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ORIGINAL ARTICLE

Truncated variable algorithm using DUS-neutrosophic Weibull distribution

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Abstract

The existing truncated variable method to generate random variate cannot be applied when indeterminacy is presented in either the parameters or observations. This paper addresses the truncated variable simulation under the indeterminate environment. The truncated variable simulation method will be introduced using the DUS-neutrosophic Weibull distribution. The algorithm to generate random variate will be presented and applied in random variate generation. Extensive simulation tables for various values of indeterminacy and truncated variables are presented. The proposed study for other neutrosophic statistical distribution can be extended as future research.

Keywords Uncertainty · Statistics · Simulation · Random variate · Algorithm · Neutrosophy · Distribution

Introduction

Sometimes, the algorithms are run to generate the random variate between the specific intervals rather. The partition of the original interval of the underlying distribution is done to the behavior of random variate between the specified intervals. Another reason to partition or truncated the original random variable is due to the highly reliable product. The truncation of the variable is done to save time, and cost to check the randomness in the variate. The truncated variable method utilizes the specified interval to generate the random variate other than the original range of the interval associated with the underlying distribution. Michael et al. [1] worked on the random generation method with multiple roots and transformations. Kachitvichyanukul and Schmeiser [2] present a method to generate random variate from the hypergeometric distribution. Hörmann [3] used the binomial distribution in generating the random variate. Kundu and Gupta [4] presented a method to generate random variate from the generalized exponential distribution. Bergman [5] generated the random variate from the new statistical distribution. Mohazzabi and Connolly [6] applied the normal distribution in random number generation. Qu et al. [7] worked on the generation of random variate for the gamma and exponential distributions. For more details, the reader may refer to [8–10].

Smarandache [11] presented work on neutrosophic logic and showed that neutrosophic logic is more efficient than fuzzy logic. Neutrosophic logic provides information about the measure of indeterminacy while fuzzy logic does not provides such information, see [11]. More information about fuzzy logic can be seen in [12–15]. Smarandache [16] introduced neutrosophic statistics (NS) to analyze and interpret the results from the data having neutrosophic numbers. Neutrosophic statistics was found to be more efficient than classical statistics in terms of information and flexibility. Neutrosophic statistics is the generalization of classical statistics and can be applied when imprecise observations are available in the data. Neutrosophic statistics reduce to classical statistics when no imprecise observations in the data. Chen et al. [17], Chen et al. [18] and Aslam [19] showed that NS is more efficient than classical statistics (CS). Recently, Nayana et al. [20] introduced the DUS-neutrosophic Weibull distribution and compared the performance with the DUS-Weibull distribution under CS. More information on NS can be seen in [21–23]. The importance of neutrosophic theory and the difference between neutrosophic theory and fuzzy set theory can be seen in [24–28]. Recently, Smarandache [29] showed the efficiency of neutrosophic statistics over interval statistics and classical multivariate statistics. More applications of neutrosophic logic and sets can be seen in [30–33].

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The existing truncated variable random variate generate method was introduced under CS. The existing simulation method cannot be applied in an indeterminate environment. By exploring the literature and according to the best of the author's knowledge, there is no work on the algorithm for generating random variate from the DUS-neutrosophic Weibull distribution. To fill this gap, in this paper, the DUS-neutrosophic Weibull distribution will be applied in designing the truncated variable simulation method. The main contribution and novelty of the work are to introduce the algorithm for generating random variate using various values of shape and scale parameters of the DUS-neutrosophic Weibull distribution. We will introduce the algorithm to generate random variate from the DUS-neutrosophic Weibull distribution. The rest of the paper is organized as follows: a brief introduction to DUS-neutrosophic Weibull distribution will be given in the next section. In the subsequent section, the neutrosophic truncated variable method to generate random variate will be introduced followed by which the simulation study and comparative study are given, respectively. The concluding remarks are given in the last section.

