ON THE THIRD SMARANDACHE CONJECTURE ABOUT PRIMES

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Abstract . In this paper we basically verify the third Smarandache conjecture on prime.

Key words . Smarandache third conjecture, prime , gap.

For any positive integre n , let $P(n)$ be the n-th prime. Let k be a positive integer with $k>1$, and let

(1) $$c(n,k)=(P(n+1))^{1/k}-(P(n))^{1/k} .$$

Smarandache [3] has been conjectured that

(2) $$C(n,k)< \frac{2}{k} .$$

In [2], Russo verified this conjecture for $P(n)<2^{25}$ and $2 \leqslant k \leqslant 10$. In this paper we prove a general result as follows .

Theorem . If $k>2$ and $n>C$, where C is an effectively computable absolute constant, then the inequality (2) holds.

Proof . Since $k>2$, we get from (1) that

$$C(n,k)= \frac{P(n+1)-P(n)}{(P(n+1))^{(k-1)/k}+(P(n+1))^{(k-2)/k}(P(n))^{1/k}+...+(P(n))^{(k-1)/k}}$$

(3) $$< \frac{P(n+1)-P(n)}{k(P(n))^{(k-1)/k}} \leqslant \frac{2}{k}\left(\frac{(P(n+1)-P(n)}{2(P(n)^{2/3}} \right) .$$

By the result of [1] , we have

(4) $$P(n+1)-P(n))<C(a)(P(n))^{11/20+a} ,$$

for any positive number a, where $C(a)$ is an effectively computable constant depending on a. Put a=1/20 . Since $k \geq 3$ and $(k-1)/k \geq 2/3$, we see from (3) and (4) that

(5) $$C(n,k) < \frac{2}{k}\left(\frac{C(1/20)}{2(P(n))^{1/15}}\right) .$$

Since $C(1/20)$ is an effectively computable absolute constant, if $n>C$, then $2(P(n))^{1/15}>C(1/20)$. Thus, by (5), the inequality (2) holds. The theorem is proved.

References

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