Some results about four Smarandache U-product sequences

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Abstract

In this paper four Smarandache product sequences have been studied: Smarandache Square product sequence, Smarandache Cubic product sequence, Smarandache Factorial product sequence and Smarandache Palprime product sequence. In particular the number of primes, the convergence value for Smarandache Series, Smarandache Continued Fractions, Smarandache Infinite product of the mentioned sequences has been calculated utilizing the Ubasic software package. Moreover for the first time the notion of Smarandache Continued Radicals has been introduced. One conjecture about the number of primes contained in these sequences and new questions are posed too.

Introduction

In [1] Iacobescu describes the so called Smarandache U-product sequence.
Let u_n $n \geq 1$, be a positive integer sequence. Then a U-sequence is defined as follows:

$$U_n = 1 + u_1 \cdot u_2 \cdot \ldots \cdot u_n$$

In this paper differently from [1], we will call this sequence a U-sequence of the first kind because we will introduce for the first time a U-sequence of the second kind defined as follows:

$$U_n = \left|1 - u_1 \cdot u_2 \cdots u_n\right|$$

In this paper we will discuss about the "Square product", "Cubic product", "Factorial product" and "Primorial product" sequences. In particular we will analyze the question posed by Iacobescu in [1] on the number of primes contained in those sequences. We will also analyze the convergence values of the Smarandache Series [2], Infinite product [3], Simple Continued Fractions [4] of the four sequences. Moreover for the first time we will introduce the notion of Smarandache Continued Radicals and we will analyse the convergence of sequences reported above.

Sequences details

o ***Smarandache square product sequence of the first and second kind.***

In this case the sequence u_n is given by:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144........

that is the square of n. The first 20 terms of the sequence U_n ($1 \leq n \leq 20$) both the first and second kind are reported in the table below:

Smarandache Square product sequence (first kind)	Smarandache Square product sequence (second kind)
2	0
5	3
37	35
577	575
14401	14399
518401	518399
25401601	25401599
1625702401	1625702399
131681894401	131681894399
13168189440001	13168189439999
1593350922240001	1593350922239999
229442532802560001	229442532802559999
38775788043632640001	38775788043632639999
7600054456551997440001	7600054456551997439999
1710012252724199424000001	1710012252724199423999999
437763136697395052544000001	437763136697395052543999999
126513546505547170185216000001	126513546505547170185215999999
40990389067797283140009984000001	40990389067797283140009983999999
14797530453474819213543604224000001	14797530453474819213543604223999999
5919012181389927685417441689600000001	5919012181389927685417441689599999999

o ***Smarandache cubic product sequence of the first and second kind.***

In this case the sequence u_n is given by:

1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728......

that is the cube of n. Here the first 17 terms for the sequence U_n of the first and second kind.

Smarandache Cubic product sequence (first kind)	Smarandache Cubic product sequence (second kind)
2	0
9	7
217	215
13825	13823
1728001	1727999
373248001	373247999
128024064001	128024063999
65548320768001	65548320767999
47784725839872001	47784725839871999
47784725839872000001	47784725839871999999
63601470092869632000001	63601470092869631999999
109903340320478724096000001	109903340320478724095999999
241457638684091756838912000001	241457638684091756838911999999
662559760549147780765974528000001	662559760549147780765974527999999
2236139191853373760085164032000000001	2236139191853373760085164031999999999
9159226129831418921308831875072000000001	9159226129831418921308831875071999999999
44999277975861761160390291002228736000000001	44999277975861761160390291002228735999999999

o Smarandache factorial product sequence of the first and second kind.

In this case the sequence u_n is given by:

1, 2, 6, 24. 120, 720. 5040, 40320, 362880…….

that is the factorial of n. The first 13 terms of the U_n sequence of the first and second kind follow.