Preliminaries

Let $X_N \epsilon [X_L, X_U]$ be a neutrosophic random variable and suppose that $x_1 = x_{1L} + x_{1U} I_N$ and $x_2 = x_{2L} + x_{2U} I_N$ be two neutrosophic numbers (NNs), where $I_N \epsilon [I_L, I_U]$ is the measure of indeterminacy. By following [17] and [34], the basic operations of NNs:

$$x_1 + x_2 = (x_{1L} + x_{2L}) + (x_{1U} + x_{2U}) I_N = [x_{1L} + x_{2L} + x_{1L} I_L + x_{2L} I_L, x_{1L} + x_{2L} + x_{1L} I_U + x_{2L} I_U]$$

$$x_1 - x_2 = x_{1L} - x_{2L} + (x_{1U} - x_{2U}) I_N = [x_{1L} - x_{2L} + x_{1U} I_L - x_{2U} I_L, x_{1L} - x_{2L} + x_{1U} I_U - x_{2U} I_U]$$

$$x_1 \times x_2 = x_{1L} x_{2L} + (x_{1L} x_{2U} + x_{2L} x_{1U}) I_N + (x_{1U} x_{2U}) I_N^2$$

$$= \begin{bmatrix} \min\begin{pmatrix} (x_{1L} + x_{1U} I_L)(x_{2L} + x_{2U} I_L), (x_{1L} + x_{1U} I_L)(x_{2L} + x_{2U} I_U), \\ (x_{1L} + x_{1U} I_U)(x_{2L} + x_{2U} I_L), (x_{1L} + x_{1U} I_U)(x_{2L} + x_{2U} I_U) \end{pmatrix} \\ \max\begin{pmatrix} (x_{1L} + x_{1U} I_L)(x_{2L} + x_{2U} I_L), (x_{1L} + x_{1U} I_L)(x_{2L} + x_{2U} I_U), \\ (x_{1L} + x_{1U} I_U)(x_{2L} + x_{2U} I_L), (x_{1L} + x_{1U} I_U)(x_{2L} + x_{2U} I_U) \end{pmatrix} \end{bmatrix}$$

$$\frac{x_1}{x_2} = \frac{x_{1L} + x_{1U} I_N}{x_{2L} + x_{2U} I_N} = \frac{[x_{1L} + x_{1U} I_L, x_{1L} + x_{1U} I_U]}{[x_{2L} + x_{2U} I_L, x_{2L} + x_{2U} I_U]}$$

$$= \begin{bmatrix} \min\left(\frac{x_{1L}+x_{1U} I_L}{x_{2L}+x_{2U} I_U}, \frac{x_{1L}+x_{1U} I_L}{x_{2L}+x_{2U} I_L}, \frac{x_{1L}+x_{1U} I_U}{x_{2L}+x_{2U} I_U}, \frac{x_{1L}+x_{1U} I_U}{x_{2L}+x_{2U} I_L}\right), \\ \max\left(\frac{x_{1L}+x_{1U} I_L}{x_{2L}+x_{2U} I_U}, \frac{x_{1L}+x_{1U} I_L}{x_{2L}+x_{2U} I_L}, \frac{x_{1L}+x_{1U} I_U}{x_{2L}+x_{2U} I_U}, \frac{x_{1L}+x_{1U} I_U}{x_{2L}+x_{2U} I_L}\right) \end{bmatrix}.$$

The explanations of these operations with examples can be seen in [17, 34].

DUS-neutrosophic Weibull distribution

Suppose that $x_N \epsilon [x_L, x_U] > 0$ is neutrosophic random variable follows the DUS-neutrosophic Weibull distribution. The neutrosophic form of $x_N \epsilon [x_L, x_U]$ is expressed as $x_N = x_L + x_U I_N$; $I_N \epsilon [I_L, I_U]$, where x_L is a random variable presents classical statistics and $x_U I_N$ is the indeterminate part and $I_N \epsilon [I_L, I_U]$ is the measure of indeterminacy. Nayana et al. [20] introduced the DUS-neutrosophic Weibull distribution with the following neutrosophic probability density (npdf) having shape parameter $\beta > 0$ and scale parameter $\alpha > 0$.

$$f(x_N) = \frac{\beta}{\alpha^\beta} x_N^{\beta-1} \exp\left(-\left(\frac{x_N}{\alpha}\right)^\beta\right)(1 + I_N);\ I_N \epsilon [I_L, I_U]. \tag{1}$$

The corresponding neutrosophic cumulative distribution function (ncdf) is expressed as follows:

$$F(x_N) = \left(1 - \exp\left(-\left(\frac{x_N}{\alpha}\right)^\beta\right)\right)(1 + I_N);\ x_N > 0, \beta > 0, \alpha > 0, I_N \epsilon [I_L, I_U]. \tag{2}$$

Note that $I_N \epsilon [I_L, I_U]$ denotes the measure of the indeterminacy. The npdf and ncdf reduce to pdf and cdf DUS-Weibull distribution under classical statistics when $I_N = 0$.

