Two conjectures on Smarandache's divisor products sequence

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Abstract. In this paper I make the following two conjectures on the *Smarandache's divisor products sequence* where a term P(n) of the sequence is defined as the product of the positive divisors of n: (1) there exist an infinity of n composites such that the number m = P(n) + n – 1 is prime; (2) there exist an infinity of n composites such that the number m = P(n) – n + 1 is prime.

The *Smarandache's divisor products sequence* (see A007955 in OEIS):
: 1, 2, 3, 8, 5, 36, 7, 64, 27, 100, 11, 1728, 13, 196, 225, 1024, 17, 5832, 19, 8000, 441, 484, 23, 331776, 125, 676, 729, 21952, 29, 810000, 31, 32768, 1089, 1156, 1225, 10077696, 37, 1444, 1521, 2560000, 41, 3111696, 43, 85184, 91125, 2116, 47, 254803968 (...)

Conjecture 1:

Let P(n) be the *Smarandache's divisor products sequence* where a term P(n) of the sequence is defined as the product of the positive divisors of n: there exist an infinity of n composites such that the number m = P(n) + n – 1 is prime.

Note that for n primes, because P(n) = n, P(n) + n – 1 = 2*n – 1 and is already conjectured that there exist an infinity of primes of the form 2*q – 1, where q prime.

The sequence of primes m:
: m = 3, prime, for (n, P(n)) = (2, 2);
: m = 11, prime, for (n, P(n)) = (4, 8);
: m = 41, prime, for (n, P(n)) = (6, 36);
: m = 71, prime, for (n, P(n)) = (8, 64);
: m = 109, prime, for (n, P(n)) = (10, 100);
: m = 1739, prime, for (n, P(n)) = (12, 1728);
: m = 239, prime, for (n, P(n)) = (15, 225);
: m = 1039, prime, for (n, P(n)) = (16, 1024);
: m = 5849, prime, for (n, P(n)) = (18, 5832);
: m = 461, prime, for (n, P(n)) = (21, 441);
: m = 149, prime, for (n, P(n)) = (25, 125);
: m = 701, prime, for (n, P(n)) = (26, 676);
: m = 1259, prime, for (n, P(n)) = (35, 1225);
: m = 1481, prime, for (n, P(n)) = (38, 1444);
: m = 2560039, prime, for (n, P(n)) = (40, 2560000);

: m = 2161, prime, for (n, P(n)) = (46, 2116);
(...)

Examples of larger m:
: m = 46656000059, prime, for (n, P(n)) = (60, 46656000000);
: m = 782757789791, prime, for (n, P(n)) = (96, 782757789696);
: m = 1586874323051, prime, for (n, P(n)) = (108, 1586874322944);
: m = 634562281237119143, prime, for (n, P(n)) = (168, 634562281237118976).

Note that m is prime for n = 12, 60, 96, 108, 168. I conjecture that m is prime for an infinity of n of the form 12*k.

Conjecture 2:

Let P(n) be the *Smarandache's divisor products sequence* where a term P(n) of the sequence is defined as the product of the positive divisors of n: there exist an infinity of n composites such that the number m = P(n) - n + 1 is prime.

Note that for n primes, because P(n) = n, P(n) - n + 1 = 1.

The sequence of primes m:
: m = 5, prime, for (n, P(n)) = (4, 8);
: m = 31, prime, for (n, P(n)) = (6, 36);
: m = 19, prime, for (n, P(n)) = (9, 27);
: m = 211, prime, for (n, P(n)) = (15, 225);
: m = 1009, prime, for (n, P(n)) = (16, 1024);
: m = 421, prime, for (n, P(n)) = (21, 441);
: m = 463, prime, for (n, P(n)) = (22, 484);
: m = 331753, prime, for (n, P(n)) = (24, 331776);
: m = 149, prime, for (n, P(n)) = (25, 125);
: m = 1123, prime, for (n, P(n)) = (34, 1156);
: m = 254803921, prime, for (n, P(n)) = (48, 254803968);
(...)

Examples of larger m:
: m = 531440999911, prime, for (n, P(n)) = (90, 531441000000);
: m = 389328928561, prime, for (n, P(n)) = (208, 389328928768).

Note that m is prime for n = 24, 48. I conjecture that m is prime for an infinity of n of the form 12*k.