AN INFINITY OF UNSOLVED PROBLEMS CONCERNING A FUNCTION IN THE NUMBER THEORY

§1. Abstract

W. Sierpiński has asserted to an international conference that if mankind lasted for ever and numbered the unsolved problems, then in the long run all these unsolved problems would be solved.

The purpose of our paper is that making an infinite number of unsolved problems to prove his supposition is not true. Moreover, the author considers the unsolved problems proposed in this paper can never be all solved!

Every period of time has its unsolved problems which were not previously recommended until recent progress. Number of new unsolved problems are exponentially increasing in comparison with ancient unsolved ones which are solved at present. Research into one unsolved problem may produce many new interesting problems. The reader is invited to exhibit his works about them.

§2. Introduction

We have constructed (*) a function η which associates to each non-null integer n the smallest positive integer m such that m! is a multiple of n. Thus, if n has the standard form:

$$n = \epsilon\, p_1^{a_1} \cdots p_r^{a_r}$$, with all p_i distinct primes,

all $a_i \in N^*$, and $\epsilon = \pm 1$, then $\eta(n) = \max_{1 \le i \le r} \{\eta_{p_i}(a_i)\}$, and

$\eta(\pm 1) = 0$.

Now, we define the η_p functions: let p be a prime and $a \in N^*$; then $\eta_p(a)$ is the smallest positive integer b such that b! is a multiple of p^a. Constructing the sequence:

$$\alpha_k^{(p)} = \frac{p^k - 1}{p-1}, \quad k = 1, 2, \ldots$$

we have $\eta_p(\alpha_k^{(p)}) = p^k$, for all prime p, and all k = 1, 2, ... Because any $a \in N^*$ is uniquely written in the form:

$$a = t_1 \alpha_{n_1}^{(p)} + \ldots + t_e \alpha_{n_e}^{(p)}, \text{ where } n_1 > n_2 > \ldots > n_e > 0,$$

and $1 \le t_j \le p - 1$ for $j = 0, 1, \ldots, e - 1$, and $1 \le t_e \le p$, with all n_i, t_i from N, the author proved that

$$\eta_p(a) = \sum_{i=1}^{e} t_i\, \eta_p(\alpha_{n_i}^{(p)}) = \sum_{i=1}^{e} t_i p^{n_i}.$$

§3. Some Properties of the Function η

Clearly, the function η is even: $\eta(-n) = \eta(n)$, $n \in Z^*$. If $n \in N^*$ we have:

$$(1) \quad \frac{-1}{(n-1)!} \le \frac{\eta(n)}{n} \le 1 ,$$

and: $\frac{\eta(n)}{n}$ is maximum if and only if n is prime or n = 4;

$\frac{\eta(n)}{n}$ is minimum if and only if n = k! .

Clearly η is not a periodical function. For p prime, the functions η_p are increasing, not injective but on $N^* - \{p^k \mid k = 1, 2, \ldots\}$ they are surjective. From (1) we find that $\eta = o(n^{1+\epsilon})$, $\epsilon > 0$, and $\eta = O(n)$.

The function η is <u>generally increasing</u> on N*, that is: $(\forall)\ n \in N^*$, $(\exists)\ m_0 \in N^*$, $m_0 = m_0(n)$, such that for all $m \ge m_0$ we have $\eta(m) \ge \eta(n)$ (and generally decreasing on Z_-^*); it is not injective, but it is surjective on $Z \setminus \{0\} \to N \setminus \{1\}$.

The number n is called a <u>barrier</u> for a number-theoretic function f(m) if, for all m < n, m + f(m) $\le$ n (P. Erdös and J. L. Selfridge). Does $\epsilon\, \eta(m)$ have infinitely many barriers, with $0 < \epsilon \le 1$? [No, because there is a $m_0 \in N$ such that for all $n - 1 \ge m_0$ we have $\eta(n-1) \ge \frac{2}{\epsilon}$ (η is generally increasing), whence $n - 1 + \epsilon\, \eta(n-1) \ge n + 1$.]

$\sum_{n \ge 2} 1/\eta(n)$ is divergent, because $1/\eta(n) \ge 1/n$.

$$\eta\left(\underbrace{2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}}_{k\text{ times}}\right) = 2 + \underbrace{2^{\cdot^{\cdot^{\cdot^{2^{n}}}}}}_{k-1\text{ times}}$$

Proof: Let

$a_m^{(2)} = 2^m - 1$, where $m = \underbrace{2^{\cdot^{\cdot^{\cdot}}}}_{k-2\text{ times}}$;

then $\eta\left(2^{2^m}\right) = \eta_2(2^m) = \eta_2(1 + a_m^{(2)}) = \eta_2(1) + \eta_2(a_m^{(2)}) =$
$= 2 + 2^m$.

§4. Glossary of Symbols and Notions

A-sequence: an integer sequence $1 \le a_1 < a_2 < \ldots$ so that no a_i is the sum of distinct members of the sequence other than a_i (R. K. Guy);

Average Order: if $f(n)$ is an arithmetical function and $g(n)$ is any simple function of n such that $f(1) + \ldots + f(n) \sim g(1) + \ldots + g(n)$ we say that $f(n)$ is of the average order of $g(n)$;

$d(x)$: number of positive divisors of x;

d_x: difference between two consecutive primes: $p_{x+1} - p_x$;

Dirichlet Series: a series of the form $F(s) = \sum_{n=1}^{\infty} \frac{\alpha_n}{n^s}$, s may be real or complex;

Generating Function: any function $F(s) = \sum_{n=1}^{\infty} \alpha_n u_n(s)$ is considered as a generating function of α_n; the most usual form of $u_n(s)$ is: $u_n(s) = e^{-\lambda_n \cdot s}$, where λ_n is a sequence of positive numbers which increases steadily to infinity;

Log x: Napierian logarithm of x, to base e;

Normal Order: f(n) has the normal order F(n) if f(n) is approximately F(n) for almost all values of n, i.e. (2), $(\forall)\ \epsilon > 0$, $(1 - \epsilon) \cdot F(n) < f(n) < (1 + \epsilon) \cdot F(n)$ for almost all values of n; "almost all" n means that the numbers less than n which do not possess the property (2) is o (x);