Neutrosophic truncated variable method

In this section, we will introduce, a neutrosophic truncated variable method to generate random variate from the DUS-Neutrosophic Weibull distribution by following [35]. In the truncated method, sometimes, the decision-makers are interested to generate random variate from the specified interval rather than using the original range of the variables. For example, the neutrosophic variable x_N from the DUS-Neutrosophic Weibull distribution has ranged from 0 to ∞. Suppose that the decision-makers are interested to generate random variate from c_N to d_N. Note that limits c_N and d_N lie within the original limits of DUS-Neutrosophic Weibull distribution. The ncdf is given in Eq. (2) for lower truncated value is given by

$$F(c_N) = \left(1 - \exp\left(-\left(\frac{c_N}{\alpha}\right)^\beta\right)\right)(1 + I_N);\ c_N > 0,\ \beta > 0,\ \alpha > 0,\ I_N \epsilon [I_L, I_U]. \tag{3}$$

The ncdf is given in Eq. (2) for upper truncated value is given by

$$F(d_N) = \left(1 - \exp\left(-\left(\frac{d_N}{\alpha}\right)^{\beta}\right)\right)(1 + I_N);\ d_N > 0,\ \beta > 0,\ \alpha > 0,\ I_N \epsilon [I_L,\ I_U]. \quad (4)$$

The new density of DUS-Neutrosophic Weibull distribution using Eq. (1), Eqs. (3) and (4) when $a \leq c \leq x_N \leq d \leq b$, where $a = 0$ and $b = \infty$ is given by

$$g(x_N) = \frac{\frac{\beta}{\alpha^{\beta}} x_N^{\beta-1} \exp\left(-\left(\frac{x_N}{\alpha}\right)^{\beta}\right)(1 + I_N)}{\left[\left(1 - \exp\left(-\left(\frac{d_N}{\alpha}\right)^{\beta}\right)\right)(1 + I_N)\right] - \left[\left(1 - \exp\left(-\left(\frac{c_N}{\alpha}\right)^{\beta}\right)\right)(1 + I_N)\right]};\ I_N \epsilon [I_L,\ I_U]. \quad (5)$$

The value of v_N based on Eqs. (3) and (4) can be calculated as follows:

$$v_N = \left[\left(1 - \exp\left(-\left(\frac{c_N}{\alpha}\right)^{\beta}\right)\right)(1 + I_N)\right] + u_N\left[\left[\left(1 - \exp\left(-\left(\frac{d_N}{\alpha}\right)^{\beta}\right)\right)(1 + I_N)\right] - \left[\left(1 - \exp\left(-\left(\frac{c_N}{\alpha}\right)^{\beta}\right)\right)(1 + I_N)\right]\right], \quad (6)$$

where u_N is a uniform random variate from intervals 0 and 1.

Algorithm

To find the random variate using the truncated variable simulation under indeterminacy, the following routine can be run. Note that the following routine does not need $g(x_N)$ described in Eq. (5).

Step-1: Specify the scale parameter $\alpha > 0$ and shape parameter $\beta > 0$ of the DUS-Weibull distribution.

Step-2: Specify the values of I_N

Step-3: Generate a uniform variate, say u_N between 0 and 1.

Step-4: Compute $F(c_N) = \left(1 - \exp\left(-\left(\frac{c_N}{\alpha}\right)^{\beta}\right)\right)(1 + I_N)$ and $F(d_N) = \left(1 - \exp\left(-\left(\frac{d_N}{\alpha}\right)^{\beta}\right)\right)(1 + I_N)$.

Step-5: Compute the values of v_N using Eq. (6)

Step-6: Set $v_N = F(x_N)$

Step-7: Record the random variate such that $F_N^{-1}(v_N)$

Step-8: Return x_N

The proposed algorithm is an extension of an algorithm given in [35]. The proposed algorithm under indeterminacy reduces to [35] algorithm when no indeterminacy is found. The proposed algorithm is shown in Fig. 1.

