Neutrosophic Masses & Indeterminate Models.

Applications to Information Fusion

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***Abstract*—In this paper we introduce the *indeterminate models* in information fusion, which are due either to the existence of some *indeterminate elements* in the fusion space or to some *indeterminate masses*. The best approach for dealing with such models is the neutrosophic logic.**

Keywords: neutrosophic logic; indeterminacy; indeterminate model; indeterminate element; indeterminate mass; indeterminate fusion rules; DSmT; DST; TBM;

I. INTRODUCTION

Let Θ be a frame of discernment, defined as:

$$\Theta = \{\phi_1, \phi_2, \ldots, \phi_n\}, n \geq 2, \tag{1}$$

and its Super-Power Set (or fusion space):

$$S^\Theta = (\Theta, \cup, \cap, C) \tag{2}$$

which means the set Θ closed under union, intersection, and respectively complement.

This paper is organized as follows: we present the neutrosophic logic, the indeterminate masses, elements and models, and give an example of indeterminate intersection.

II. INDETERMINATE MASS

A. Neutrosophic Logic

Neutrosophic Logic (NL) [1] started in 1995 as a generalization of the fuzzy logic, especially of the intuitionistic fuzzy logic. A logical proposition P is characterized by three neutrosophic components:

$$NL(P) = (T, I, F) \tag{3}$$

where T is the degree of truth, F the degree of falsehood, and I the degree of indeterminacy (or neutral, where the name "neutro-sophic" comes from, i.e. neither truth nor falsehood but in between – or included-middle principle), and with:

$$T, I, F \subseteq]^-0, 1^+[\tag{4}$$

where $]^-0, 1^+[$ is a non-standard interval.

In this paper, for technical proposal, we can reduce this interval to the standard interval $[0, 1]$.

The main distinction between neutrosophic logic and intuitionistic fuzzy logic (IFL) is that in NL the sum $T+I+F$ of the components, when T, I, and F are crisp numbers, does not need to necessarily be 1 as in IFL, but it can also be less than 1 (for incomplete/missing information), equal to 1 (for complete information), or greater than 1 (for paraconsistent/contradictory information).

The combination of neutrosophic propositions is done using the neutrosophic operators (especially $\wedge$, $\vee$).

B. Neutrosophic Mass

We recall that a classical mass $m(.)$ is defined as:

$$m : S^\Theta \rightarrow [0,1] \tag{5}$$

such that

$$\sum_{X \in S^\Theta} m(X) = 1 \tag{6}$$

We extend this classical basic belief assignment (mass) $m(.)$ to a neutrosophic basic belief assignment (nbba) (or neutrosophic mass) $m_n(.)$ in the following way.

$$m_n : S^\Theta \rightarrow [0,1]^3 \tag{7}$$

with

$$m_n(A) = (T(A), I(A), F(A)) \tag{8}$$

where $T(A)$ means the (local) chance that hypothesis A occurs, $F(A)$ means the (local) chance that hypothesis A does not occur (nonchance), while $I(A)$ means the (local) indeterminate chance of A (i.e. knowing neither if A occurs nor if A doesn't occur),

such that:

$$\sum_{X \in S^\Theta} [T(X) + I(X) + F(X)] = 1. \tag{9}$$

In a more general way, the summation (9) can be less than 1 (for incomplete neutrosophic information), equal to 1 (for complete neutrosophic information), or greater than 1 (for paraconsistent/conflicting neutrosophic information). But in this paper we only present the case when summation (9) is equal to 1.

Of course,

$$0 \le T(A), I(A), F(A) \le 1. \qquad (10)$$

A *basic belief assignment* (or *mass*) is considered *indeterminate* if there exist at least an element $A \in S^{\Theta}$ such that $I(A) > 0$, i.e. there exists some indeterminacy in the chance of at least an element A for occurring or for not occurring. Therefore, a neutrosophic mass which has at least one element A with $I(A) > 0$ is an indeterminate mass.

A classical mass $m(.)$ as defined in equations (5) and (6) can be extended under the form of a neutrosophic mass $m_n'(.)$ in the following way:

$$m_n' : S^{\Theta} \to [0,1]^3 \qquad (11)$$

with

$$m_n'(A) = (m(A), 0, 0) \qquad (12)$$

but reciprocally it does not work since $I(A)$ has no correspondence in the definition of the classical mass.

