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New Results of Intuitionistic Fuzzy Soft Set

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Abstract—In this paper, three new operations are introduced on intuitionistic fuzzy soft sets .They are based on concentration, dilatation and normalization of intuitionistic fuzzy sets. Some examples of these operations were given and a few important properties were also studied.

Index Terms—Soft Set, Intuitionistic Fuzzy Soft Set, Concentration, Dilatation, Normalization.

I. Introduction

The concept of the intuitionistic fuzzy (IFS, for short) was introduced in 1983 by K. Aanassov [1] as an extension of Zadeh's fuzzy set. All operations, defined over fuzzy sets were transformed for the case the IFS case .This concept is capable of capturing the information that includes some degree of hesitation and applicable in various fields of research. For example, in decision making problems, particularly in the case of medical diagnosis, sales analysis, new product marketing, financial services, etc. Atanassov et.al [2,3] have widely applied theory of intuitionistic sets in logic programming, Szmidt and Kacprzyk [4] in group decision making , De et al [5] in medical diagnosis etc. Therefore in various engineering application, intuitionistic fuzzy sets techniques have been more popular than fuzzy sets techniques in recent years. Another important concept that addresses uncertain information is the soft set theory originated by Molodtsov [6]. This concept is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Molodtsov has successfully applied the soft set theory in many different fields such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, and probability. In recent years, soft set theory has been received much attention since its appearance. There are many papers devoted to fuzzify the concept of soft set theory which leads to a series of mathematical models such as fuzzy soft set [7,8,9,10,11], generalized fuzzy soft set [12,13], possibility fuzzy soft set [14] and so on. Thereafter, P.K.Maji and his coauthor [15] introduced the notion of intuitionistic fuzzy soft set which is based on a combination of the intuitionistic fuzzy sets and soft set models and they studied the properties of intuitionistic fuzzy soft set. Then, a lot of extensions of intuitionistic fuzzy soft have appeared such as generalized intuitionistic fuzzy soft set [16], possibility intuitionistic fuzzy soft set [17] etc.

In this paper our aim is to extend the two operations defined by Wang et al. [18] on intuitionistic fuzzy set to the case of intuitionistic fuzzy soft sets, then we define the concept of normalization of intuitionistic fuzzy soft sets and we study some of their basic properties.

This paper is arranged in the following manner .In section 2, some definitions and notions about soft set, fuzzy soft set, intuitionistic fuzzy soft set and several properties of them are presented. In section 3, we discuss the normalization intuitionistic fuzzy soft sets. In section 4, we conclude the paper.

II. Preliminaries

In this section, some definitions and notions about soft sets and intutionistic fuzzy soft set are given. These will be useful in later sections.

Let U be an initial universe, and E be the set of all possible parameters under consideration with respect to U. The set of all subsets of U, i.e. the power set of U is denoted by P(U) and the set of all intuitionistic fuzzy subsets of U is denoted by IF^U. Let A be a subset of E.

2.1 Definition

A pair (F, A) is called a soft set over U, where F is a mapping given by F: A→ P (U).

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in$ A, F (ε) may be considered as the set of ε -approximate elements of the soft set (F, A).

2.2 Definition

Let U be an initial universe set and E be the set of parameters. Let IF^U denote the collection of all intuitionistic fuzzy subsets of U. Let. A ⊆ E pair (F, A) is called an intuitionistic fuzzy soft set over U where F is a mapping given by F: A→ IF^U.

2.3 Defintion

Let F: A→ IF^U then F is a function defined as F (ε) ={ x, $\boldsymbol{\mu_{F(\varepsilon)}(x)}$, $\boldsymbol{\nu_{F(\varepsilon)}(x) : x \in U}$, $\varepsilon \in \boldsymbol{E}$} where μ , ν denote the degree of membership and degree of non-membership respectively.

