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Extension of Inagaki General Weighted Operators and A New Fusion Rule Class of Proportional Redistribution of Intersection Masses

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Abstract

In this paper we extend Inagaki Weighted Operators fusion rule (WO) [see 1, 2] in information fusion by doing redistribution of not only the conflicting mass, but also of masses of non-empty intersections, that we call Double Weighted Operators (DWO).

Then we propose a new fusion rule Class of Proportional Redistribution of Intersection Masses (CPRIM), which generates many interesting particular fusion rules in information fusion.

Both formulas are presented for 2 and for n ≥ 3 sources.

An application and comparison with other fusion rules are given in the last section.

Keywords: Inagaki Weighted Operator Rule, fusion rules, proportional redistribution rules, DSm classic rule, DSm cardinal, Smarandache codification, conflicting mass

Mathematics Subject Classification: 99C99, 68P99

ACM Classification: I.4.8.

1. Introduction.

Let $\theta = \{\theta_1, \theta_2, ..., \theta_n\}$, for $n \geq 2$, be the frame of discernment, and $S^\theta = (\theta, \cup, \cap, \tau)$ its super-power set, where τ(x) means complement of x with respect to the total ignorance.

Let I_t = total ignorance = $\theta_1 \chi\ \theta_2 \chi \dots \chi \theta_n$.

$S^\theta = 2\text{\textasciicircum}\theta_{\text{refined}} = 2\text{\textasciicircum}(2\text{\textasciicircum}\theta) = D^{\theta\chi\theta c}$, when refinement is possible, where $\theta_c = \{\tau(\theta_1), \tau(\theta_2), \dots, \tau(\theta_n)\}$.

We consider the general case when the domain is S^θ, but S^θ can be replaced by $D^\theta = (\theta, \chi, 1)$ or by $2^\theta = (\theta, \chi)$ in all formulas from below.

Let $m_1(\cdot)$ and $m_2(\cdot)$ be two normalized masses defined from S^θ to $[0,1]$.

We use the conjunction rule to first combine $m_1(\cdot)$ with $m_2(\cdot)$ and then we redistribute the mass of $m(X \cap Y) \neq 0$, when $X \cap Y = \Phi$.

Let's denote $m_{2\cap}(A)=\left(m_1\oplus m_2\right)(A)=\sum_{\substack{X,Y\in S^\theta\\(X\cap Y)=A}} m_1(X)m_2(Y)$ using the conjunction rule.

Let's note the set of intersections by:

$$S_\cap=\left\{\begin{array}{l}X\in S^\theta \mid X=y\cap z,\ \text{where } y,z\in S^\theta\setminus\{\Phi\},\\ X \text{ is in a canonical form, and}\\ X \text{ contains at least an } \cap \text{ symbol in its formula}\end{array}\right\}. \quad (1)$$

In conclusion, S1 is a set of formulas formed with singletons (elements from the frame of discernment), such that each formula contains at least an intersection symbol 1, and each formula is in a canonical form (easiest form).

For example: $A\cap A\notin S_\cap$ since $A\cap A$ is not a canonical form, and $A\cap A=A$. Also, $\left(A\cap B\right)\cap B$ is not in a canonical form but $\left(A\cap B\right)\cap B=A\cap B\in S_\cap$.

Let

$S_\cap^\Phi =$ the set of all empty intersections from $S_\cap$,

and

$S_{\cap,r}^{non\Phi} =$ {the set of all non-empty intersections from $S_\cap^{non\Phi}$ whose masses are redistributed to other sets, which actually depends on the sub-model of each application}.

2. Extension of Inagaki General Weighted Operators (WO).

Inagaki general weighted operator (WO) is defined for two sources as:

$$\forall A\in 2^\theta\setminus\{\Phi\},\ m_{(WO)}(A)=\sum_{\substack{X,Y\in 2^\theta\\(X\cap Y)=A}} m_1(X)m_2(Y)+W_m(A)\cdot m_{2\cap}(\Phi), \quad (2)$$

where

$$\sum_{X\in 2^\theta} W_m(X)=1 \text{ and all } W_m(\cdot)\in[0,1]. \quad (3)$$

So, the conflicting mass is redistributed to non-empty sets according to these weights $W_m(\cdot)$.

In the extension of this WO, which we call the Double Weighted Operator (DWO), we redistribute not only the conflicting mass $m_{2\cap}(\Phi)$ but also the mass of some (or all) non-empty intersections, i.e. those from the set $S_{\cap,r}^{non\Phi}$, to non-empty sets from S^{θ} according to some weights $W_m(\cdot)$ for the conflicting mass (as in WO), and respectively according to the weights $V_m(.)$ for the non-conflicting mass of the elements from the set $S_{\cap,r}^{non\Phi}$:

$$\forall A \in \left(S^{\theta} \setminus S_{\cap,r}^{non\Phi}\right) \setminus \{\Phi\},\ m_{DWO}(A) = \sum_{\substack{X,Y \in S^{\theta} \\ (X \cap Y) = A}} m_1(X) m_2(Y) + W_m(A) \cdot m_{2\cap}(\Phi) + V_m(A) \cdot \sum_{z \in S_{\cap,r}^{non\Phi}} m_{2\cap}(z), \quad (4)$$

where

$$\sum_{X \in S^{\theta}} W_m(X) = 1 \text{ and all } W_m(\cdot) \in [0,1], \text{ as in (3)}$$

and

$$\sum_{z \in S_{\cap,r}^{non\Phi}} V_m(z) = 1 \text{ and all } V_m(\cdot) \in [0,1]. \quad (5)$$

In the free and hybrid modes, if no non-empty intersection is redistributed, i.e. $S_{\cap,r}^{non\Phi}$ contains no elements, DWO coincides with WO.