We just have $T(A) = m(A)$ and $F(A) = m(C(A))$, where $C(A)$ is the complement of A. The non-null $I(A)$ can, for example, be roughly approximated by the total ignorance mass m(Θ), or better by the partial ignorance mass $m(\Theta_I)$ where Θ_I is the union of all singletons that have some non-zero indeterminacy, but these mean less accuracy and less refinement in the fusion.

If $I(X) = 0$ for all $X \in S^{\Theta}$, then the neutrosophic mass is simply reduced to a classical mass.

III. INDETERMINATE ELEMENT

We have two types of elements in the fusion space S^{Θ}, *determinate elements* (which are well-defined), and *indeterminate elements* (which are not well-defined; for example: a geographical area whose frontiers are vague; or let's say in a murder case there are two suspects, *John* – who is known/determinate element – but he acted together with another man X (since the information source saw *John* together with an unknown/unidentified person) – therefore X is an indeterminate element).

Herein we gave examples of singletons as indeterminate elements just in the frame of discernment Θ, but indeterminate elements can also result from the combinations (unions, intersections, and/or complements) of determinate elements that form the super-power set S^{Θ}. For example, A and B can be determinate singletons (we call the elements in Θ as singletons), but their intersection $A \cap B$ can be an indeterminate (unknown) element, in the sense that we might not know if $A \cap B = \phi$ or $A \cap B \neq \phi$.

Or A can be a determinate element, but its complement $C(A)$ can be indeterminate element (not well-known), and similarly for determinate elements A and B, but their $A \cup B$ might be indeterminate.

Indeterminate elements in S^{Θ} can, of course, result from the combination of indeterminate singletons too. All depends on the problem that is studied.

A frame of discernment which has at least an indeterminate element is called *indeterminate frame of discernment*. Otherwise, it is called *determinate frame of discernment*. Similarly we call an *indeterminate fusion space* (S^{Θ}) that fusion space which has at least one indeterminate element. Of course an indeterminate frame of discernment spans an indeterminate fusion space.

An *indeterminate source of information* is a source which provides an indeterminate mass or an indeterminate fusion space. Otherwise it is called a *determinate source of information*.

IV. INDETERMINATE MODEL

An *indeterminate model* is a model whose fusion space is indeterminate, or a mass that characterizes it is indeterminate.

Such case has not been studied in the information fusion literature so far. In the next sections we'll present some examples of indeterminate models.

V. CLASSIFICATION OF MODELS

In the classical fusion theories all elements are considered determinate in the Closed World, except in Smets' Open World where there is some room (i.e. mass assigned to the empty set) for a possible unknown missing singleton in the frame of discernment. So, the Open World has a probable indeterminate element, and thus its frame of discernment is indeterminate. While the Closed World frame of discernment is determinate.

In the Closed World in Dezert-Smarandache Theory there are three models classified upon the types of singleton intersections: Shafer's Model (where all intersections are empty), Hybrid Model (where some intersections are empty, while others are non-empty), and Free Model (where all intersections are non-empty).

We now introduce a fourth category, called *Indeterminate Model* (where at least one intersection is indeterminate/unknown, and in general at least one element of the fusion space is indeterminate). We do this because in practical problems we don't always know if an intersection is empty or nonempty. As we still have to solve the problem in the real time, we have to work with what we have, i.e. with indeterminate models.

The *indeterminate intersection* cannot be refined (because not knowing if $A \cap B$ is empty or nonempty, we'd get two different refinements: $\{A, B\}$ when intersection is empty, and $\{A \backslash B, B \backslash A, A \cap B\}$ when intersection is nonempty).

The *percentage of indeterminacy* of a model depends on the number of indeterminate elements and indeterminate masses.

By default: the sources, the masses, the elements, the frames of discernment, the fusion spaces, and the models are supposed determinate.

VI. AN EXAMPLE OF INFORMATION FUSION WITH AN INDETERMINATE MODEL

We present the below example.