2.4 Definition

For two intuitionistic fuzzy soft sets (F , A) and (G, B) over a common universe U , we say that (F , A) is an intuitionistic fuzzy soft subset of (G, B) if

(1) A ⊆ B and

(2) F (ε) ⊆ G(ε) for all $\varepsilon \in$ A. i.e $\boldsymbol{\mu_{F(\varepsilon)}(x)} \leq \boldsymbol{\mu_{G(\varepsilon)}(x)}$, $\boldsymbol{\nu_{F(\varepsilon)}(x)} \geq \boldsymbol{\nu_{G(\varepsilon)}(x)}$ for all $\varepsilon \in$ E and

We write (F, A) ⊆ (G, B).

2.5 Definition

Two intuitionistic fuzzy soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

2.6 Definition

Let U be an initial universe, E be the set of parameters, and A ⊆ E.

(a) (F, A) is called a null intuitionistic fuzzy soft set (with respect to the parameter set A), denoted by φ_A, if F (a) = φ for all a ∈ A.

(b) (G, A) is called an absolute intuitionistic fuzzy soft set (with respect to the parameter set A), denoted by U_A, if G(e) = U for all e ∈ A.

2.7Definition

Let (F, A) and (G, B) be two IFSSs over the same universe U. Then the union of (F, A) and (G, B) is denoted by '(F, A) ∪ (G, B)' and is defined by (F, A) ∪ (G, B) = (H, C), where C=A ∪ B and the truth-membership, falsity-membership of (H, C) are as follows:

$$\boldsymbol{H}(\varepsilon) = \begin{cases} \{(\boldsymbol{\mu_{F(\varepsilon)}(x)}, \boldsymbol{\nu_{F(\varepsilon)}(x)} : \boldsymbol{x} \in \boldsymbol{U}\} & \text{, if } \varepsilon \in \text{A} - \text{B}, \\ \{(\boldsymbol{\mu_{G(\varepsilon)}(x)}, \boldsymbol{\nu_{G(\varepsilon)}(x)} : \boldsymbol{x} \in \boldsymbol{U}\} & \text{, if } \varepsilon \in \text{B} - \text{A} \\ \left\{\max\left(\boldsymbol{\mu_{F(\varepsilon)}(x)}, \boldsymbol{\mu_{G(\varepsilon)}(x)}\right), \min\left(\boldsymbol{\nu_{F(\varepsilon)}(x)}, \boldsymbol{\nu_{G(\varepsilon)}(x)}\right) : \boldsymbol{x} \in \boldsymbol{U}\right\}, & \text{if } \varepsilon \in \text{A} \cap \text{B} \end{cases}$$

Where $\boldsymbol{\mu_{H(\varepsilon)}(x)} = \max(\boldsymbol{\mu_{F(\varepsilon)}(x)}, \boldsymbol{\mu_{G(\varepsilon)}(x)})$ and $\boldsymbol{\nu_{H(\varepsilon)}(x)} = \min(\boldsymbol{\nu_{F(\varepsilon)}(x)}, \boldsymbol{\nu_{G(\varepsilon)}(x)})$

2.8 Definition

Let (F, A) and (G, B) be two IFSSs over the same universe U such that A ∩ B≠0. Then the intersection of (F, A) and (G, B) is denoted by '(F, A) ∩ (G, B)' and is defined by (F, A) ∩ (G, B) = (K, C),where C =A ∩B and the truth-membership, falsity-membership of (K, C) are related to those of (F, A) and (G, B) by:

$$\boldsymbol{K}(\varepsilon) = \begin{cases} \{(\boldsymbol{\mu_{F(\varepsilon)}(x)}, \boldsymbol{\nu_{F(\varepsilon)}(x)} : \boldsymbol{x} \in \boldsymbol{U}\} & \text{, if } \varepsilon \in \text{A} - \text{B}, \\ \{(\boldsymbol{\mu_{G(\varepsilon)}(x)}, \boldsymbol{\nu_{G(\varepsilon)}(x)} : \boldsymbol{x} \in \boldsymbol{U}\} & \text{, if } \varepsilon \in \text{B} - \text{A} \\ \left\{\min\left(\boldsymbol{\mu_{F(\varepsilon)}(x)}, \boldsymbol{\mu_{G(\varepsilon)}(x)}\right), \max\left(\boldsymbol{\nu_{F(\varepsilon)}(x)}, \boldsymbol{\nu_{G(\varepsilon)}(x)}\right) : \boldsymbol{x} \in \boldsymbol{U}\right\}, & \text{if } \varepsilon \in \text{A} \cap \text{B} \end{cases}$$