Lipschitz-Condition: a function f verifies the Lipschitz-condition of order $\alpha \in (0, 1]$ if $(\exists)\ k > 0$: $|f(x)-f(y)| \leq k\ |x-y|^\alpha$; if $\alpha = 1$, f is called a k Lipschitz-function; if $k < 1$, f is called a contractant function;

Multiplicative Function: a function $f: N^* \to C$ for which $f(1) = 1$, and $f(m \cdot n) = f(m) \cdot f(n)$ when $(m, n) = 1$;

p(x): largest prime factor of x;

Uniformly Distributed: a set of points in (a, b) is uniformly distributed if every sub-interval of (a, b) contains its proper quota of points;

Incongruent Roots: two integers x, y which satisfy the congruence $f(x) \equiv f(y) \equiv 0 \pmod{m}$ and so that $x \not\equiv y \pmod{m}$;

s-additive sequence: a sequence of the form: $a_1 = \ldots = a_s = 1$ and $a_{n+s+1} = a_{n+1} + \ldots + a_{n+s}$, $n \in N^*$ (R. Queneau);

$s(n)$: sum of aliquot parts (divisors of n other than n) of n; $\sigma(n) - n$;

$s^k(n)$: k^{th} iterate of $s(n)$;

$s^*(n)$: sum of unitary aliquot parts of n;

$r_k(n)$: least number of numbers not exceeding n, which must contain a k-term arithmetic progression;

$\pi(x)$: number of primes not exceeding x;

$\pi(x; a, b)$: number of primes not exceeding x and congruent to a modulo b;

$\sigma(n)$: sum of divisors of n; $\sigma_1(n)$;

$\sigma_k(n)$: sum of k-th powers of divisors of n;

$\sigma^k(n)$: k-th iterate of $\sigma(n)$;

$\sigma^*(n)$: sum of unitary divisors of n;

$\varphi(n)$:	Euler's totient function; number of numbers not exceeding n and prime to n;
$\varphi^k(n)$:	k-th iterate of $\varphi(n)$;
$\bar{\varphi}(n)$:	$= n \prod (1 + p^{-1})$, where the product is taken over the distinct prime divisors of n;
$\Omega(n)$:	number of prime factors of n, counting repetitions;
$\omega(n)$:	number of distinct prime factors of n;
$\lfloor a \rfloor$:	floor of a; greatest integer not greater than a;
(m, n):	g.c.d. (greatest common divisor) of m and n;
[m, n]:	l.c.d. (least common multiple) of m and n;
$\|f\|$:	modulus or absolute value of f;
$f(x) \sim g(x)$:	$f(x)/g(x) \to 1$ as $x \to \infty$; f is asymptotic t g;
$f(x) = o(g(x))$:	$f(x)/g(x) \to 0$ as $x \to \infty$;
$f(x) = O(g(x))$, $f(x) \ll g(x)$;	there is a constant c such that $\|f(x)\| < cg(x)$, for any x;
$\Gamma(x)$:	Euler's function of first case (gamma function); $\Gamma : R^*_+ \to R$, $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$. We have $\Gamma(x+1) = x\,\Gamma(x)$. If $x \in N^*$, $\Gamma(x) = (x-1)!$

$\beta(x)$: Euler's function of second degree (beta function); $\beta : R^*_+ \times R^*_+ \to R$,

$$\beta(u, v) = \Gamma(u)\,\Gamma(v)/\Gamma(u+v) = \int_0^1 t^{u-1} \cdot (1-t)^{v-1}\,dt;$$

$\mu(x)$: Möbius' function; $\mu : N \to N$ $\mu(1) = 1$; $\mu(n) = (-1)^k$ if n is the product of $k > 1$ distinct primes; $\mu(n) = 0$ in all other cases;

$\theta(x)$: Tchebycheff θ-function; $\theta : R_+ \to R$,

$\theta(x) = \Sigma \log p$

where the summation is taken over all primes p not exceeding x;

$\Psi(x)$: Tchebycheff's Ψ-function; $\Psi(x) =$

$$= \sum_{n \leq x} \Lambda(n), \text{ with}$$

$$\Lambda(n) = \begin{cases} \log p, & \text{if n is an integer power of the prime p;} \\ 0, & \text{in all other cases.} \end{cases}$$

This glossary can be continued with OTHER (ARITHMETICAL) FUNCTIONS.

§5. General Unsolved Problems Concerning the Function η

(1) Is there a closed expression for $\eta(n)$?

(2) Is there a good asymptotic expression for $\eta(n)$? (If yes, find it.)

(3) For a fixed non-null integer m, does $\eta(n)$ divide n-m? (Particularly when m = 1.) Of course, for m = 0 it is trivial: we find n = k!, or n is a squarefree, etc.

(4) Is η an algebraic function? (If no, is there the max Card $\{n \in Z^* \mid (\exists)\ p \in R[x, y]$, p non-null polynomial, with $p(n, \eta(n)) = 0$ for all these n$\}$?) More generally we introduce the notion: g is a <u>f-function</u> if $f(x, g(x)) = 0$ for all x, and $f \in R[x, y]$, f non-null. Is η a f-function? (If no, is there the max Card $\{n \in Z^* \mid (\exists)\ f \in R[x, y]$, f non-null, $f(n, \eta(n)) = 0$ for all these n$\}$?)

(5) Let A be a set of consecutive integers from N*. Find max Card A for which η is monotonous. For example, Card A $\geq$ 5, because for A = {1, 2, 3, 4, 5} η is 0, 2, 3, 4, 5, respectively.

(6) A number is called an <u>η-algebraic number of degree n</u> $\in$ N* if it is a root of the polynomial

(p) $p_\eta(x) = \eta(n)\ x^n + \eta(n-1)\ x^{n-1} + \ldots + \eta(1)\ x^1 = 0.$

An <u>η-algebraic field</u> $\mathbb{M}$ is the aggregate of all numbers

$$R_\eta\ (\upsilon) = \frac{A(\upsilon)}{B(\upsilon)},$$

where υ is a given η-algebraic number, and $A(\upsilon)$, $B(\upsilon)$ are polynomials in υ of the form (p) with $B(\upsilon) \neq 0$. Study $\mathbb{M}$.