Suppose we have two sources, $m_1(.)$ and $m_2(.)$, such that:

	A	B	C	$A \cup B \cup C$	$A \cap B$ = Ind.	$A \cap C$ = ϕ	$B \cap C$ = Ind.
m_1	0.4	0.2	0.3	0.1			
m_2	0.1	0.3	0.2	0.4			
m_{12}	.21	.17	.20	.04	.14	.11	.13

Table 1

Applying the conjunction rule to m_1 and m_2 we get $m_{12}(.)$ as shown in Table 1.

The frame of discernment is $\Theta = \{A, B, C\}$. We know that $A \cap C$ is empty, but we don't know the other two intersections: we note them as $A \cap B$ = *ind.* and $B \cap C$ = *ind,.* where *ind.* means indeterminate.

Using the Conjunctive Rule to fusion m_1 and m_2, we get $m_{12}(.)$:

$$\forall A \in S^{\Theta} \setminus \phi, m_{12}(A) = \sum_{\substack{X,Y \in S^{\Theta} \\ A = X \cap Y}} m_1(X) m_2(Y) . \tag{13}$$

Whence: $m_{12}(A)=0.21$, $m_{12}(B)=0.17$, $m_{12}(C)=0.20$, $m_{12}(A \cup B \cup C)=0.04$, and for the intersections:

$m_{12}(A \cap B)=0.14$, $m_{12}(A \cap C)=0.11$, $m_{12}(B \cap C)=0.13$.

We then use the *PCR5* fusion rule style to redistribute the masses of these three intersections. We recall *PCR5* for two sources:

$$\forall A \in S^{\Theta} \setminus \phi,$$
$$m_{12PCR5}(A) = m_{12}(A) + \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A = \phi}} \left[\frac{m_1(A)^2 m_2(X)}{m_1(A) + m_2(X)} + \frac{m_2(A)^2 m_1(X)}{m_2(A) + m_1(X)}\right] \tag{14}$$

a) $m_{12}(A \cap C)=0.11$ is redistributed back to A and C because $A \cap C = \phi$, according to the PCR5 style.

Let $\alpha 1$ and $\alpha 2$ be the parts of mass *0.11* redistributed back to A, and $\gamma 1$ and $\gamma 2$ be the parts of mass *0.11* redistributed back to C.
We have the following proportionalizations:

$$\frac{\alpha 1}{0.4} = \frac{\gamma 1}{0.2} = \frac{0.4 \cdot 0.2}{0.4 + 0.2} = 0.133333 ,$$

whence $\alpha 1 = 0.4(0.133333) \approx 0.053333$
and $\gamma 1 = 0.2(0.13333) \approx 0.026667$.
Similarly:

$$\frac{\alpha 2}{0.1} = \frac{\gamma 2}{0.3} = \frac{0.1 \cdot 0.3}{0.1 + 0.3} = 0.075 ,$$

whence $\alpha 2 = 0.1(0.075) = 0.0075$
and $\gamma 2 = 0.3(0.075) = 0.0225$.
Therefore the mass of A, which can also be noted as $T(A)$ in a neutrosophic mass form, receives from 0.11 back:
$\alpha 1+\alpha 2 = 0.053333+0.0075 = 0.060833$,
while the mass of C, or $T(C)$ in a neutrosophic form, receives from 0.11 back:
$\gamma 1+\gamma 2 = 0.026667+0.0225= 0.049167$.
We verify our calculations: 0.060833+0.049167=0.11.

b) $m_{12}(A \cap B)=0.14$ is redistributed back to the indeterminate parts of the masses of A and B respectively, namely $I(A)$ and $I(B)$ as noted in the neutrosophic mass form, because $A \cap B$=*Ind.* We follow the same PCR5 style as done in classical PCR5 for empty intersections (as above).

Let $\alpha 3$ and $\alpha 4$ be the parts of mass *0.14* redistributed back to $I(A)$, and $\beta 1$ and $\beta 2$ be the parts of mass *0.14* redistributed back to $I(B)$.
We have the following proportionalizations:

$$\frac{\alpha 3}{0.4} = \frac{\beta 1}{0.3} = \frac{0.4 \cdot 0.3}{0.4 + 0.3} = 0.171429 ,$$

whence $\alpha 3 = 0.4(0.171429) \approx 0.068572$
and $\beta 1 = 0.3(0.171429) \approx 0.051428$.
Similarly:

$$\frac{\alpha 4}{0.1} = \frac{\beta 2}{0.2} = \frac{0.1 \cdot 0.2}{0.1 + 0.2} = 0.066667$$

whence $\alpha 4 = 0.1(0.066667) \approx 0.006667$
and $\beta 2 = 0.2(0.066667) \approx 0.013333$.
Therefore, the indeterminate mass of A, $I(A)$ receives from 0.14 back:

$\alpha 3+ \alpha 4 = 0.068572+0.006667=0.075239$

and the indeterminate mass of B, $I(B)$, receives from 0.14 back:

$\beta 1+ \beta 2 = 0.051428+0.013333=0.064761$.

c) Analougously, $m_{12}(B \cap C)=0.13$ is redistributed back to the indeterminate parts of the masses of B and C respectively, namely $I(B)$ and $I(C)$ as noted in the neutrosophic mass form, because $B \cap C$=*Ind.* also following the PCR5 style. Whence $I(B)$ gets back 0.065 and $I(C)$ also gets back 0.065.

Finally we sum all results obtained from firstly using the Conjunctive Rule [Table 1] and secondly redistributing the intersections masses with PCR5 [sections a), b), and c) from above]:

	T(A)	T(B)	T(C)	T(Θ)	I(A)	I(B)	I(C)
m_{12}	.21	.17	.20	.04			
addi- tions	.0075 .053 333		.022 5 .026 667		.068 572 .006 667	.051 428 .013 333 .02 .045	.04 .045
$m_{12PCR5I}$	.270 833	.17	.249 167	.04	.075 239	.129 761	.065

Table 2

where $\Theta = A \cup B \cup C$ is the total ignorance.

VII. BELIEF, DISBELIEF, AND UNCERTAINTY

In classical fusion theory there exist the following functions:

Belief in *A* with respect to the bba *m(.)* is:

$$Bel(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \subseteq A}} m(X) \tag{15}$$

Disbelief in *A* with respect to the bba *m(.)* is:

$$Dis(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A = \phi}} m(X) \tag{16}$$

Uncertainty in *A* with respect to the bba *m(.)* is:

$$U(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi \\ X \cap C(A) \neq \phi}} m(X), \tag{17}$$

where *C(A)* is the complement of A with respect to the total ignorance Θ.

Plausability of *A* with respect to the bba *m(.)* is:

$$Pl(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi}} m(X) \tag{18}$$

VIII. NEUTROSOPHIC BELIEF, NEUTROSOPHIC DISBELIEF, AND NEUTROSOPHIC UNDECIDABILITY

Let's consider a neutrosophic mass $m_n(.)$ as defined in formulas (7) and (8), $m_n(X) = (T(X), I(X), F(X))$ for all $X \in S^{\Theta}$.

We extend formulas (15)-(18) from *m(.)* to $m_n(.)$:

Neutrosophic Belief in *A* with respect to the nbba $m_n(.)$ is:

$$NeutBel(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \subseteq A}} T(X) + \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A = \phi}} F(X) \tag{19}$$

Neutrosophic Disbelief in *A* with respect to the nbba $m_n(.)$ is:

$$NeutDis(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A = \phi}} T(X) + \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \subseteq A}} F(X) \tag{20}$$

Neutrosophic Uncertainty in *A* with respect to the nbba $m_n(.)$ is

$$NeutU(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi \\ X \cap C(A) \neq \phi}} T(X) + \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi \\ X \cap C(A) \neq \phi}} F(X) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi \\ X \cap C(A) \neq \phi}} [T(X) + F(X)] \tag{21}$$

We now introduce the **Neutrosophic Global Indeterminacy in *A*** with respect to the nbba $m_n(.)$ as a sum of local indeterminacies of the elements included in *A*:

$$NeutGlobInd(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \subseteq A}} I(X) \tag{22}$$

And afterwards we define another function called **Neutrosophic Undecidability about *A*** with respect to the nbba $m_n(.)$:

$$NeutUnd(A) = NeutU(A) + NeutGlobInd(A) \tag{23}$$

or

$$NeutUnd(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi \\ X \cap C(A) \neq \phi}} [T(X) + F(X)] + \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \subseteq A}} I(X) \tag{24}$$