III. CONCENTRATION OF INTUITIONISTIC FUZZY SOFT SET

3.1 Definition

The concentration of an intuitionistic fuzzy soft set (F, A) of universe U, denoted by **CON** (F, A), and is defined as a unary operation on IF^U:

$$\textbf{Con}: IF^U \to IF^U$$

Con (F, A) =
{Con {F(ε) } = {<x, $\boldsymbol{\mu^2_{F(\varepsilon)}}(x)$, 1- $(1 - \boldsymbol{\nu}_{(\boldsymbol{F(\varepsilon)})}(\boldsymbol{x}))^2$ > | $\boldsymbol{x}$ ∈ U and $\varepsilon \in$ A}. where

From $0 \leq \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \leq 1$

and $\mu_{F(\varepsilon)}(x) + \nu_{F(\varepsilon)}(x) \leq 1$,

we obtain $0 \leq \mu^2_{F(\varepsilon)}(x) \leq \mu_{F(\varepsilon)}(x)$

$1 \geq 1 - (1 - \boldsymbol{\nu}_{(\boldsymbol{F(\varepsilon)})}(\boldsymbol{x}))^2 \geq \nu_{F(\varepsilon)}(x)$

and **Con** (F, A) ∈ IF^U, i.e **Con** (F, A) ⊆ (F, A) this means that concentration of a intuitionistic fuzzy soft set leads to a reduction of the degrees of membership.

In the following theorem, The operator "**Con** "reveals nice distributive properties with respect to intuitionistic union and intersection.

3.2 Therorem

i. **Con** (F, A) ⊆ (F, A)

ii. **Con** ((F, A) ∪ (G,B)) = **Con** (F, A) ∪ **Con** (G, B)

iii. **Con** ((F, A) ∩ (G,B)) = **Con** (F, A) ∩ **Con** (G, B)

iv. **Con** ((F, A) ⊗ (G,B))= **Con** (F, A) ⊗ Con (G,B)

v. **Con** (F, A) ⊕ **Con** (G, B) ⊆ **Con** ((F, A) ⊕ (G,B))

vi. (F, A) ⊆ (G, B) ⇒ **Con** (F, A) ⊆ Con(G, B)

Proof , we prove only (v) ,i.e

$\mu^2_{F(\varepsilon)}(x)$ + $\mu^2_{G(\varepsilon)}(x)$ - $\mu^2_{F(\varepsilon)}(x)\ \mu^2_{G(\varepsilon)}(x) \leq (\ \mu_{F(\varepsilon)}(x) + \mu_{G(\varepsilon)}(x) - \mu_{F(\varepsilon)}(x)\ \mu_{G(\varepsilon)}(x))^2$,

(1- $(1-\ \nu_{F(\varepsilon)}\ (x))^2$). (1- $(1-\nu_{G(\varepsilon)}(x))^2$) ≥ 1- $(1-\nu_{F(\varepsilon)}(x).\nu_{G(\varepsilon)}(x))^2$ or, putting

a= $\mu_{F(\varepsilon)}(x)$, b= $\mu_{G(\varepsilon)}(x)$, c = $\nu_{F(\varepsilon)}(x)$, d = $\nu_{G(\varepsilon)}(x)$

$a^2 + b^2$ - $a^2\ b^2 \leq (\ a + \ b - \ a\ b)^2$,

(1- $(1-\ c)^2$) . (1- $(1-\ d)^2$) ≥ 1- $(1-c.d)^2$

The last inequality follows from 0 ≤ a, b, c, d ≤ 1.