(7) Are the points $p_n = \eta(n)/n$ uniformly distributed in the interval (0, 1)?

(8) Is 0.023453746511 4..., where the sequence of digits is $\eta(n)$, $n \geq 1$, an irrational number?

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Is it possible to represent all integer n under the form:

(9) $n = \pm \eta(a_1)^{a_2} \pm \eta(a_2)^{a_3} \pm \ldots \pm \eta(a_k)^{a_1}$, where the integers k, $a_1, \ldots, a_k$, and the signs are conveniently chosen?

(10) But as $n = \pm a_1^{\eta(a_1)} \pm \ldots \pm a_k^{\eta(a_k)}$?

(11) But as $n = \pm a_1^{\eta(a_2)} \pm a_2^{\eta(a_3)} \pm \ldots \pm a_k^{\eta(a_1)}$?

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Find the smallest k for which: $(\forall)\ n \in N^*$ at least one of the numbers $\eta(n), \eta(n+1), \ldots, \eta(n+k-1)$ is:

(12) A perfect square.

(13) A divisor of k^n.

(14) A multiple of a fixed nonzero integer p.

(15) A factorial of a positive integer.

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(16) Find a general form of the continued fraction expansion of $\eta(n)/n$, for all $n \geq 2$.

(17) Are there integers m, n, p, q, with $m \neq n$ or $p \neq q$, for which: $\eta(m) + \eta(m+1) + \ldots + \eta(m+p) = \eta(n) + \eta(n+1) + \ldots + \eta(n+q)$?

(18) Are there integers m, n, p, k with $m \neq n$ and $p > 0$, such that:

$$\frac{\eta(m)^2 + \eta(m+1)^2 + \ldots + \eta(m+p)^2}{\eta(n)^2 + \eta(n+1)^2 + \ldots + \eta(n+p)^2} = k\ ?$$

(19) How many primes have the form:

$\overline{\eta(n)\ \eta(n+1)\ \ldots\ \eta(n+k)}$,

for a fixed integer k? For example:

$\overline{\eta(2)\ \eta(3)} = 23$, $\overline{\eta(5)\ \eta(6)} = 53$ are primes.

(20) Prove that $\eta(x^n) + \eta(y^n) = \eta(z^n)$ has an infinity of integer solutions, for any $n \geq 1$. Look, for example, at the solution (5, 7, 2048) when n = 3. (On Fermat's last theorem.) More generally: the diophantine equation $\sum_{i=1}^{k} \eta(x_i^s) = \sum_{j=1}^{m} \eta(y_j^t)$ has an infinite number of solutions.

(21) Are there m, n, k non-null positive integers, $m \neq 1 \neq n$, for which $\eta(m \cdot n) = m^k \cdot \eta(n)$? Clearly, η is not homogenous to degree k.

(22) Is it possible to find two distinct numbers k, n for which $\log_{\eta(k^n)} \eta(n^k)$ be an integer? (The base is $\eta(k^n)$.)

(23) Let the congruence be: $h_\eta(x) = c_n x^{\eta(n)} + \ldots + c_1 \cdot x^{\eta(1)} \equiv 0 \pmod{m}$. How many incongruent roots has h_η, for some given constant integers n, $c_1, \ldots, c_n$?

(24) We know that $e^x = \sum_{n=0}^{\infty} x^n/n!$. Calculate $\sum_{n=1}^{\infty} x^{\eta(n)} / n!$, $\sum_{n=1}^{\infty} x^n / \eta(n)!$ and eventually some of their properties.

(25) Find the average order of $\eta(n)$.

(26) Find some $u_n(s)$ for which F(s) be a generating function of $\eta(n)$, and F(s) have at all a simple form.

Particularly, calculate Dirichlet series $F(s) = \sum_{n=1}^{\infty} \eta(n)/n^s$, with $s \in R$ (or $s \in C$).

(27) Does $\eta(n)$ have a normal order?

(28) We know that Euler's constant is

$$\gamma = \lim_{n\to\infty} \left(1 + \frac{1}{2} + \dots + \frac{1}{n} - \log n \right).$$

Is $\lim_{n\to\infty} \left[1 + \sum_{k=2}^{n} 1/\eta(k) - \log \eta(n) \right]$ a constant? If yes, find it.

(29) Is there an m for which $\eta^{-1}(m) = \{a_1, a_2, \dots, a_{pq}\}$ such that the numbers $a_1, a_2, \dots, a_{pq}$ can constitute a matrix of p rows and q columns with the sum of elements on each row and each column is constant? Particularly when the matrix is square.

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(30) Let $\{x_n^{(s)}\}_{n\geq 1}$ be a s-additive sequence. Is it possible to have $\eta(x_n^{(s)}) = x_m^{(s)}$, $n \neq m$? But $x_{\eta(n)}^{(s)} = \eta(x_n^{(s)})$?

(31) Does η verify a Lipschitz Condition?

(32) Is η a k-Lipschitz Condition?

(33) Is η a contractant function?

(34) Is it possible to construct an A-sequence $a_1, \ldots, a_n$ such that $\eta(a_1), \ldots, \eta(a_n)$ be an A-sequence, too? Yes, for example 2, 3, 7, 31, ... Find such an infinite sequence.

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Find the greatest n such that: if $a_1, \ldots, a_n$ constitute a p-sequence then $\eta(a_1), \ldots, \eta(a_n)$ constitute a p-sequence, too; where a p-sequence means:

(35) Arithmetical progression.

(36) Geometrical progression.

(37) A complete system of modulo n residues.

Remark: let p be a prime, and $p, p^2, \ldots, p^p$ a geometrical progression, then $\eta(p^i) = ip$, $i \in \{1, 2, \ldots, p\}$, constitute an arithmetical progression of length p. In this case $n \to \infty$.

