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$$e^{i\pi} = -1$$

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A Group-Permutation Algorithm to Solve the Generalized SUDOKU[1]

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Abstract. Sudoku can be generalized to squares whose dimensions are $n^2 \times n^2$, where $n \geq 2$, using various symbols (numbers, letters, mathematical symbols, etc.), written just one time on each row and on each column; and the large square is divided into n^2 small squares with the side $n \times n$ and each will contain all n^2 symbols written only once. In this paper we present an elementary solution for the generalized sudoku based on a group-permutation algorithm.

Keywords: permoutation, group, sudoku.

MSC 2000: 00A08, 97A20.

Sudoku is a game with numbers, formed by a square with the side of 9, and on each row and column are placed the digits $1, 2, 3, 4, 5, 6, 7, 8, 9$, written only one time; the square is subdivided in 9 smaller squares with the side of 3×3, which, also, must satisfy the same condition, i.e. each square to contain all digits from 1 to 9 written only once.

The Japanese company Nikoli has popularized this game in 1986 under the name of *sudoku*, meaning "single number".

Sudoku can be generalized to squares whose dimensions are $n^2 \times n^2$, where $n \geq 2$, using various symbols (numbers, letters, mathematical symbols, etc.), written just one time on each row and on each column; and the large square is divided into n^2 small squares with the side $n \times n$ and each will contain all n^2 symbols written only once.

An *elementary solution* of one of these *generalized Sudokus*, with elements (symbols) from the set

$$S = \{s_1, s_2, \ldots, s_n, s_{n+1}, \ldots, s_{2n}, \ldots, s_{n^2}\}$$

(supposing that their placement represents the relation of total order on the set of elements S), is:

Row 1: all elements in ascending order

$$s_1, s_2, \ldots, s_n, s_{n+1}, \ldots, s_{2n}, \ldots, s_{n^2}$$

On the next rows we will use circular permutations, considering groups of n elements from the first row as follows:

Row 2:

$$s_{n+1}, s_{n+2}, \ldots, s_{2n}; s_{2n+1}, \ldots, s_{3n}; \ldots, s_{n^2}; s_1, s_2, \ldots, s_n$$

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Row 3:

$$s_{2n+1}, \ldots, s_{3n}; \ldots, s_{n^2}; s_1, s_2, \ldots, s_n; s_{n+1}, s_{n+2}, \ldots, s_{2n},$$

....
Row n:

$$s_{n^2-n+1}, \ldots, s_{n^2}; s_1, \ldots, s_n; s_{n+1}, s_{n+2}, \ldots, s_{2n}; \ldots, s_{3n}; \ldots, s_{n^2-n}.$$

Now we start permutations of the elements of row n +1 considering again groups of n elements.

Row $n+1$:

$$s_2, \ldots, s_n, s_{n+1}; s_{n+2}, \ldots, s_{2n}, s_{2n+1}; \ldots; s_{n^2-n+2}, \ldots, s_{n^2}, s_1$$

Row $n+2$:

$$s_{n+2}, \ldots, s_{2n}, s_{2n+1}; \ldots; s_{n^2-n+2}, \ldots, s_{n^2}, s_1; s_2, \ldots, s_n, s_{n+1}$$

....
Row $2n$:

$$s_{n^2-n+2}, \ldots, s_{n^2}, s_1; s_2, \ldots, s_n, s_{n+1}; s_{n+2}, \ldots, s_{2n}, s_{2n+1}; \ldots$$

Row $2n+1$:

$$s_3, \ldots, s_{n+2}; s_{n+3}, \ldots, s_{2n+2}; \ldots; s_{n^2+3}, \ldots, s_{n^2}, s_1, s_2$$

and so on.

Replacing the set S by any *permutation* of its symbols, which we'll note by S', and applying the same procedure as above, we will obtain a new solution.

The *classical Sudoku* is obtained for $n = 3$.

Below is an example of this group-permutation algorithm for the classical case:

1	2	3	4	5	6	7	8	9
4	5	6	7	8	9	1	2	3
7	8	9	1	2	3	4	5	6
2	3	4	5	6	7	8	9	1
5	6	7	8	9	1	2	3	4
8	9	1	2	3	4	5	6	7
3	4	5	6	7	8	9	1	2
6	7	8	9	1	2	3	4	5
9	1	2	3	4	5	6	7	8

For a $4^2 \times 4^2$ square we use the following 16 symbols:

$$\{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P\}$$

and use the same group-permutation algorithm to solve this Sudoku.

From one solution to the generalized Sudoku we can get *more solutions* by simply doing permutations of columns or/and of rows of the first solution.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
E	F	G	H	I	J	K	L	M	N	O	P	A	B	C	D
I	J	K	L	M	N	O	P	A	B	C	D	E	F	G	H
M	N	O	P	A	B	C	D	E	F	G	H	I	J	K	L
B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	A
F	G	H	I	J	K	L	M	N	O	P	A	B	C	D	E
J	K	L	M	N	O	P	A	B	C	D	E	F	G	H	I
N	O	P	A	B	C	D	E	F	G	H	I	J	K	L	M
C	D	E	F	G	H	I	J	K	L	M	N	O	P	A	B
G	H	I	J	K	L	M	N	O	P	A	B	C	D	E	F
K	L	M	N	O	P	A	B	C	D	E	F	G	H	I	J
O	P	A	B	C	D	E	F	G	H	I	J	K	L	M	N
D	E	F	G	H	I	J	K	L	M	N	O	P	A	B	C
H	I	J	K	L	M	N	O	P	A	B	C	D	E	F	G
L	M	N	O	P	A	B	C	D	E	F	G	H	I	J	K
P	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O

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