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A Method of Resolving in Integer Numbers of Certain Nonlinear Equations

In Florentin Smarandache: "Collected Papers", vol. I (second edition). Ann Arbor (USA): InfoLearnQuest, 2007.

A METHOD OF RESOLVING IN INTEGER NUMBERS OF CERTAIN NONLINEAR EQUATIONS

Let's consider a polynomial with integer coefficients, of degree m

$$P(X_1,...,X_n)=\sum_{\substack{0\le i_1+...+i_n\le m\\ 0\le i_j\le m,\, j=\overline{1,n}}} a_{i_1...i_n}X_1^{i_1}...X_n^{i_n}$$

which can be decomposed in linear factors (which can eventually be established through the undetermined coefficients method):

$$P(X_1,...,X_n)=\left(A_1^{(1)}X_1+...+A_n^{(1)}X_n+A_{n+1}^{(1)}\right)\cdots\left(A_1^{(m)}X_1+...+A_n^{(m)}X_n+A_{n+1}^{(m)}\right)+B$$

with all $A_j^{(k)}$, B in $\mathbb{Q}$, but which by bringing to the same common denominator and by eliminating it from the equation $P(X_1,...,X_n)=0$ they can be considered integers.. Thus the equation transforms in the following system:

$$\begin{cases} A_1^{(1)}X_1+...+A_n^{(1)}X_n+A_{n+1}^{(1)}=D_1\\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\ A_1^{(m)}X_1+...+A_n^{(m)}X_n+A_{n+1}^{(m)}=D_m \end{cases}$$

where $D_1,...,D_m$ are the divisors for B and $D_1\cdots D_m=B$.

We resolve separately each linear Diophantine equation and then we intersect the equations.

Example. Resolve in integer numbers the equation:

$$-2x^3+5x^2y+4xy^2-3y^3-3=0.$$

We'll write the equation in another format

$$(x+y)(2x-y)(-x+3y)=3.$$

Let m, n and p be the divisors of $3, m\cdot n\cdot p=3$. Thus

$$\begin{cases} x+y\ =m\\ 2x-y\ =n\\ -x+3y=p \end{cases}$$

For this system to be compatible it is necessary that

$$\begin{pmatrix} 1 & 1 & m\\ 2 & -1 & n\\ -1 & 3 & p \end{pmatrix}=0,$$

or

$$5m-4n-3p=0 \qquad (1)$$

In this case

$$x=\frac{m+n}{3} \text{ and } y=\frac{2m-n}{3} \qquad (2)$$

Because $m,n,p\in\mathbb{Z}$, from (1) it results – by resolving in integer numbers – that:

$$\begin{cases} m = 3k_1 - k_2 \\ n = \qquad k_2 \\ p = 5k_1 - 3k_2 \end{cases} \qquad k_1, k_2 \in \mathbb{Z}$$

which substituted in (2) will give us $x = k_1$ and $y = 2k_1 - k_2$. But $k_2 \in D(3) = \{\pm 1, \pm 3\}$; thus the only solution is obtained for $k_2 = 1$, $k_1 = 0$ from where $x = 0$ and $y = -1$.

Analogue it can be shown that, for example the equation:

$$-2x^3 + 5x^2y + 4xy^2 - 3y^3 = 6$$

does not have solutions in integer numbers.

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