(38) Let's use the sequence $a_n = \eta(n)$, $n \geq 1$. Is there a recurring relation of the form $a_n = f(a_{n-1}, a_{n-2}, \ldots)$ for any n?

(39) Are there blocks of consecutive composite numbers $m + 1, \ldots, m + n$ such that $\eta(m + 1), \ldots, \eta(m + n)$ be composite numbers, too? Find the greatest n.

(40) Find the number of partitions of n as sum of $\eta(m)$, $2 < m \leq n$.

MORE UNSOLVED GENERAL PROBLEMS CONCERNING THE FUNCTION η

§6. Unsolved Problems Concerning the Function η and Using the Number Sequences

41-2065) Are there non-null and non-prime integers a_1, a_2, ..., a_n in the relation P, so that $\eta(a_1)$, $\eta(a_2)$, ..., $\eta(a_n)$ be in the relation R? Find the greatest n with this property. (Of course, all a_i are distinct.) Where each P, R can represent one of the following number sequences:

(1) Abundant numbers; $a \in N$ is abundant if $\sigma(a) > 2a$.

(2) Almost perfect numbers; $a \in N$, $\sigma(a) = 2a - 1$.

(3) Amicable numbers; in this case we take n = 2; a, b are called amicable if $a \neq b$ and $\sigma(a) = \sigma(b) = a + b$.

(4) Augmented amicable numbers; in this case n = 2; a, b are called augmented amicable if $\sigma(a) = \sigma(b) = a + b - 1$ (Walter E. Beck and Rudolph M. Najar).

(5) Bell numbers: $b_n = \sum_{k=1}^{n} S(n, k)$, where S(n, k) are stirling numbers of second case.

(6) Bernoulli numbers (Jacques 1st): B_n, the coefficients of the development in integer sequence of

$$\frac{t}{e^t-1} = 1 - \frac{t}{2} + \frac{B_1}{2!}t^2 - \frac{B_2}{4!}t^4 + \ldots + (-1)^{n-1}\frac{B_n}{(2n)!}t^{2n} + \ldots,$$

for $0 < |t| < 2\pi$; (here we always take $\lfloor 1/B_n \rfloor$).

(7) Catalan numbers: $c_1 = 1$, $c_n = \frac{1}{n}\binom{2n-2}{n-1}$ for $n \geq 2$.

(8) Carmichael numbers; an odd composite number a, which is a pseudoprime to base b for every b relatively prime to a, is called a Carmichael number.

(9) Congruent numbers; let n = 3, and the numbers a, b, c; we must have $a \equiv b \pmod{c}$.

(10) Cullen numbers: $C_n = n \cdot 2^n + 1$, $n \geq 0$.

(11) C_1-sequence of integers; the author introduced a sequence $a_1, a_2, \ldots$ so that:

$(\forall)\ i \in N^*,\ (\exists)\ j, k \in N^*,\ j \neq i \neq k \neq j,\ :\ a_i \equiv a_j \pmod{a_k}$

(12) C_2-sequence of integers; the author defined other sequence $a_1, a_2, \ldots$ so that:

$(\forall)\ i \in N^*,\ (\exists)\ j, k \in N^*,\ i \neq j \neq k \neq i,\ :\ a_j \equiv a_k \pmod{a_i}$.

(13) Deficient numbers; $a \in N^*$, $\sigma(a) < 2a$.

(14) Euler numbers: the coefficients E_n in the expansion of $\sec x = \sum_{n\geq 0} E_n x^n/n!$; here we will take $|E_n|$.

(15) Fermat numbers: $F_n = 2^{2^n} + 1$, $n \geq 0$.

(16) Fibonacci numbers: $f_1 = f_2 = 1$, $f_n = f_{n-1} + f_{n-2}$, $n \geq 3$.

(17) Genocchi numbers: $G_n = 2\,(2^{2n} - 1)\,B_n$, where B_n are Bernoulli numbers; always $G_n \in Z$.

(18) Harmonic mean; in this case every member of the sequence is the harmonic mean of the preceding members.

(19) Harmonic numbers; a number n is called harmonic if the harmonic mean of all divisors of n is an integer (C. Pomerance).

(20) Heteromeous numbers: $h_n = n(n+1)$, $n \in N^*$.

(21) K-hyperperfect numbers; a is k-hyperperfect if $a = 1 + \Sigma d_i$, where the numeration is taken over all proper divisors, $1 < d_i < a$, or $k\,\sigma(a) = (k+1)\,a + k - 1$ (Daniel Minoli and Robert Bear).

(22) Kurepa numbers: $!n = 0! + 1! + 2! + \ldots + (n-1)!$

(23) Lucas numbers: $L_1 = 1$, $L_2 = 3$, $L_n = L_{n-1} + L_{n-2}$, $n \geq 3$.

(24) Lucky numbers: from the natural numbers strike out all even numbers, leaving the odd numbers; apart from 1, the first remaining number is 3; strike out every third member in the new sequence; the next member remaining is 7; strike out every seventh member in this sequence; next 9 remains; etc. (V. Gardiner, R. Lazarus, N. Metropolis, S. Ulam).

(25) Mersenne numbers: $M_p = 2^p - 1$.

(26) m-perfect numbers; a is m-perfect if $\sigma^m(a) = 2a$ (D. Bode).

(27) Multiply perfect (or k-fold perfect) numbers; a is k-fold perfect if $\sigma(a) = k\,a$.

(28) Perfect numbers; a is perfect if $\sigma(a) = 2a$.

(29) Polygonal numbers (represented on the perimeter of a polygon): $p_n^k = k\,(n - 1)$.

(30) Polygonal numbers (represented on the closed surface of a polygon): $p_n^k = \frac{(k-2)\,n^2 - (k-4)\,n}{2}$.

(31) Primitive abundant numbers; a is primitive abundant if it is abundant, but none of its proper divisors are.

(32) Primitive pseudoperfect numbers; a is primitive pseudoperfect if it is pseudoperfect, but none of its proper divisors are.