Simulation study

In this section, we will present the simulation study using the proposed algorithm for the DUS-Neutrosophic Weibull distribution. We will consider various values of u_N and I_N. We also used various truncated variable values c_N and d_N to generate random variate from the DUS-Neutrosophic Weibull distribution. Using the above-mentioned algorithm, Table 1 is presented for random variate when $\alpha = 0.5$ and $\beta = 3.0$, Table 2 for random variate when $\alpha = 2$ and $\beta = 2$. Table 3 for random variate when $\alpha = 3$ and $\beta = 5$. Similar tables can be constructed for other values of the parameters with the R codes that are given in the supplementary file. From Tables 1, 2, 3, it can be noted that for other the same parameters, there are no specific trends in random variate for various values of I_N. From Tables 2 and 3, it can be noted the values of x_N increase when $\alpha = 2$ and $\beta = 2.0$ increase to $\alpha = 3$ and $\beta = 5.0$. For example, when c_N=0.2, d_N=1 and I_N=0.1, the value of x_N from Table 2 is 0.7036 and the value of x_N from Table 3 is 0.7753. From the example, it is clear that there is an increasing trend in random variate x_N when $\alpha > 1$ and $\beta > 1$.

Comparative study

To see the behavior of random variate, a simulated data of random variate x_N is generated using $\alpha = 3$, $\beta = 5$. The histograms at different values of indeterminacy are presented in Figs. 2, 3, 4, and 5. Figures 2, 3, 4, and 5 are presented for various values of $\alpha = 3$, $\beta = 5$ and various truncated variable values. Figure 2 shows the histograms for $I_N = 0$ to $I_N = 1$ when the variable is truncated at $c_N = 0$ and $d_N = 1$. Figure 3 shows the histograms for $I_N = 0$ to $I_N = 1$ when the variable is truncated at $c_N = 0.2$ and $d_N = 1$. Figure 4 shows the histograms for $I_N = 0$ to $I_N = 1$ when the variable is truncated at $c_N = 0.4$ and $d_N = 1$. Figure 5 shows the histograms for $I_N = 0$ to $I_N = 1$ when the variable is truncated at $c_N = 0.5$ and $d_N = 1$. The histograms show that the low frequencies occur at lower tail whereas the higher frequencies occur at the upper tail of truncated variable. From