In the Shafer's model, always DWO coincides with WO.

For $s \geq 2$ sources, we have a similar formula:

$$\forall A \in \left(S^{\theta} \setminus S_{\cap,r}^{non\Phi}\right) \setminus \{\Phi\},\ m_{DWO}(A) = \sum_{\substack{X_1, X_2, \ldots, X_n \in S^{\theta} \\ \bigcap_{i=1}^{s} X_i = A}} \prod_{i=1}^{s} m_i(X_i) + W_m(A) \cdot m_{s\cap}(\Phi) + V_m(A) \cdot \sum_{z \in S_{\cap,r}^{non\Phi}} m_{s\cap}(z) \quad (6)$$

with the same restrictions on $W_m(\cdot)$ and $V_m(\cdot)$.

3. A Fusion Rule Class of Proportional Redistribution of Intersection Masses

For $A \in \left(S^{\theta} \setminus S_{\cap,r}^{non\Phi}\right) \setminus \{\Phi, I_t\}$ for two sources we have:

$$m_{CPR \setminus M}(A) = m_{2\cap}(A) + f(A) \cdot \sum_{\substack{X,Y \in S^{\theta} \\ \{\Phi = X \cap Y \text{ and } A \subseteq M\} \\ \text{or } \{\Phi \neq X \cap Y \in S_{\cap,r}^{non\Phi} \text{ and } A \subseteq N\}}} \frac{m_1(X) m_2(Y)}{\sum_{z \subseteq M} f(z)}, \quad (7)$$

where $f(X)$ is a function directly proportional to X, $f: S^{\theta} \to [0, \infty]$. (8)

For example, $f(X) = m_{2\cap}(X)$, or (9)

$f(X) = card(X)$, or

$f(X) = \dfrac{card(X)}{card(M)}$ (ratio of cardinals), or

$f(X) = m_{2\cap}(X) + card(X)$, etc.;

and M is a subset of S^{θ}, for example: (10)

$M = \tau\left(X \bigcup Y\right)$, or

$M = \left(X \bigcup Y\right)$, or

M is a subset of $X \bigcup Y$, etc.,

where N is a subset of S^{θ}, for example: (11)

$N = X \bigcup Y$, or

N is a subset of $X \bigcup Y$, etc.

And

$$m_{CPR\setminus M}(I_t) = m_{2\cap}(I_t) + \sum_{\substack{X,Y \in S^{\theta} \\ \left\{X \cap Y = \Phi \text{ and } (M = \Phi \text{ or } \sum_{z \subseteq M} f(z) = 0)\right\}}} m_1(X) m_2(Y). \quad (12)$$

These formulas are easily extended for any $s \geq 2$ sources $m_1(\cdot), m_2(\cdot), ..., m_s(\cdot)$.

Let's denote, using the conjunctive rule:

$$m_{s\cap}(A) = \left(m_1 \oplus m_2 \oplus ... \oplus m_s\right)(A) = \sum_{\substack{X_1, X_2, ..., X_s \in S^{\wedge}\Theta \\ \bigcap_{i=1}^{s} X_i = A}} \prod_{i=1}^{s} m_i(x_i) \quad (13)$$

$$m_{s\cap}(A) = m_{s\cap}(A) + \text{f(A)} \cdot \sum_{\substack{X_1, X_2, ..., X_n \in S^{\theta} \\ \left\{\Phi = \bigcap_{i=1}^{s} X_i \text{ and } A \subseteq M\right\} \\ \text{or } \left\{\Phi \neq \bigcap_{i=1}^{s} X_i \in S_{\cap, r}^{non\Phi} \text{ and } A \subseteq N\right\}}} \frac{\prod_{i=1}^{s} m_i(X_i)}{\sum_{z \subseteq M} f(z) \neq 0} \quad (14)$$

where $f(\cdot)$, M, and N are similar to the above where instead of $X \cup Y$ (for two sources) we take $X_1 \cup X_2 \cup ... \cup X_s$ (for s sources), and instead of $m_{2\cap}(X)$ for two sources we take $m_{s\cap}(X)$ for s sources.

