PROPOSED PROBLEM

by Thomas Martin

Let $\eta : Z^* \to N$ Smarandache Function: $\eta(m)$ is the smallest integer n such that $n!$ is divisible by m.

a) Prove that for any number $k \in R_*$ there exist a series $\{p_i\}_i$ of positive integer numbers such that :

$$L = \lim_{i \to \infty} \frac{p_i}{\eta(p_i)} > k$$

b) Does $L = \lim_{n \to \infty} \frac{m}{\eta(m)}$ diverge to $+\infty$.

Solution:

a) Let p_j be a prime number greater than k. Index j is fixed. We construct $p_i = p_j p_{j+i}$, for $i = 1,2,3\ldots$.

Lemma 1. If $u < v$ are prime numbers, then $\eta(uv) = v$.

Of course $v! = 1 \cdot 2 \cdot \ldots \ldots \cdot u \cdot \ldots \cdot v = \mathcal{M}_v = \mathcal{M}_u$.

Hence $\eta(p_i) = p_{j+i}$, for any $i = 1,2,3\ldots$ where p_{j+i} is the $j+i^{th}$ prime number. Then $L = p_j > k$.

b) Because there exists an infinity of primes : $p_j, p_{j+1}, \ldots,$ greater than k, we find an infinity of limits for each $\{p_i(j)\}_i$ series, i.e. $L = p_{j+1}$ or $L = p_{j+2}$ etc.

Therefore $L = \lim_{m \to \infty} \frac{m}{\eta(m)}$ does not exist!

Reference:

R. Muller, "Smarandache Function Journal", Vol. 1. No. 1, 1990.