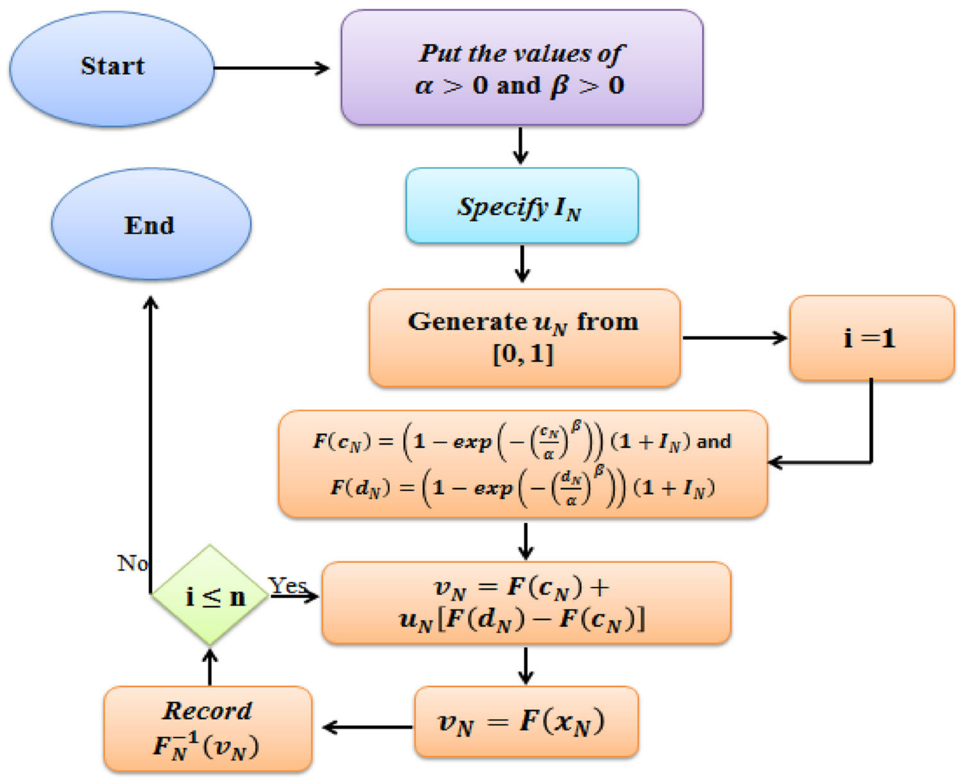

Fig. 1 The algorithm of the proposed method

Table 1 Random variate when $\alpha = 0.5$ and $\beta = 3.0$

I_N	Truncated variable values c_N and d_N							
	0, 1		0.2, 1		0.4, 1		0.5, 1	
	u_N	x_N	u_N	x_N	u_N	x_N	u_N	x_N
0	0.6836	0.5238	0.7029	0.5424	0.1103	0.4284	0.0704	0.5119
0.1	0.8729	0.6363	0.0776	0.2625	0.5718	0.5539	0.6466	0.6340
0.2	0.6901	0.5270	0.2349	0.3461	0.7907	0.6376	0.1021	0.5173
0.3	0.1159	0.2488	0.9595	0.7415	0.6383	0.5759	0.9558	0.8002
0.4	0.1950	0.3004	0.7951	0.5906		0.4800	0.7482	0.6672
0.5	0.4612	0.4259	0.3866	0.4103	0.6927	0.5957	0.1502	0.5258
0.6	0.2035	0.3052	0.9763	0.7796	0.6746	0.5889	0.3076	0.5549
0.7	0.5908	0.4815	0.0953	0.2737	0.7582	0.6225	0.9079	0.7501
0.8	0.3739	0.3882	0.5819	0.4890	0.8413	0.6647	0.1780	0.5307
0.9	0.1413	0.2670	0.4723	0.4446	0.2405	0.4616	0.2762	0.5489
1	0.0962	0.2329	0.3023	0.3756	0.6798	0.5908	0.4049	0.5747

Figs. 2, 3, 4, it can be noted that the behavior of random variate x_N seems to be negatively skewed. From Fig. 5, it is observed that the distribution of random variate x_N tends to be uniform distribution. From Figs. 2, 3, 4, 5, it is concluded that as the interval reduces between the truncated values c_N and d_N, the trend of the random variable x_N move towards the uniform distribution.

Merits and demerits

The proposed method of simulation can be applied in those situations where it is difficult to gather the original data due to complexity or uncertainty. The data obtained from the proposed simulation method can be applied to testing the lifetime of the product. In addition, the proposed simulated data can be applied to reliability studies. This random variate can be

Table 2 Random variate when $\alpha = 2$ and $\beta = 2.0$

I_N	Truncated variable values c_N and d_N							
	0, 1		0.2, 1		0.4, 1		0.5, 1	
	u_N	x_N	u_N	x_N	u_N	x_N	u_N	x_N
0	0.4396	0.6397	0.2032	0.4662	0.1142	0.4975	0.3062	0.6820
0.1	0.5592	0.7268	0.5040	0.7036	0.4918	0.7424	0.4391	0.7498
0.2	0.7856	0.8738	0.8393	0.9107	0.0758	0.4667	0.1452	0.5921
0.3	0.0739	0.2567	0.4123	0.6390	0.1231	0.5044	0.8931	0.9554
0.4	0.6045	0.7577	0.1919	0.4553	0.9052	0.9552	0.1427	0.5906
0.5	0.0074	0.0810	0.3144	0.5637	0.4391	0.7123	0.3106	0.6843
0.6	0.6734	0.8032	0.1532	0.4158	0.5037	0.7491	0.1445	0.5917
0.7	0.6316	0.7759	0.5286	0.7202	0.3198	0.6403	0.1625	0.6022
0.8	0.5923	0.7495	0.9252	0.9590	0.7339	0.8711	0.3655	0.7128
0.9	0.7456	0.8491	0.4533	0.6685	0.4965	0.7450	0.2441	0.6485
1	0.3586	0.5750	0.2377	0.4982	0.3847	0.6802	0.4819	0.7707