(33) Pseudoperfect numbers; a is pseudoperfect if it is equal to the sum of some of its proper divisors (W. Sierpiński).

(34) Pseudoprime numbers to base b; a is pseudoprime to base b if a is an odd composite number for which $b^{a-1} \equiv 1 \pmod{a}$ (C. Pomerance, J. L. Selfridge, S. Wagstaff).

(35) Pyramidal numbers: $\pi_n = \frac{1}{6}\,n\,(n + 1)\,(n + 2)$, $n \in N^*$.

(36) Pythagorian numbers; let n = 3 and a, b, c be integers; then it must have the relation: $a^2 = b^2 + c^2$.

(37) Quadratic residues of a fixed prime p: the nonzero numbers r for which the congruence $r \equiv x^2 \pmod{p}$ has solutions.

(38) Quasi perfect numbers; a is quasi perfect if $\sigma(a) = 2a + 1$.

(39) Reduced amicable numbers; we take n = 2; two integers a, b for which $\sigma(a) = \sigma(b) = a + b + 1$ are called reduced amicable numbers (Walter E. Beck and Rudolph M. Najar).

(40) Stirling numbers of first case: $s(0, 0) = 1$, and $s(n, k)$ is the coefficient of x^k from the development $x(x - 1) \dots (x - n + 1)$.

(41) Stirling numbers of second case: $S(0, 0) = 1$, and $S(n, k)$ is the coefficient of the polynom $x^{(k)} = x(x - 1) \dots (x - k + 1)$, $1 \le k \le n$, from the development (which is uniquely written):

$$x^n = \sum_{k=1}^{n} S(n, k)\, x^{(k)} .$$

(42) Superperfect numbers; a is superperfect if $\sigma^2(a) = 2a$ (D. Suryanarayana).

(43) Untouchable numbers; a is untouchable if $s(x) = 1$ has no solution (Jack Alanen).

(44) U-numbers: starting from arbitrary u_1 and u_2 continues with those numbers which can be expressed in just

one way as the sum of two distinct earlier members of the sequence (S. M. Ulam).

(45) Weird numbers; a is called weird if it is abundant but not pseudoperfect (S. J. Benkoski).

MORE NUMBER SEQUENCES

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The unsolved problem No. 41 is obtained by taking P = = (1) and R = (1).

The unsolved problem No. 42 is obtained by taking P = = (1), R = (2).

The unsolved problem No. 2065 is obtained by taking P = (45) and R = (45).

OTHER UNSOLVED PROBLEMS CONCERNING THE FUNCTION η AND USING NUMBER SEQUENCES

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§7. Unsolved Diophantine Equations Concerning the Function η

2066) Let $0 < k \leq 1$ be a rational number. Does the diophantine equation $\eta(n)/n = k$ always have solutions? Find all k so that this equation has an infinite number of solutions. (For example, if $k = 1/r$, $r \in N^*$, then $n = rp_{a+h}$, $h = 1, 2, \ldots$, all p_{a+h} are primes, and a is a chosen index such that $p_{a+1} > r$.)

2067) Let $\{a_n\}_{n\geq 0}$ be a sequence, $a_0 = 1$, $a_1 = 2$, and $a_{n+1} = a_{\eta(n)} + \eta(a_n)$. Are there infinitely many pairs (m, n), $m \neq n$, for which $a_m = a_n$? (For example: $a_9 = a_{13} = 16$.)

2068) Conjecture: the equation $\eta(x) = \eta(x + 1)$ has no solution.

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Let m, n be fixed integers. Solve the diophantine equations:

2069) $\eta(m\,x + n) = x$.

2070) $\eta(m\,x + n) = m + n\,x$.

2071) $\eta(m\,x + n) = x!$

2072) $\eta(x^m) = x^n$.

2073) $\eta(x)^m = \eta(x^n)$.

2074) $\eta(m\,x + n) = \eta(x)^y$.

2075) $\eta(x) + y = x + \eta(y)$, x and y are not primes.

2076) $\eta(x) + \eta(y) = \eta(x + y)$, x and y are not twin primes. (Generally, η is not additive.)

2077) $\eta(x + y) = \eta(x)\cdot\eta(y)$. (Generally, η is not an exponential function.)

2078) $\eta(xy) = \eta(x)\eta(y)$. (Generally, η is not a multiplicative function.)

2079) $\eta(m\,x + n) = x^y$.

2080) $\eta(x)\,y = x\,\eta(y)$, x and y are not primes.

2081) $\eta(x)/y = x/\eta(y)$, x and y are not primes. (Particularly when $y = 2^k$, $k \in N$, i.e., $\eta(x)/2^k$ is a dyadic rational number.)

2082) $\eta(x)^{y} = x^{\eta(y)}$, x and y are not primes.

2083) $\eta(x)^{\eta(y)} = \eta(x^y)$.

2084) $\eta(x^y) - \eta(z^w) = 1$, with $y \neq 1 \neq w$. (On Catalan's problem.)

2085) $\eta(x^y) = m$, $y \geq 2$.

2086) $\eta(x^x) = y^y$. (A trivial solution: $x = y = 2$.)

2087) $\eta(x^y) = y^x$. (A trivial solution: $x = y = 2$.)

2088) $\eta(x) = y!$ (An example: $x = 9$, $y = 3$.)

2089) $\eta(m\ x) = m\ \eta(x)$, $m \geq 2$.

2090) $m^{\eta(x)} + \eta(x)^n = m^n$.

2091) $\eta(x^2)/m \pm \eta(y^2)/n = 1$.

2092) $\eta(x_1^{y_1} + \ldots + x_r^{y_r}) = \eta(x_1)^{y_1} + \ldots + \eta(x_r)^{y_r}$.

2093) $\eta(x_1! + \ldots + x_r!) = \eta(x_1)! + \ldots + \eta(x_r)!$.

2094) $(x, y) = (\eta(x), \eta(y))$, x and y are not primes.

2095) $[x, y] = [\eta(x), \eta(y)]$, x and y are not primes.