Smarandache Factorial product sequence (first kind)	Smarandache Factorial product sequence (second kind)
2	0
3	1
13	11
289	287
34561	34559
24883201	24883199
125411328001	125411327999
5056584744960001	5056584744959999
1834933472251084800001	1834933472251084799999
6658606584104736522240000001	6658606584104736522239999999
265790267296391946810949632000000001	265790267296391946810949631999999999
127313963299399416749559771247411200000000001	127313963299399416749559771247411199999999999
792786697595796795607377086400871488552960000000000001	792786697595796795607377086400871488552959999999999999

o Smarandache primorial product sequence of the first and second kind.

In this case the sequence u_n is given by:

2, 3, 5, 7, 11, 101, 121,131, 151, 181, 191, 313, 353, 353, 373......

that is the sequence of palindromic primes. Below the first 17 terms of the U_n sequence of the first and second kind.

Smarandache Palprime product sequence (first kind)	Smarandache Palprime product sequence (second kind)
3	1
7	5
31	29
211	209
2311	2309
233311	233309
28230511	28230509
3698196811	3698196809
558427718311	558427718309
101075417014111	101075417014109
19305404649695011	19305404649695009
6042591655354538131	6042591655354538129
2133034854340151959891	2133034854340151959889
795622000668876681038971	795622000668876681038969
304723222625617976883792551l	30472322262561797688379255O9
221533785488242691945171845771	221533785488242691945171845769
16770107561459971780249508724789l	16770107561459971780249508724788 9

Results

For all above sequences the following qestions have been studied:

1. How many terms are prime?
2. Is the Smarandache Series convergent?
3. Is the Smarandache Infinite product convergent?
4. Is the Smarandache Simple Continued Fractions convergent?
5. Is the Smarandache Continued Radicals convergent?

For this purpose the software package Ubasic Rev. 9 has been utilized. In particular for the item n. 1, a strong pseudoprime test code has been written [5]. Moreover, as already mentioned above, the item 5 has been introduced for the first time; a Smarandache Continued Radicals is defined as follows:

$$\sqrt{a(1)+\sqrt{a(2)+\sqrt{a(3)+\sqrt{a(4)+\ldots\ldots}}}}$$

where a(n) is the nth term of a Smarandache sequence. Here below a summary table of the obtained results:

	# Primes	SS_cv	SIP_cv	SSCF_cv	SCR_cv
Square 1st kind	12/456=0.026	0.7288315379.....	0	2.1989247812....	2.3666079803....
Square 2nd kind	1/463=0.0021	∞	∞	0.3301888340....	1.8143775546....
Cubic 1st kind	@	0.6157923201.....	0	2.1110542477....	2.6904314681....
Cubic 2nd kind	@	∞	∞	0.1427622842....	2.2446613806....
Factorial 1st kind	5/70=0.071	0.9137455924.....	0	2.3250021620....	2.2332152218....
Factorial 2nd kind	2/66=0.033	∞	∞	0.9166908563....	1.6117607295....
Palprime 1st kind	10/363=0.027	0.5136249121.....	0	3.1422019345....	2.5932060878....
Palprime 2nd kind	9/363=0.024	1.2397048573.....	0	1.1986303614....	2.1032632883....

Legend:

\# primes (Number of primes/number of sequence terms checked)
SS_cv (Smarandache Series convergence value)
SIP_cv (Smarandache Infinite Product convergence value)
SSCF_cv (Smarandache Simple Continued Fractions convergence value)
SCR _cv (Smarandache Continued Radicals convergence value)
@ (This sequence contain only one prime as proved by M. Le and K. Wu [6])

About the items 2,3,4 and 5 according to these results the answer is: yes, all the analyzed sequences converge except the Smarandache Series and the Smarandache Infinite product for the square product (2nd kind), cubic product (2nd kind) and factorial product (2nd kind). In particular notice the nice result obtained with the convergence of Smarandache Simple Continued Fractions of Smarandache palprime product sequence of the first kind.
The value of convergence is roughly π with the first two decimal digits correct.