4. Application and Comparison with other Fusion Rules.

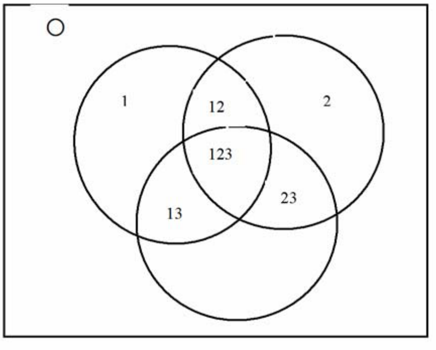
Let's consider the frame of discernment Θ = {A, B, C}, and two independent sources $m_1(.)$ and $m_2(.)$ that provide the following masses:

	A	B	C	A∪B∪C
$m_1(.)$	0.3	0.4	0.2	0.1
$m_2(.)$	0.5	0.2	0.1	0.2

Now, we apply the conjunctive rule and we get:

	A	B	C	A∪B∪C	A∩B	A∩C	B∩C
$m_{12\cap}(.)$	0.26	0.18	0.07	0.02	0.26	0.13	0.08

Suppose that all intersections are non-empty {this case is called: free DSm (Dezert-Smarandache) Model}. See below the Venn Diagram using the Smarandache codification [3]:



Applying DSm Classic rule, which is a generalization of classical conjunctive rule from the fusion space (Θ, ∪), called *power set*, when all hypotheses are supposed exclusive (i.e. all intersections are empty) to the fusion space (Θ, ∪, ∩), called *hyper-power set*, where hypotheses are not necessarily exclusive (i.e. there exist non-empty intersections), we just get:

	A	B	C	$A\cup B\cup C$	$A\cap B$	$A\cap C$	$B\cap C$
$m_{DSmC}(.)$	0.26	0.18	0.07	0.02	0.26	0.13	0.08

DSmC and the Conjunctive Rule have the same formula, but they work on different fusion spaces.

Inagaki rule was defined on the fusion space $(\Theta, \cup)$. In this case, since all intersections are empty, the total conflicting mass, which is $m_{12\cap}(A\cap B) + m_{12\cap}(A\cap C) + m_{12\cap}(B\cap C) = 0.26 + + 0.13 + 0.08 = 0.47$, and this is redistributed to the masses of A, B, C, and $A\cup B\cup C$ according to some weights w_1, w_2, w_3, and w_4 respectively, depending to each particular rule, where:

$0 \le w_1, w_2, w_3, w_4 \le 1$ and $w_1 + w_2 + w_3 + w_4 = 1$. Hence

	A	B	C	$A\cup B\cup C$
$m_{Inagaki}(.)$	$0.26+0.47w_1$	$0.18+0.47w_2$	$0.07+0.47w_3$	$0.02+0.47w_4$

Yet, Inagaki rule can also be straightly extended from the power set to the hyper-power set.

Suppose in DWO the user finds out that the hypothesis $B\cap C$ is not plausible, therefore

$m_{12\cap}(B\cap C) = 0.08$ has to be transferred to the other non-empty elements: A, B, C, $A\cup B\cup C$, $A\cap B$, $A\cap C$, according to some weights v_1, v_2, v_3, v_4, v_5, and v_6 respectively, depending to the particular version of this rule is chosen, where:

$0 \le v_1, v_2, v_3, v_4, v_5, v_6 \le 1$ and $v_1 + v_2 + v_3 + v_4 + v_5 + v_6 = 1$. Hence

	A	B	C	$A\cup B\cup C$	$A\cap B$	$A\cap C$
$m_{DWO}(.)$	$0.26+0.08v_1$	$0.18+0.08v_2$	$0.07+0.08v_3$	$0.02+0.08v_4$	$0.26+0.08v_5$	$0.13+0.08v_6$

Now, since CPRIM is a particular case of DWO, but CPRIM is a class of fusion rules, let's consider a sub-particular case for example when the redistribution of $m_{12\cap}(B\cap C) = 0.08$ is done proportionally with respect to the DSm cardinals of B and C which are both equal to 4. DSm cardinal of a set is equal to the number of disjoint parts included in that set upon the Venn Diagram (see it above).

Therefore 0.08 is split equally between B and C, and we get:

	A	B	C	$A \cup B \cup C$	$A \cap B$	$A \cap C$
$m_{CPRIMcard}(.)$	0.26	0.18+0.04=0.22	0.07+0.04=0.11	0.02	0.26	0.13

Applying one or another fusion rule is still debating today, and this depends on the hypotheses, on the sources, and on other information we receive.

5. Conclusion.

A generalization of Inagaki rule has been proposed in this paper, and also a new class of fusion rules, called **Class of Proportional Redistribution of Intersection Masses (CPRIM)**, which generates many interesting particular fusion rules in information fusion.

References:

[1] T. Inagaki, Independence Between Safety-Control Policy and Multiple-Sensors Schemes via Dempster-Shafer Theory, IEEE Transaction on Reliability, 40, 182-188, 1991.

[2] E. Lefèbvre, O. Colot, P. Vannoorenberghe, Belief Function Combination and Conflict Management, Information Fusion 3, 149-162, 2002.

[3] F. Smarandache, J. Dezert (editors), Advances and Applications of DSmT for Information Fusion, Collective Works, Vol. 2, Am. Res. Press, 2004.