Table 3 Random variate when $\alpha = 3$ and $\beta = 5.0$

I_N	Truncated variable values c_N and d_N							
	0, 1		0.2, 1		0.4, 1		0.5, 1	
	u_N	x_N	u_N	x_N	u_N	x_N	u_N	x_N
0	0.3078	0.7898	0.8822	0.9752	0.5383	0.8849	0.9542	0.9909
0.1	0.2577	0.7622	0.2804	0.7753	0.7490	0.9444	0.6953	0.9323
0.2	0.5523	0.8879	0.3985	0.8318	0.4201	0.8429	0.8895	0.9776
0.3	0.0564	0.5624	0.7626	0.9472	0.1714	0.7094	0.1804	0.7289
0.4	0.4685	0.8591	0.6690	0.9227	0.7703	0.9496	0.6294	0.9148
0.5	0.4838	0.8646	0.2046	0.7280	0.8820	0.9754	0.9896	0.9980
0.6	0.8124	0.9592	0.3575	0.8139	0.5491	0.8883	0.1303	0.6907
0.7	0.3703	0.8196	0.3595	0.8148	0.2777	0.7778	0.3307	0.8111
0.8	0.5466	0.8860	0.6903	0.9285	0.4883	0.8681	0.8651	0.9723
0.9	0.1703	0.7016	0.5358	0.8826	0.9285	0.9854	0.7776	0.9525
1	0.6250	0.9101	0.7108	0.9339	0.3487	0.8129	0.8273	0.9640

used for testing hypotheses in statistics. The proposed simulation method is simple and easy to apply for the generation of random variate under indeterminacy. The proposed simulation method has the limitation that it cannot be applied when the data follows the normal distribution. Another limitation is that the proposed method of simulation can be applied in uncertain environment.

Concluding remarks

The main objective was to introduce the truncated variable algorithm for DUS-Neutrosophic Weibull distribution. A simulation method for DUS-Neutrosophic Weibull distribution was presented in the paper. An algorithm to perform the proposed truncated variable simulation method was pre-