Neutrosophic Plausability of A with respect to the nbba $m_n(.)$ is:

$$NeutPl(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi}} T(X) + \sum_{\substack{Y \in S^{\Theta} \setminus \{\phi\} \\ C(Y) \cap A \neq \phi}} F(Y) \tag{25}$$

In the previous example let's compute *NeutBel(.), NeutDis(.),* and *NeutUnd(.)*:

	A	*B*	*C*	$A \cup B \cup C$
NeutBel	*0.270833*	*0.17*	*0.249167*	*0.73*
NeutDis	*0.419167*	*0.52*	*0.440833*	*0*
NeutGlobInd	*0.115239*	*0.169761*	*0.105*	*0*
Total	*0.805239* $\neq$ *1*	*0.859761* $\neq$ *1*	*0.795* $\neq$ *1*	*0.73* $\neq$ *1*

Table 3

As we see, for indeterminate model we cannot use the intuitionistic fuzzy set or intuitionistic fuzzy logic since the sum *NeutBel(X)+NeutDis(X)+NeutGlobInd(X)* is less than 1. In this case we use the neutrosophic set or logic which can deal with incomplete information.

The sum is less than 1 because there is missing information (we don't know if some intersections are empty or not).

For example:
NeutBel(A)+NeutDis(A)+NeutGlobInd(A)=0.805239 =1-I(B)-I(C).
Similarly,
NeutBel(B)+NeutDis(B)+NeutGlobInd(B)=0.859761 =1-I(A)-I(C).
NeutBel(C)+NeutDis(C)+NeutGlobInd(C)=0.795 =1-I(A)-I(B)
and
NeutBel(A ∪ B ∪ C)+NeutDis(A ∪ B ∪ C) +NeutGlobInd(A ∪ B ∪ C)=0.73=1-I(A)-I(B)-I(C).

IX. NEUTROSOPHIC DYNAMIC FUSION

A Neutrosophic Dynamic Fusion is a dynamic fusion where some indeterminacy occurs: with respect to the mass or with respect to some elements.

The solution of the above indeterminate model which has missing information, using the neutrosophic set, is consistent in the classical dynamic fusion in the case we receive part (or total) of the missing information.

In the above example, let's say we find out later in the fusion process that $A \cap B = \phi$. That means that the mass of indeterminacy of *A, I(A)=0.075239*, is transferred to *A*, and the masses of indeterminacy of *B* (resulted from $A \cap B$ only) - i.e. *0.051428* and *0.13333* - are transferred to *B*. We get:

	A	B	C	Θ	I(A)	I(B)	I(C)	A∩B	A∩C
m	.270833	.17	.249167	.04	0	.065	.065	0	0
+	.075239	.051428 .013333							
m_N	.346072	.234761	.249167	.04	0	.065	.065	0	0

Table 4

where $\Theta = A \cup B \cup C$ is the total ignorance.

The sum *NeutBel(X)+NeutDis(X)+NeutBlogInd(X)* increases towards 1, as indeterminacy *I(X)* decreases towards 0, and reciprocally.

When we have complete information we get *NeutBel(X)+NeutDis(X)+NeutGlobInd(X)=1* and in this case we have an intuitionistic fuzzy set, which is a particular case of the neutrosophic set.

Let's suppose once more, considering the neutrosophic dynamic fusion, that afterwards we find out that $B \cap C \neq \phi$. Then, from *Table 4* the masses of indeterminacies of *B, I(B)* (0.065 = 0.02 + 0.045, resulted from $B \cap C$ which was considered indeterminate at the beginning of the neutrosophic dynamic fusion), and that of *C, I(C)=0.065*, go now to $B \cap C$. Thus, we get:

	A	B	C	Θ	I(A)	I(B)	I(C)	A∩B	A∩C	B∩C
m_N	.346072	.234761	.249167	.04	0	.065	.065	0	0	0
-/+						-.065	-.065			+.065 +.065
m_{NN}	.346072	.234761	.249167	.04	0	0	0	0	0	.13