Example

Let U={a, b, c} and E ={ e_1, e_2 , e_3 , e_4} , A ={ e_1, e_2, e_4} ⊆ E, B={ e_1, e_2 , e_3} ⊆ E

(F, A) ={ **F(e_1)** ={((a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)}, **F(e_2)** ={((a, 0.7, 0.1), (b, 0, 0.8), (c, 0.3, 0.5)}, **F(e_4)** ={((a, 0.6, 0.3), (b, 0.1, 0.7), (c, 0.9, 0.1)}}

(G, B) ={ **G(e_1)** ={((a, 0.2, 0.6), (b, 0.7, 0.1), (c, 0.8, 0.1)}, **G(e_2)** ={((a, 0.4, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)}, **G(e_3)** ={((a, 0, 0.6), (b, 0, 0.8), (c, 0.1, 0.5)}}

Con (F, A)={ **con(F(e_1))** ={ (a, 0.25, 0.19), (b, 0.01, 0.96), (c, 0.04, 0.75)}, **con(F(e_2)**)={ (a, 0.49, 0.19), (b, 0, 0.96), (c, 0.09, 0.75) }, **con(F(e_4)**)={ (a, 0.36, 0.51), (b, 0.01, 0.91), (c, 0.81, 0.19) }

Con (G, B)={ **con(G(e_1)**)={ (a, 0.04, 0.84), (b, 0.49, 0.19), (c, 0.64, 0.75)},

con(G(e_2))={ (a, 0.16, 0.19), (b, 0.25, 0.51), (c, 0.16, 0.51) }, **con(G(e_3)**)={ (a, 0, 0.84), (b, 0, 0.96), (c, 0.01, 0.75) }

(F, A) ∩ (G, B) = (H, C) = {**H (e_1)** ={(a, 0.2, 0.6), (b, 0.1, 0.8), (c, 0.2, 0. 5)}, **H (e_2)** ={(a, 0.4, 0.1), (b, 0, 0.8), (c, 0.3, 0. 5)}}

Con ((F, A) ∩ (G,B))= {**con H(e_1)** ={(a, 0.04, 0.84), (b, 0.01, 0.96), (c, 0.04, 0. 75)}, **con H(e_2)** ={(a, 0.16, 0.19), (b, 0, 0.96), (c, 0.09, 0. 75)}}

Con (F, A) ∩ **Con** (G, B) =(K,C) ={**con K(e_1)** ={(a, 0.04, 0.84), (b, 0.01, 0.96), (c, 0.04, 0. 75)}, **conK(e_2)** ={(a, 0.16, 0.19), (b, 0, 0.96), (c, 0.09, 0. 75)}}.

Then

Con ((F, A) ∩ (G,B)) = **Con** (F, A) ∩ **Con** (G, B)

IV. Dilatation of intuitionistic fuzzy soft set

4.1 Definition

The dilatation of an intuitionistic fuzzy soft set (F, A) of universe U, denoted by **DIL** (F, A), and is defined as a unary operation on IF^U:

$$\textbf{DIL}: IF^U \rightarrow IF^U$$

(F, A)= {<x, $\mu_{F(\varepsilon)}(x)$, $\nu_{F(\varepsilon)}(x)$ >|$\boldsymbol{x}$ ∈ U and ε∈ A}.

DIL(F, A) ={ **DIL** {F(ε) } =
{<x, $\boldsymbol{\mu}^{\frac{1}{2}}_{\boldsymbol{F}(\varepsilon)}(x)$, 1- $(1-\boldsymbol{\nu}_{(\boldsymbol{F}(\varepsilon))}(\boldsymbol{x}))^{\frac{1}{2}}$ >|$\boldsymbol{x}$ ∈ U and ε∈ A}.

where
From 0 ≤ $\mu_{F(\varepsilon)}(x)$, $\nu_{F(\varepsilon)}(x)$ ≤ 1,

and $\mu_{F(\varepsilon)}(x)$ +$\nu_{F(\varepsilon)}(x)$ ≤ 1,

we obtain 0 ≤ $\mu_{F(\varepsilon)}(x) \leq \mu^{\frac{1}{2}}_{F(\varepsilon)}(x)$

$$0 \leq (1-\boldsymbol{\nu}_{(\boldsymbol{F}(\varepsilon))}(\boldsymbol{x}))^{\frac{1}{2}} \leq \nu_{F(\varepsilon)}(x)$$

and **DIL**(F, A) ∈ IF^U, i.e (F, A) ⊆ **DIL**(F, A) this means that dilatation of an intuitionistic fuzzy soft set leads to an increase of the degrees of membership.