OTHER UNSOLVED DIOPHANTINE EQUATIONS CONCERNING THE FUNCTION η ONLY

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§8. Unsolved Diophantine Equations Concerning the Function η in Correlation with Other Functions

Let m, n be fixed integers. Solve the diophantine equations:

2096-2102) $\eta(x) = d(m\ x + n)$

$\eta(x)^m = d(x^n)$

$\eta(x) + y = x + d(y)$

$$\eta(x) \cdot y = x \cdot d(y)$$

$$\eta(x)/y = d(y)/x$$

$$\eta(x)^{y} = x^{d(y)}$$

$$\eta(x)^{y} = d(y)^{x}$$

2103-2221) Same equations as before, but we substitute the function $d(x)$ with d_x, $p(x)$, $s(x)$, $s^k(x)$, $s^*(x)$, $r_k(x)$, $\pi(x)$, $\pi(x;\ m,\ n)$, $\sigma_k(x)$, $\sigma^k(x)$, $\sigma^*(x)$, $\varphi(x)$, $\varphi^k(x)$, $\bar{\varphi}(x)$, $\Omega(x)$, $\omega(x)$ respectively.

2222) $\eta(s(x,\ y)) = s(\eta^{(x)},\ \eta(y))$.

2223) $\eta(S(x,\ y)) = S(\eta(x),\ \eta(y))$.

2224) $\eta(\lfloor x \rfloor) = \lfloor \Gamma(x) \rfloor$.

2225) $\eta(\lfloor x - y \rfloor) = \lfloor \beta(x,\ y) \rfloor$.

2226) $\beta(\eta(\lfloor x \rfloor),\ y) = \beta(x,\ \eta(\lfloor y \rfloor))$.

2227) $\eta(\lfloor \beta(x,\ y) \rfloor) = \lfloor \beta(\eta(\lfloor x \rfloor),\ \eta(\lfloor y \rfloor)) \rfloor$.

2228) $\mu(\eta(x)) = \mu(\varphi(x))$.

2229) $\eta(x) = \lfloor \theta(x) \rfloor$.

2230) $\eta(x) = \lfloor \psi(x) \rfloor$.

2231) $\eta(m\ x + n) = A_x^n = x(x - 1)\ \ldots\ (x - n + 1)$.

2232) $\eta(m\ x + n) = A_x^m$.

2233) $\eta(m\ x + n) = \binom{x}{n} = \dfrac{x!}{n!(x-n)!}$.

2234) $\eta(m\ x + n) = \binom{x}{m}$.

2235) $\eta(m\ x + n) = p_x$ = the x-th prime.

2236) $\eta(m\ x + n) = \lfloor 1/B_x \rfloor$.

2237) $\eta(m\ x + n) = G_x$.

2238) $\eta(m\ x + n) = k_x = \binom{\ }{n}$.

2239) $\eta(m\ x + n) = k_x^m$.

2240) $\eta(m\ x + n) = s(m, x)$.

2241) $\eta(m\ x + n) = s(x, n)$.

2242) $\eta(m\ x + n) = S(m, x)$.

2243) $\eta(m\ x + n) = S(x, n)$.

2244) $\eta(m\ x + n) = \pi_x$.

2245) $\eta(m\ x + n) = b_x$.

2246) $\eta(m\ x + n) = |E_x|$.

2247) $\eta(m\ x + n) = !\ x$.

2248) $\eta(x) \equiv \eta(y) \pmod{m}$.

2249) $\eta(xy) \equiv x \pmod{y}$.

2250) $\eta(x)\ (x + m) + \eta(y)\ (y + m) = \eta(z)\ (z + m)$.

2251) $\eta(m\ x + n) = f_x$.

2252) $\eta(m\ x + n) = F_x$.

2253) $\eta(m\ x + n) = M_x$.

2254) $\eta\ (m\ x + n) = c_x$.

2255) $\eta\ (m\ x + n) = C_x$.

2256) $\eta\ (m\ x + n) = h_x$.

2257) $\eta\ (m\ x + n) = L_x$.

More unsolved diophantine equations concerning the function η in correlation with other functions.

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§9. Unsolved Diophantine Equations Concerning the Function η in Composition with Other Functions

2258) $\eta(d(x)) = d(\eta(x))$, x is not prime.

2259-2275) Same equations as this, but we substitute the function d(x) with d_x, p(x), ..., ω(x) respectively.

More unsolved diophantine equations concerning the function η in composition with other functions. (For example: $\eta(\pi(4(x))) = \varphi(\eta(\pi(x)))$, etc.)

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§10. Unsolved Diophantine Inequations Concerning the Function η

Let m, n be fixed integers. Solve the following diophantine inequalities:

2276) $\eta(x) \geq \eta(y)$.

2277) is $0 < \{x/\eta(x)\} < \{\eta(x)/x\}$ infinitely often? where {a} is the fractional part of a.

2278) $\eta(m x + n) < d(x)$.

2279-2300) Same (or similar) inequations as this, but we substitute the function d(x) with d_x, p(x), ..., ω(x), Γ(x), β(x, x), μ(x), θ(x), Ψ(x), respectively.

More unsolved diophantine inequations concerning the function η in correlation (or composition, etc.) with other functions. (For example: $\theta(\eta(\lfloor x \rfloor)) < \eta(\lfloor \theta(x) \rfloor)$, etc.)

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§11. Arithmetic Functions Constructed by Means of the Function η

UNSOLVED PROBLEMS CONCERNING THESE NEW FUNCTIONS

I. The function $S_\eta : N^* \to N$, $S_\eta(x) = \sum_{0<n\leq x} \eta(n)$.

2301) Is $\sum_{x\geq 2} S_\eta(x)^{-1}$ a convergent series?

2302) Find the smallest k for which $(\underbrace{S_\eta \circ \ldots \circ S_\eta}_{k \text{ times}})(m) \geq n$, for m, n fixed integers.

2303-4602) Study S_η. The same (or similar questions for S_η as for η.

II. The function $C_\eta : N^* \to Q$, $C_\eta(x) = \frac{1}{x}(\eta(1) + \eta(2) + \ldots + \eta(x))$ (sum of Cesaro concerning the function η).