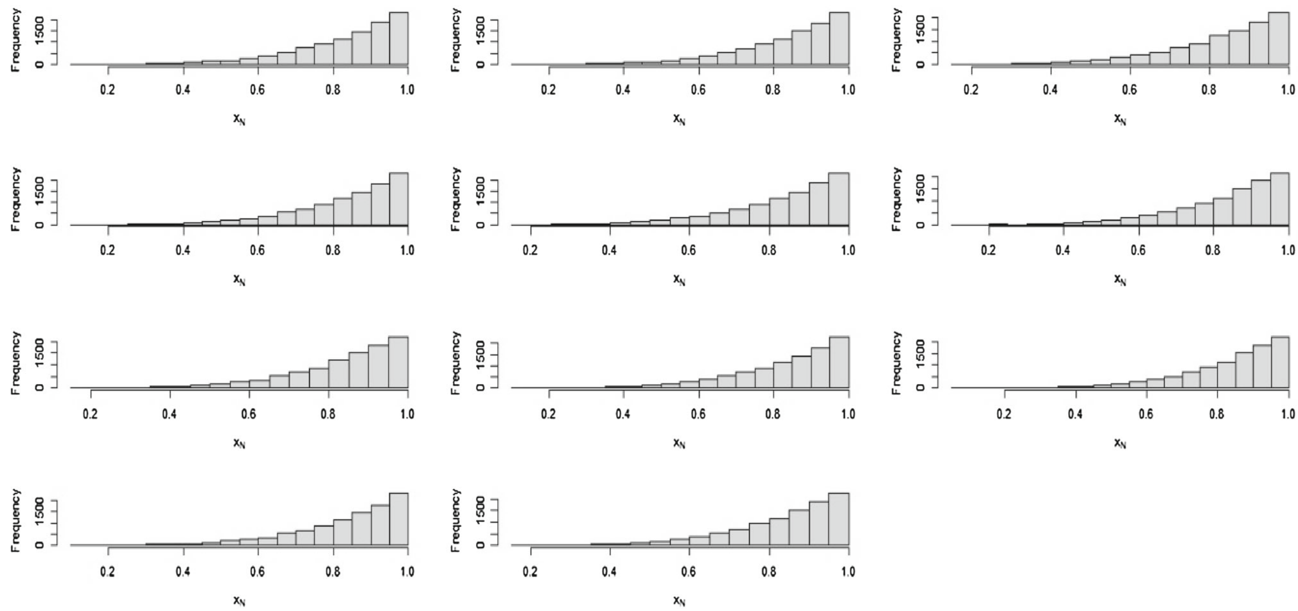

Fig. 2 The histogram at $c_N = 0$ and $d_N = 1$

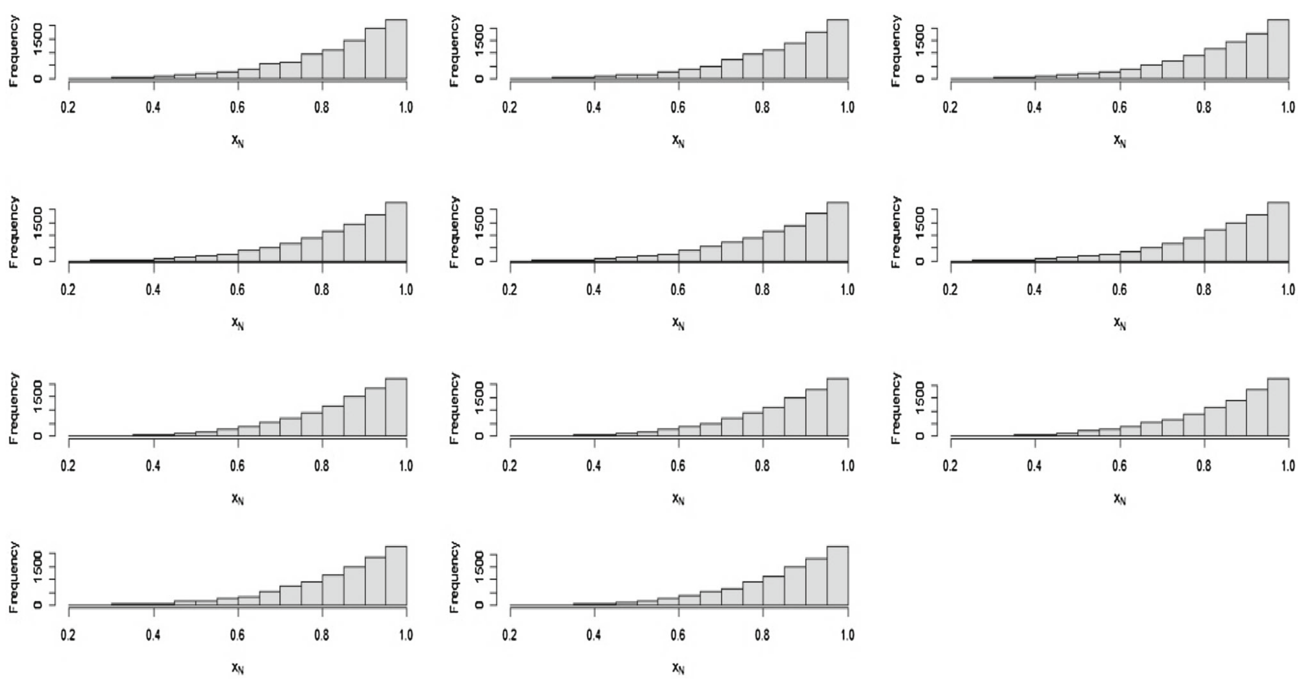

Fig. 3 The histogram at $c_N = 0.2$ and $d_N = 1$

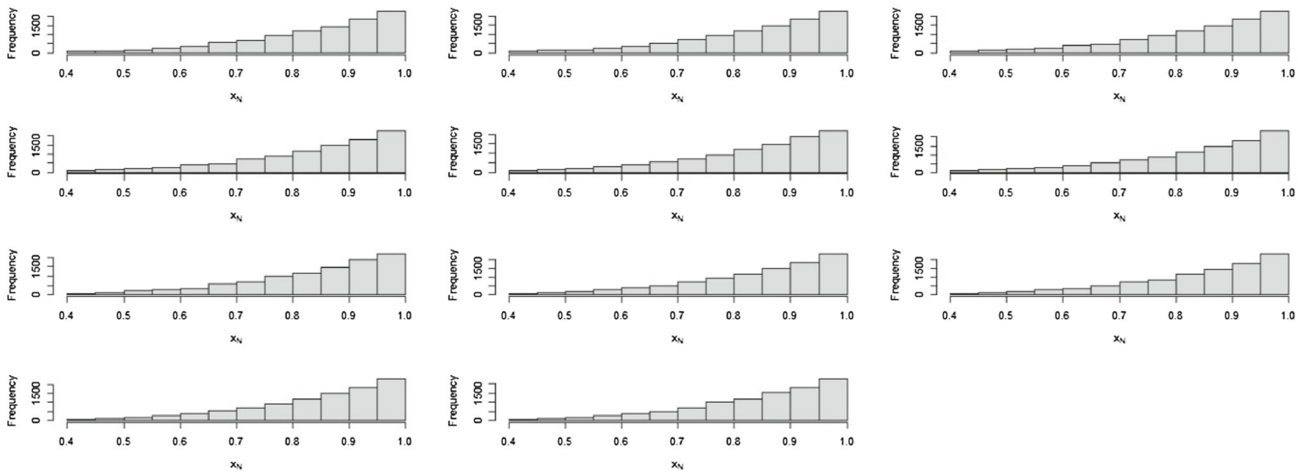

Fig. 4 The histogram at $c_N = 0.4$ and $d_N = 1$

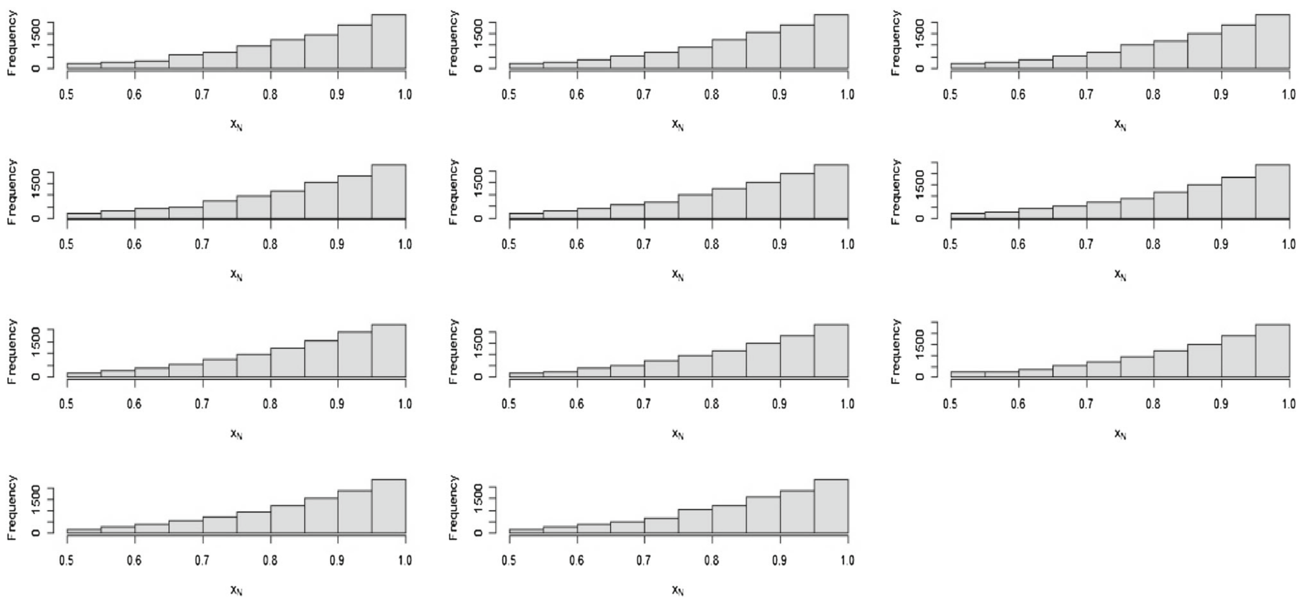

Fig. 5 The histogram at $c_N = 0.5$ and $d_N = 1$

sented. The extensive tables for various parameters and truncated values are present to see the behavior of random variate generating using the proposed algorithm. From the simulation study, it was concluded that when the difference between truncated values decreases the distribution of random variate tends to uniform. The proposed simulation method can be applied in computer science, medical science, industrial statistics and the automobile industry. The proposed simulation can be applied when real data under a complex system cannot be obtained. The proposed simulation method for other statistical distributions can be extended for future research.

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Declarations

Data availability The data are given in the paper.

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