4.2 Theorem

i. (F, A) ⊆ **DIL**(F, A)

ii. **DIL** ((F, A) ∪ (G, B)) = **DIL** (F, A) ∪ **DIL** (G, B)

iii. **DIL** ((F, A) ∩ (G, B)) = **DIL**(F, A) ∩ **DIL** (G, B)

iv. **DIL**((F, A) ⊗ (G, B))= **DIL** (F, A) ⊗ **DIL** (G,B)

v. **DIL** (F, A) ⊕ **DIL** (G, B) ⊆ **DIL** ((F, A) ⊕ (G,B))

vi. **CON** (**DIL** (F, A)) = (F,A)

vii. **DIL** (**CON** (F, A) = (F,A)

viii. (F, A) ⊆ (G, B) ⇒ **DIL** (F, A) ⊆ DIL(G, B)

Proof .we prove only (v), i.e

$\mu^{\frac{1}{2}}_{F(\varepsilon)}(x) + \mu^{\frac{1}{2}}_{G(\varepsilon)}(x) - \mu^{\frac{1}{2}}_{F(\varepsilon)}(x)\, \mu^{\frac{1}{2}}_{G(\varepsilon)}(x) \geq (\mu_{F(\varepsilon)}(x) + \mu_{G(\varepsilon)}(x) - \mu_{F(\varepsilon)}(x)\ \mu_{G(\varepsilon)}(x))^{\frac{1}{2}}$,

$(1-(1-\nu_{F(\varepsilon)}(x))^{\frac{1}{2}}).(1-(1-\nu_{G(\varepsilon)}(x))^{\frac{1}{2}}) \leq 1-(1-\nu_{F(\varepsilon)}(x).\nu_{G(\varepsilon)}(x))^{\frac{1}{2}}$ or, putting

a= $\mu_{F(\varepsilon)}(x)$, b= $\mu_{G(\varepsilon)}(x)$, c = $\nu_{F(\varepsilon)}(x)$, d = $\nu_{G(\varepsilon)}(x)$

$a^{\frac{1}{2}} + b^{\frac{1}{2}} - a^{\frac{1}{2}}\, b^{\frac{1}{2}} \leq (a + b - a\ b)^{\frac{1}{2}}$,

$(1-(1-c)^{\frac{1}{2}}).(1-(1-d)^{\frac{1}{2}}) \leq 1-(1-c.d)^{\frac{1}{2}}$, or equivalently : a+ b – a b ≤ 1 ,$\sqrt{1-cd} \leq 1$.

The last inequality follows from 0 ≤ a, b,c,d ≤ 1.

Example

Let U={a, b, c} and E ={ e_1, e_2, e_3, e_4}, A ={ e_1, e_2, e_4} ⊆ E, B={ e_1, e_2, e_3} ⊆ E

(F, A) ={ $\mathbf{F(e_1)}$ ={((a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)}, $\mathbf{F(e_2)}$ ={((a, 0.7, 0.1), (b, 0, 0.8), (c, 0.3, 0.5)}, $\mathbf{F(e_4)}$ ={((a, 0.6, 0.3), (b, 0.1, 0.7), (c, 0.9, 0.1)}} and

(G, B) ={ $\mathbf{G(e_1)}$ ={((a, 0.2, 0.6), (b, 0.7, 0.1), (c, 0.8, 0.1)}, $\mathbf{G(e_2)}$ ={((a, 0.4, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)}, $\mathbf{G(e_3)}$ ={((a, 0, 0.6), (b, 0, 0.8), (c, 0.1, 0.5)}}

