonal elements are all unity.
2. The k^{th} column is nothing but the SLRS(k).
3. The first four rows are the same as that of the Pascal's Triangle.
4. II^{nd} column contains natural numbers.
5. III^{rd} column elements are the triangular numbers.
6. If p is a prime then p divides all the terms of the p^{th} row except the first and the last which are unity. In other words Σ p^{th} **row** $\equiv$ **2 (mod p)**

Some keen observation opens up vistas of challenging problems:

In the 9^{th} row 42 appears at three consecutive places.

OPEN PROBLEM:

(1) Can there be arbitrarily large lengths of equal values appear in a row.?

2. To find the sum of a row.

3. Explore for congruence properties for composite n.

SMARANDACHE LCM FUNCTION:

The Smarandache function S(n) is defined as S(n) = k where is the smallest integer such that n divies k! . Here we define another function as follows:

Smarandache Lcm Function denoted by S_L(n) = k , where k is the smallest integer such that n divide LCM (1, 2, ,3 . . . k).

Let $n = p_1^{a1} p_2^{a2} p_3^{a3} \ldots p_r^{ar}$

Let p_m^{am} be the largest divisor of n with only one prime factor, then

We have $S_L(n) = p_m^{am}$

If n =k! then S(n) = k and S_L(**n) > k**

If n is a prime then we have S_L(n) = S(n) = n

Clearly S_L(n) $\geq$ S(n) the equality holding good for n a prime or n = 4 , n=12.

Also S_L(**n) = n if n is a prime power. (n = p^a)**

OPEN PROBLEMS:

(1) **Are there numbers n >12 for which S_L(n)** = S(n).

(2) **Are there numbers n for which S_L(n)** = S(n) $\neq$ n

REFERENCE:

[1] Amarnath Murthy, Some new smarandache type sequences, partitions and set, SNJ, VOL 1-2-3 , 2000.