$$\pi \approx 3+\cfrac{1}{7+\cfrac{1}{31+\cfrac{1}{211+\cfrac{1}{2311+\cfrac{1}{233311+\cfrac{1}{28230511+\cdots}}}}}}$$

Analogously for the cubic product sequence of the second kind the simple continued fraction converge roughly to $\pi-3$, while for the factorial product sequence of the second kind the continued radical converge roughly (two first decimal digits correct) to the golden ratio ϕ, that is:

$$\pi \approx 3 + \left[0 + \cfrac{1}{7 + \cfrac{1}{215 + \cfrac{1}{13823 + \cfrac{1}{1727999 + \cfrac{1}{373247999 + \cdots}}}}} \right]$$

$$\phi \approx \sqrt{0 + \sqrt{1 + \sqrt{11 + \sqrt{287 + \sqrt{34559 + \cdots}}}}}$$

About the item 1, the following table reports the values of n in the sequence that generate a strong pseudoprime number and its digit's number.

	n	d
Square 1st kind	1/2/3/4/5/9/10/11/1324/65/76	1/1/2/3/5/12/14/16/20/48/182/223
Square 2nd kind	2	1
Cubic 1st kind	1	1
Cubic 2nd kind	2	1
Factorial 1st kind	1/2/3/7/14	1/1/2/125/65
Factroial 2nd kind	3/7	2/12..
Palprime 1st kind	1/2/3/4/5/7/10/19/57/234	1/1/2/3/4/8/15/39/198/1208
Palprime 2nd kind	2/3/4/5/7/10/19/57/234	1/2/3/4/8/15/39/198/1208

Please note that the primes in the sequence of palprime of the first and second kind generate pairs of twin primes. The first ones follow:

(3.5) (5.7) (29.31) (209,211) (2309,2311) (28230509,28230511) (101075417014109,101075417014111) ....

Due to the fact that the percentage of primes found is very small and that according to Prime Number Theorem, the probability that a randomly chosen number of size n is prime decreases as 1/d (where d is the number of digits of n) we are enough confident to pose the following conjecture:

- ***The number of primes contained in the Smarandache Square product sequence (1st and 2nd kind), Smarandache Factorial product sequence (1st and 2nd kind) and Smarandache Palprime product sequence (1st and 2nd kind) is finite.***

New Questions

- Is there any Smarandache sequence whose SS, SIP, SSCF and SCR converge to some known mathematical constants?

- Are all the estimated convergence values irrational or trascendental?

- Is there for each prime inside the Smarandache Palprime product sequence of the second kind the correspondent twin prime in the Smarandache Palprime product sequence of the first kind?

- Are there any two Smarandache sequences a(n) and b(n) whose Smarandache Infinite Product ratio converge to some value k different from zero?

$$\lim_{n \to \infty} \frac{\prod_n \frac{1}{a(n)}}{\prod_n \frac{1}{b(n)}} = k$$

- Is there any Smarandache sequence a(n) such that:

$$\lim_{n\to\infty} e^{\sum_n \frac{1}{a(n)}} \cong \pi$$

- For the four sequences of first kind a(n), study:

$$\lim_{n\to\infty} \sum_n \frac{\mathrm{a(n)}}{\mathrm{R(a(n))}}$$

where R(a(n)) is the reverse of a(n). (For example if a(n)=17 then R(a(n))=71 and so on).

References

[1] F. Iacobescu, Smarandache partition type and other sequences, Bull. Pure Appl. Sci. Sec. E 16(1997), No. 2, 237-240.

[2] C. Ashbacher, Smarandache Series convergence, to appear

[3] See http://www.gallup.unm.edu/~smarandache/product .txt

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[5] D.M. Bressoud, Factorization and primality testing, Springer Verlag, 1989, p. 77

[6] M. Le and K. Wu , The primes in the Smarandache Power product Sequence, Smarandache Notions Journal, Vol. 9, No. 1-2, 1998, 97-97