DIL(F, A)={ **DIL**(F(e_1))={ (a, 0.70, 0.05), (b, 0.31, 0.55), (c, 0.44, 0.29)}, DIL (F(e_2))={ (a, 0.83, 0.05), (b, 0, 0.55), (c, 0.54, 0.29) }, **DIL**(F(e_4))={ (a, 0.77, 0.05), (b, 0.31, 0.45), (c, 0.94, 0.05) } and

DIL (G, B) = {**DIL** (G (e_1)) = {(a, 0.44, 0.36), (b, 0.83, 0.05), (c, 0.89, 0.05)},

DIL(G(e_2)) ={ (a, 0.63, 0.05), (b, 0.70, 0.05), (c, 0.63, 0.29) }, **DIL**(G(e_3))={ (a, 0, 0.36), (b, 0, 0.55), (c, 0.31, 0.29) }

(F, A) ∩ (G, B) = (H, C) = {$\mathbf{H\ (e_1)}$ = {(a, 0.2, 0.6), (b, 0.1, 0.8), (c, 0.2, 0. 5)}, $\mathbf{H\ (e_2)}$ = {(a, 0.4, 0.1), (b, 0, 0.8), (c, 0.3, 0. 5)}}

DIL ((F, A) ∩ (G,B))= {$\mathbf{DILH(e_1)}$ ={(a, 0.44, 0.36), (b, 0.31, 0.55), (c, 0.44, 0. 29)}, $\mathbf{DILH(e_2)}$ ={(a, 0.63, 0.05), (b, 0, 0.55), (c, 0.54, 0. 29)}}

DIL (F, A) ∩ **DIL** (G, B) =(K,C) ={ **DIL** $\mathbf{K(e_1)}$ ={(a, 0.04, 0.84), (b, 0.01, 0.96), (c, 0.04, 0. 75)}, **DIL** $\mathbf{K(e_2)}$ ={(a, 0.16, 0.19), (b, 0, 0.96), (c, 0.09, 0. 75)}}

Then

DIL ((F, A) ∩ (G,B)) = **DIL**(F, A) ∩ **DIL** (G, B)

V. Normalization of Intuitionistic Fuzzy Soft Set

In this section, we shall introduce the normalization operation on intuitionistic fuzzy soft set.

5.1 Definition:

The normalization of an intuitionistic fuzzy soft set (F, A) of universe U ,denoted by

NORM (F, A) is defined as:

NORM (F, A) ={ Norm {F(ε)} = {<x, $\boldsymbol{\mu}_{\boldsymbol{Norm}\,(\boldsymbol{F}(\varepsilon))}(\boldsymbol{x})$, $\boldsymbol{\nu}_{\boldsymbol{Norm}\,(\boldsymbol{F}(\varepsilon))}(\boldsymbol{x})$, > $|\boldsymbol{x} \in$ U and $\varepsilon \in$ A}. where

$\boldsymbol{\mu}_{\boldsymbol{Norm}\,(\boldsymbol{F}(\varepsilon))}(\boldsymbol{x}) = \frac{\mu_{F(\varepsilon)}(x)}{\sup(\mu_{F(\varepsilon)}(x))}$ and $\boldsymbol{\nu}_{\boldsymbol{Norm}\,(\boldsymbol{F}(\varepsilon))}(\boldsymbol{x}) = \frac{\nu_{F(\varepsilon)}(x) - inf(\nu_{F(\varepsilon)}(x))}{1-\inf(\nu_{F(\varepsilon)}(x))}$ and

Inf $(\boldsymbol{\nu}_{\boldsymbol{F}(\varepsilon)}(\boldsymbol{x})) \neq \mathbf{0}$.