4603) Is $\sum_{x\geq 1} C_\eta(x)^{-1}$ a convergent series?

4604) Find the smallest k for which $(\underbrace{C_\eta o \ldots o C_\eta}_{k \text{ times}})(m) \geq n$, for m, n fixed integers.

4605)-6904) Study C_η. The same (or similar) questions for C_η as for η.

III. The function $E_\eta : N^* \to N$, $E_\eta(x) = \sum_{k=1}^{k_0} \eta^{(k)}(x)$, where

$\eta^{(1)} = \eta$ and $\eta^{(k)} = \eta o \ldots o \eta$ of k times, and k_0 is the smallest integer k for which $\eta^{(k+1)}(x) = \eta^{(k)}(x)$.

6905) Is $\sum_{x \geq 2} E_\eta(x)^{-1}$ a convergent series?

6906) Find the smallest x for which $E_\eta(x) > m$, where m is a fixed integer.

6907-9206) Study E_η. The same (or similar) questions for S_η as for η.

IV. The function $F_\eta : N \setminus \{0, 1\} \to N$, $F_\eta(x) = \sum_{\substack{0<p\leq x \\ p \text{ prime}}} \eta_p(x)$.

9207) Is $\sum_{x \geq 2} F_\eta(x)^{-1}$ a convergent series?

9208-11507) Study the function F_η. The same (or similar questions for F_η as for η.

V. The function $\alpha_\eta : N^* \to N$, $\alpha_\eta(x) = \sum_{n=1}^{x} \beta(n)$, where

$$\beta(n) = \begin{cases} 0, & \text{if } \eta(n) \text{ is even;} \\ 1, & \text{if } \eta(n) \text{ is old.} \end{cases}$$

11508) Let $n \in N^*$. Find the smallest k for which

$$\underbrace{(\alpha_\eta o \ldots o \alpha_\eta)}_{k \text{ times}}(n) = 0.$$

11509-13808) Study α_η. The same (or similar) questions for α_η as for η.

VI. The function $m_\eta : N^* \to N$, $m_\eta(j) = a_j$, $1 \le j \le n$, fixed integers, and $m_\eta(n+1) = \min_i \{\eta(a_i + a_{n-i})\}$, etc.

13809) Is $\sum_{x \ge 1} m_\eta(x)^{-1}$ a convergent series?

13810-16109) Study m_η. The same (or similar) questions for m_η as for η.

VII. The function $M_\eta : N^* \to N$. A given finite positive integer sequence $a_1, \ldots, a_n$ is successively extended by:

$M_\eta(n+1) = \max_i \{\eta(a_i + a_{n-i})\}$, etc.

$M_\eta(j) = a_j$, $1 \le j \le n$.

16110) Is $\sum_{x \ge 1} M_\eta(x)^{-1}$ a convergent series?

16111-18410) Study M_η. The same (or similar) questions for M_η as for η.

VIII. The function $\eta_{min}^{-1} : N \setminus \{1\} \to N$, $\eta_{min}^{-1}(x) = \min\{\eta^{-1}(x)\}$, where $\eta^{-1}(x) = \{a \in N \mid \eta(a) = x\}$. For example $\eta^{-1}(6) = \{2^4, 2^4 \cdot 3, 2^4 \cdot 3^2, 3^2, 3^2 \cdot 2, 3^2 \cdot 2^2, 3^2 \cdot 2^3\}$, whence $\eta_{min}^{-1}(6) = 9$.

18411) Find the smallest k for which $(\underbrace{\eta_{min}^{-1} \circ \ldots \circ \eta_{min}^{-1}}_{k \text{ times}})$

18412-20711) Study η_{min}^{-1}. The same (or similar) questions for η_{min}^{-1} as for η.

IX. The function $\eta^{-1}_{card} : N \to N$, $\eta^{-1}_{card}(x) = \text{Card}\{\eta^{-1}(x)\}$, where Card A means the number of elements of the set A.

20712) Find the smallest k for which

$$\big(\underbrace{\eta^{-1}_{card} \circ \ldots \circ \eta^{-1}_{card}}_{k \text{ times}}\big)(m) \geq n, \text{ for } m, n \text{ fixed integers.}$$

20713-23012) Study η^{-1}_{card}. The same (or similar) questions for η^{-1}_{card} as for η.

X. The function $d_\eta : N^* \to N$, $d_\eta(x) = |\eta(x+1) - \eta(x)|$.

Let $d_\eta^{(k+1)}(x) = |d_\eta^{(k)}(x+1) - d_\eta^{(k)}(x)|$, for all $k \in N^*$, where $d_\eta^{(1)}(x) = d_\eta(x)$.

23013) Conjecture: $d_\eta^{(k)}(1) = 1$ or 0, for all $k \geq 2$. (This reminds us of Gillreath's conjecture on primes.) For example:

```
η(1) = 0
            2
η(2) = 2       1
            1     1
η(3) = 3       0     1
            1     0     0
η(4) = 4       0     1     1
            1     1     1     0
η(5) = 5       1     0     1     0
            2     1     0     0     1
η(6) = 3       2     0     1     1     0
            4     1     1     1     1     1
η(7) = 7       1     1     0     2     1     0
            3     0     1     3     2     1     1
η(8) = 4       1     0     3     0     0     1     0
            2     0     4     3     2     0     1     1
η(9) = 6       1     4     0     2     0     0     1     1
            1     4     4     1     2     0     2     0     1
η(10)= 5       5     0     1     0     0     2     1     0     1
            6     4     3     1     2     2     1     0     0     0
η(11)=11       1     3     0     2     2     1     1     0     1
            7     1     3     3     0     1     0     0     1
η(12)= 4       2     0     3     2     1     1     1     1
            9     1     0     1     1     0     1     1
η(13)=13       3     0     2     1     1     0     0
            6     1     2     0     0     0     1
η(14)= 7       4     2     2     1     1     1
            2     3     4     1     1     1
η(15)= 5       1     6     1     0     0
            1     9     5     1     1
η(16)= 6      10     1     2     1
           11    10     7     2
η(17)=17       0     8     0
           11     2     7
η(18)= 6       2     1
           13     1
η(19)=19       1
           14
η(20)= 5
  .
  .
  .
```

23014-25313) Study $d_\eta^{(k)}$. The same (or similar) questions for $d_\eta^{(k)}$ as for η.

