THE PRIMES p WITH lg (p) =1

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Abstract. In this paper we prove that if $p=\overline{a_k \ldots a_1 a_0}$ is a prime satisfying $p>10$ and $lg(p)=1$, then $a_k=\ldots=a_1=a_0=1$ and $k+1$ is a prime.

Let $n=\overline{a_k \ldots a_1 a_0}$ be a decimal integer. Then the number of distinct digits of n is called the length of Smarandache generalized period of n and denoted by lg(n) (see [1, Notion 114]). In this paper we prove the following result.

Theorem. If $p=\overline{a_k \ldots a_1 a_0}$ is a prime satisfying $p>10$ and $lg(p)=1$, then we have $a_k=\ldots=a_1=a_0=1$ and $k+1$ is a prime.

Proof. Since $lg(p)=1$, we have $a_k=\ldots=a_1=a_0$. Let $a_0=a$, where a is an integer with $0<a\leq 9$. Then we have $a|p$. Since p is a prime and $p>10$, we get $a=1$ and

(1) $$p=\overline{1 \ldots 1}1=10^k+\ldots+10+1=\frac{10^{k+1}-1}{10-1},$$

where k is a positive integer. Since $k+1>1$, if $k+1$ is not a prime, then $k+1$ has a prime factor q such that $(k+1)/q>1$.

Hence, we see from (1) that

$$p=\frac{10^{k+1}-1}{10-1}=\left(\frac{10^{q}-1}{10-1}\right)\left(\frac{10^{k+1}-1}{10^{q}-1}\right)=(10^{q-1}+\ldots+10+1)(10^{(k+1)/q-1}+\ldots+10^{q}+1).$$

It implies that p is not a prime, a contradiction. Thus, if p is a prime, then k+1 must be a prime. The theorem is proved.

Reference

1. Dumitrescu and Seleacu, Some Notions and Questions In Number Theory, Erhus Univ. Press, Glendale, 1994