Example. Let there are five objects as the universal set where U = {x_1, x_2, x_3, x_4, x_5} and the set of parameters as E = {beautiful, moderate, wooden, muddy, cheap, costly} and Let A = {beautiful, moderate, wooden}. Let the attractiveness of the objects represented by the intuitionistic fuzzy soft sets (F, A) is given as

F(beautiful)={$x_{1/(.6,.4)}$, $x_{2/(.7,\ .3)}$, $x_{3/(.5,\ .5)}$, $x_{4/(.8,\ .2)}$, $x_{5/(.9,\ .1)}$},

F(moderate)={$x_{1/(.3,\ .7)}$, $x_{2/(6,\ .4)}$, $x_{3/(.8,\ .2)}$, $x_{4/(.3,\ .7)}$, $x_{5/(1,\ .9)}$} and

F(wooden) ={ $x_{1/(.4,\ .6)}$, $x_{2/(.6,\ .4)}$, $x_{3/(.5,\ .5)}$, $x_{4/(.2,\ .8)}$, $x_{5/(.3,\ .7,)}$}.

Then,

sup($\boldsymbol{\mu}_{\boldsymbol{F}(\text{beautiful})}(\boldsymbol{x})$) = 0.9, inf($\boldsymbol{\nu}_{\boldsymbol{F}(\text{beautiful})}(\boldsymbol{x})$ = 0.1. We have

$\boldsymbol{\mu}_{\boldsymbol{Norm}\,(\boldsymbol{F}(\text{beautiful}))}(\boldsymbol{x_1}) = \frac{\mathbf{0.6}}{0.9} = 0.66$,

$\boldsymbol{\mu}_{\boldsymbol{Norm}\,(\boldsymbol{F}(\text{beautiful}))}(\boldsymbol{x_2}) = \frac{\mathbf{0.7}}{0.9} = 0.77$,

$\boldsymbol{\mu}_{\boldsymbol{Norm}\,(\boldsymbol{F}(\text{beautiful}))}(\boldsymbol{x_3}) = \frac{\mathbf{0.5}}{0.9} = 0.55$,

$\boldsymbol{\mu}_{\boldsymbol{Norm}\,(\boldsymbol{F}(\text{beautiful}))}(\boldsymbol{x_4}) = \frac{\mathbf{0.8}}{0.9} = 0.88$,

$\boldsymbol{\mu}_{\boldsymbol{Norm}\,(\boldsymbol{F}(\text{beautiful}))}(\boldsymbol{x_5}) = \frac{\mathbf{0.9}}{0.9} = 1$ and

$\boldsymbol{\nu}_{\boldsymbol{Norm}\,(\boldsymbol{F}(\text{beautiful}))}(\boldsymbol{x_1}) = \frac{\mathbf{0.3}}{0.9} = 0.33$,

$\boldsymbol{\nu}_{\boldsymbol{Norm}\,(\boldsymbol{F}(\text{beautiful}))}(\boldsymbol{x_2}) = \frac{\mathbf{0.2}}{0.9} = 0.22$,

$\nu_{Norm\,(F(\text{beautiful}))}(x_3) = \frac{0.4}{0.9} = 0.44$

$\nu_{Norm\,(F(\text{beautiful}))}(x_4) = \frac{0.1}{0.9} = 0.11,$

$\nu_{Norm\,(F(\text{beautiful}))}(x_5) = \frac{0.0}{0.9} = 0.$

Norm(F(beautiful)) $=\{ x_{1/(.66,.33)}, x_{2/(.77,\ .22)}, x_{3/(.55,\ .44)}, x_{4/(.88,\ .11)}, x_{5/(1,\ 0)} \}$.

$\sup(\mu_{F(\text{moderate})}(x)) = 0.8$, $\inf(\nu_{F(\text{moderate})}(x) = 0.2$. We have