XI. The function $\omega_\eta : N^* \to N$, $\omega_\eta(x)$ is the number of m, with $0 < m \leq x$, so that $\eta(m)$ divide x. Hence, $\omega_\eta(x) \geq \omega(x)$, and we have equality if x = 1 or x is a prime.

25314) Find the smallest k for which $(\underbrace{\omega_\eta \circ \dots \circ \omega_\eta}_{k \text{ times}})(x) =$ $= 0$, for a fixed integer x.

25315-27614) Study ω_η. The same (or similar) questions for ω_η as for η.

XII. The function $M_\eta : N^* \to N$, $M_\eta(x)$ is the number of m, with $0 < m \leq x^x$, so that $\eta(m)$ is a multiple of x. For example $M_\eta(3) = \text{Card}\{1, 3, 6, 9, 12, 27\} = 6$. If p is a prime, $M_\eta(p) = \text{Card}\{1, a_2, \dots, a_r\}$, then all a_i, $2 \leq i \leq r$, are multiples of p.

27615) Let m, n be integer numbers. Find the smallest k for which $(\underbrace{M_\eta \circ \dots \circ M_\eta}_{k \text{ times}})(m) \geq n$.

27616-29915) Study M_η. The same (or similar questions for M_η as for η.

XIII. The function $\sigma_\eta : N^* \to N$, $\sigma_\eta(x) = \sum_{\substack{d|x \\ d>0}} \eta(d)$.

For example $\sigma_\eta(18) = \eta(1) + \eta(2) + \eta(3) + \eta(6) + \eta(9) +$ $+ \eta(18) = 20$, $\sigma_\eta(9) = 9$.

29916) Are there an infinity of nonprimes n so that $\sigma_\eta(n) = n$?

29917-32216) Study σ_η. The same (or similar) questions for σ_η as for η..

XIV. The function $\pi_\eta : N \to N$, $\pi_\eta(x)$ is the number of numbers n so that $\eta(n) \le x$. If $p_1 < p_2 < \ldots < p_k \le n < p_{k+1}$ is the primes sequence, and for all $i = 1, 2, \ldots, k$ we have $p_i^{a_i}$ divides n! but $p_i^{a_i+1}$ does not divide n!, then:

$$\pi_\eta(n) = (a_1 + 1) \ldots (a_k + 1).$$

32217-34516) Study π_η. The same (or similar) question for π_η as for η.

XV. The function $\varphi_\eta : N^* \to N$, $\varphi_\eta(x)$ is the number of m, with $0 < m \le x$, having the property $(\eta(m), x) = 1$.

34517) Is always true that $\varphi_\eta(x) < \varphi(x)$?

34518) Find x for which $\varphi_\eta(x) \ge \varphi(x)$.

34519) Find the smallest k so that $(\underbrace{\varphi_\eta \circ \ldots \circ \varphi_\eta}_{k \text{ times}})(x) = 1$, for a fixed integer x.

34520-36819) Study φ_η. The same (or similar) questions for φ_η as for η.

More unsolved problems concerning these 15 functions.

More new (arithmetic) functions constructed by means of the function η, and new unsolved problems concerning them.

36820 → ∞. We can continue these recurring sequences of unsolved problems in number theory to infinity. Thus, we construct an infinity of more new functions: Using the functions S_η, C_η, ..., φ_η construct the functions f_{11}, f_{12}, ..., f_{1n_1} (by varied combinations between S_η, C_η, ..., φ_η; for example: $S_\eta^{(i+1)}(x) = \sum_{0<n\leq x} S_\eta^{(i)}$ for all $x \in N^*$,

$S_\eta^{(i)} : N^* \to N$ for all $i = 0, 1, 2, \ldots$, where $S_\eta^{(0)} = S_\eta$. Or:

$SC_\eta(x) = \frac{1}{x} \sum_{n=1}^{x} S_\eta(n)$, $SC_\eta : N^* \to Q$, SC_η being a combination

between S_η and C_η; etc.); analogously by means of the functions f_{11}, f_{12}, ... f_{1n_1} we construct the functions f_{21}, f_{22}, ..., f_{2n_2} etc. The method to obtain new functions continues to infinity. For each function we have at least 2300 unsolved problems, and we have an infinity of thus functions. The method can be represented in the following way:

$$\eta \xrightarrow{\text{produces}} S_\eta, C_\eta, \ldots, \varphi_\eta \to f_{11}, f_{12}, \ldots, f_{1n_1}$$

$$f_{11}, f_{12}, \ldots, f_{1n_1} \longrightarrow f_{21}, f_{22}, \ldots, f_{2n_2}$$

$$f_{21}, f_{22}, \ldots, f_{2n_2} \longrightarrow f_{31}, f_{32}, \ldots, f_{3n_3}$$

$$f_{i1}, f_{i2}, \ldots, f_{in_i} \longrightarrow f_{i+1,1}, f_{i+1,2}, \ldots, f_{i+1,n_{i+1}}$$

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Other recurring methods to make new unsolved problems.

§12. Conclusion

With this paper the author wants to prove that we can construct infinitely many unsolved problems, especially in number theory: you "rock and roll" the numbers until you create interesting scenarios! Some problems in this paper could effect the subsequent development of mathematics.

The world is in a general crisis. Do the unsolved problems really constitute a mathematical crisis, or contrary to that, do their absence lead to an intellectual stagnation? Mankind will always have problems to solve, they even must again solve previously solved problems(!) For example, this paper shows that people will be more and more overwhelmed by (open) unsolved problems. [It is easier to ask than to answer.]

Here, there are proposed (un)solved problems which are enough for ever!! Suppose you solve an infinite number of problems, there will always be an infinity of problems remaining. Do not assume those proposals are trivial and non-important, rather, they are very substantial.

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