Table 5

X. MORE REDISTRIBUTION VERSIONS FOR INDETERMINATE INTERSECTIONS OF DETERMINATE ELEMENTS

Besides *PCR5*, it is also possible to employ other fusion rules for the redistribution, such as follows:

a. For the masses of the empty intersections we can use *PCR1-PCR4, URR, PURR, Dempster's Rule*, etc. (in general any fusion rule that first uses the conjunctive rule, and then a redistribution of the masses of empty intersections).
b. For the masses of the indeterminate intersections we can use *DSm Hybrid (DSmH)* rule to transfer the mass $m_{12}(X \cap Y = ind.)$ to $X \cup Y$, since $X \cup Y$ is a kind of uncertainty related to X, Y. In our opinion, a better approach in this case would be to redistributing the empty intersection masses using the *PCR5* and the indeterminate intersection masses using the *DSmH*, so we can combine two fusion rules into one:

Let $m_1(.)$ and $m_2(.)$ be two masses. Then:

$$m_{12PCR5\,/\,DSmH}(A) = \sum_{\substack{X,Y \in S^\Theta \setminus \{\phi\} \\ X \cap Y = A}} m_1(X) m_2(Y)$$
$$+ \sum_{\substack{X \in S^\Theta \setminus \{\phi\} \\ X \cap A = \phi}} \left[\frac{m_1(A)^2 m_2(X)}{m_1(A) + m_2(X)} + \frac{m_2(A)^2 m_1(X)}{m_2(A) + m_1(X)}\right]$$
$$+ \sum_{\substack{X,Y \in S^\Theta \setminus \{\phi\} \\ X \cap Y = ind. \\ X \cup Y = A}} m_1(X) m_2(Y)$$
$$= \sum_{\substack{X,Y \in S^\Theta \setminus \{\phi\} \\ \{X \cap Y = A\} \vee \{(X \cap Y = ind.) \wedge (X \cup Y = A)\}}} m_1(X) m_2(Y)$$
$$+ \sum_{\substack{X \in S^\Theta \setminus \{\phi\} \\ X \cap A = \phi}} \left[\frac{m_1(A)^2 m_2(X)}{m_1(A) + m_2(X)} + \frac{m_2(A)^2 m_1(X)}{m_2(A) + m_1(X)}\right] \tag{26}$$

Yet, the best approach, for an indeterminate intersection resulted from the combination of two classical masses $m_1(.)$

and $m_2(.)$ defined on a determinate frame of discernment, is the first one:

- Use the *PCR5* to combine the two sources: formula (14).
- Use the *PCR5-ind* [adjusted from classical *PCR5* formula (14)] in order to compute the indeterminacies of each element involved in indeterminate intersections :

$$\forall A \in S^{\Theta} \setminus \phi,$$

$$m_{12PCR5Ind}(I(A)) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A = ind.}} \left[\frac{m_1(A)^2 m_2(X)}{m_1(A) + m_2(X)} + \frac{m_2(A)^2 m_1(X)}{m_2(A) + m_1(X)}\right] \quad (27)$$

- Compute *NeutBel(.), NeutDis(.), NeutGlobInd(.)* of each element.

CONCLUSION

In this paper we introduced for the first time the notions of indeterminate mass (bba), indeterminate element, indeterminate intersection, and so on. We gave an example of neutrosophic dynamic fusion using two classical masses, defined on a determinate frame of discernment, but having indeterminate intersections in the super-power set S^{Θ} (the fusion space). We adjusted several classical fusion rules (*PCR5* and *DSmH*) to work for indeterminate intersections instead of empty intersections.

Then we extended the classical *Bel(.), Dis(.)* {also called *Dou(.)*, i.e Dough} and the uncertainty *U(.)* functions to their respectively neutrosophic correspondent functions that use the neutrosophic masses, i.e. to the *NeutBel(.), NeutDis(.), NeutU(.)* and to the undecidability function *NeutUnd(.)* . We have also introduced the Neutrosophic Global Indeterminacy function, *NeutGlobInd(.)*, which together with *NeutU(.)* form the Neut*Und(.)* function.

In our first example the mass of $A \cap B$ is determined (it is equal to *0.14*), but the element $A \cap B$ is indeterminate (we don't know if it empty or not).

But there are cases when the element is determinate (let's say a suspect John), but its mass could be indeterminate as given by a source of information {for example $m_n(John) = (0.4, 0.1, 0.2)$, i.e. there is some mass indeterminacy: $I(John) = 0.2 > 0$}.

These are the distinctions between the indeterminacy of an element, and the indeterminacy of a mass.

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