$\mu_{Norm\,(F(\text{moderate}))}(x_1) = \frac{0.3}{0.8} = 0.375,$

$\mu_{Norm\,(F(\text{moderate}))}(x_2) = \frac{0.6}{0.8} = 0.75,$

$\mu_{Norm\,(F(\text{moderate}))}(x_3) = \frac{0.8}{0.8} = 1,$

$\mu_{Norm\,(F(\text{moderate}))}(x_4) = \frac{0.3}{0.8} = 0.375,$

$\mu_{Norm\,(F(\text{moderate}))}(x_5) = \frac{0.1}{0.8} = 0.125$ And

$\nu_{Norm\,(F(\text{moderate}))}(x_1) = \frac{0.5}{0.8} = 0.625,$

$\nu_{Norm\,(F(\text{moderate}))}(x_2) = \frac{0.2}{0.8} = 0.25,$

$\nu_{Norm\,(F(\text{moderate}))}(x_3) = \frac{0}{0.8} = 0,$

$\nu_{Norm\,(F(\text{moderate}))}(x_4) = \frac{0.5}{0.8} = 0.625,$

$\nu_{Norm\,(F(\text{moderate}))}(x_5) = \frac{0.7}{0.8} = 0.875.$

Norm(F(moderate)) $=\{ x_{1/(.375,.625)}, x_{2/(.75,\ .25)}, x_{3/(1,\ 0)}, x_{4/(.375,\ .625)}, x_{5/(0.125,\ 0.875)} \}$.

$\sup(\mu_{F(\text{wooden})}(x)) = 0.6$, $\inf(\nu_{F(\text{wooden})}(x) = 0.4$. We have

$\mu_{Norm\,(F(\text{wooden}))}(x_1) = \frac{0.4}{0.6} = 0.66,$

$\mu_{Norm\,(F(\text{wooden}))}(x_2) = \frac{0.6}{0.6} = 1,$

$\mu_{Norm\,(F(\text{wooden}))}(x_3) = \frac{0.5}{0.6} = 0.83,$

$\mu_{Norm\,(F(\text{wooden}))}(x_4) = \frac{0.2}{0.6} = 0.34,$

$\mu_{Norm\,(F(\text{wooden}))}(x_5) = \frac{0.3}{0.6} = 0.5$ and

$\nu_{Norm\,(F(\text{wooden}))}(x_1) = \frac{0.2}{0.6} = 0.34,$

$\nu_{Norm\,(F(\text{wooden}))}(x_2) = \frac{0}{0.6} = 0,$

$\nu_{Norm\,(F(\text{wooden}))}(x_3) = \frac{0.1}{0.6} = 0.17,$

$\nu_{Norm\,(F(\text{wooden}))}(x_4) = \frac{0.4}{0.6} = 0.66,$

$\nu_{Norm\,(F(\text{wooden}))}(x_5) = \frac{0.3}{0.6} = 0.5.$

Norm(F(wooden)) $=\{ x_{1/(.66,.34)}, x_{2/(1,\ .0)}, x_{3/(0.83,\ 0.17)}, x_{4/(.34,\ .66)}, x_{5/(0.5,\ 0.\ 5)} \}$.

Then, **Norm** (F, A) = {Norm F (beautiful), Norm F(moderate), Norm F(wooden)}

Norm (F,A)={ F(beautiful) $=\{ x_{1/(.66,.33)}, x_{2/(.77,\ .22)}, x_{3/(.55,\ .44)}, x_{4/(.88,\ .11)}, x_{5/(1,\ 0)} \}$, F(moderate)$=\{ x_{1/(.375,.625)}, x_{2/(.75,\ .25)}, x_{3/(1,\ 0)}, x_{4/(.375,\ .625)}, x_{5/(0.125,\ 0.875)} \}$, F(wooden) $=\{ x_{1/(.66,.34)}, x_{2/(1,\ .0)}, x_{3/(0.83,\ 0.17)}, x_{4/(.34,\ .66)}, x_{5/(0.5,\ 0.\ 5)} \}$}

Clearly, $\mu_{Norm\,(F(\varepsilon))}(x) + \nu_{Norm\,(F(\varepsilon))}(x) = 1$, for i = 1, 2, 3, 4, 5 which satisfies the property of intuitionistic fuzzy soft set. Therefore, **Norm** (F, A) is an intuitionistic fuzzy soft set.

VI. CONCLUSION

In this paper, we have extended the two operations of intuitionistic fuzzy set introduced by Wang et al.[18] to the case of intuitionistic fuzzy soft sets. Then we have introduced the concept of normalization of intuitionistic fuzzy soft sets and studied several properties of these operations.

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