

# SIX CONJECTURES WHICH GENERALIZE OR ARE RELATED TO ANDRICA'S CONJECTURE

Florentin Smarandache, Ph D  
 Associate Professor  
 Chair of Department of Math & Sciences  
 University of New Mexico  
 200 College Road  
 Gallup, NM 87301, USA  
 E-mail: smarand@unm.edu

Six conjectures on pairs of consecutive primes are listed below together with examples in each case.

1) The equation  $p_{n+1}^x - p_n^x = 1$ , (1)

where  $p_n$  is the  $n^{\text{th}}$  prime, has a unique solution in between 0.5 and 1. Checking the first 168 prime numbers (less than 1000), one obtains that:

- The maximum occurs, of course, for  $n = 1$ , i.e.

$$3^x - 2^x = 1, \text{ when } x = 1.$$

- The minimum occurs for  $n = 31$ , i.e.

$$127^x - 113^x = 1, \text{ when } x = 0.567148\dots = a_0 \quad (2)$$

Thus, Andrica's Conjecture

$$A_n = \sqrt{p_{n+1}} - \sqrt{p_n} < 1$$

is generalized to:

2)  $B_n = p_{n+1}^a - p_n^a < 1$ , where  $a < a_0$ . (3)

It is remarkable that the minimum  $x$  doesn't occur for  $11^x - 7^x = 1$  as in Andrica Conjecture's maximum value, but as in example (2) for  $a_0 = 0.567148\dots$ .

Also, the function  $B_n$  in (3) is falling asymptotically as  $A_n$  in (2) i.e. in Andrica's Conjecture.

Looking at the prime exponential equations solved with a TI-92 Graphing Calculator (approximately: the bigger the prime number gap is, the smaller solution  $x$  for the equation (1); for the same gap between two consecutive primes, the larger the primes, the bigger  $x$ ):

- $3^x - 2^x = 1$ , has the solution  $x = 1.000000$ .
- $5^x - 3^x = 1$ , has the solution  $x \approx 0.727160$ .
- $7^x - 5^x = 1$ , has the solution  $x \approx 0.763203$ .
- $11^x - 7^x = 1$ , has the solution  $x \approx 0.599669$ .
- $13^x - 11^x = 1$ , has the solution  $x \approx 0.807162$ .
- $17^x - 13^x = 1$ , has the solution  $x \approx 0.647855$ .
- $19^x - 17^x = 1$ , has the solution  $x \approx 0.826203$ .
- $29^x - 23^x = 1$ , has the solution  $x \approx 0.604284$ .

$37^x - 31^x = 1$ , has the solution  $x \approx 0.624992$ .  
 $97^x - 89^x = 1$ , has the solution  $x \approx 0.638942$ .  
 $127^x - 113^x = 1$ , has the solution  $x \approx 0.567148$ .  
 $149^x - 139^x = 1$ , has the solution  $x \approx 0.629722$ .  
 $191^x - 181^x = 1$ , has the solution  $x \approx 0.643672$ .  
 $223^x - 211^x = 1$ , has the solution  $x \approx 0.625357$ .  
 $307^x - 293^x = 1$ , has the solution  $x \approx 0.620871$ .  
 $331^x - 317^x = 1$ , has the solution  $x \approx 0.624822$ .  
 $497^x - 467^x = 1$ , has the solution  $x \approx 0.663219$ .  
 $521^x - 509^x = 1$ , has the solution  $x \approx 0.666917$ .  
 $541^x - 523^x = 1$ , has the solution  $x \approx 0.616550$ .  
 $751^x - 743^x = 1$ , has the solution  $x \approx 0.732707$ .  
 $787^x - 773^x = 1$ , has the solution  $x \approx 0.664972$ .  
 $853^x - 839^x = 1$ , has the solution  $x \approx 0.668274$ .  
 $877^x - 863^x = 1$ , has the solution  $x \approx 0.669397$ .  
 $907^x - 887^x = 1$ , has the solution  $x \approx 0.627848$ .  
 $967^x - 953^x = 1$ , has the solution  $x \approx 0.673292$ .  
 $997^x - 991^x = 1$ , has the solution  $x \approx 0.776959$ .

If  $x > a_0$ , the difference of x-powers of consecutive primes is normally greater than 1. Checking more versions:

$3^{0.99} - 2^{0.99} \approx 0.981037$ .  
 $11^{0.99} - 7^{0.99} \approx 3.874270$ .  
 $11^{0.60} - 7^{0.60} \approx 1.001270$ .  
 $11^{0.59} - 7^{0.59} \approx 0.963334$ .  
 $11^{0.55} - 7^{0.55} \approx 0.822980$ .  
 $11^{0.50} - 7^{0.50} \approx 0.670873$ .

$389^{0.99} - 383^{0.99} \approx 5.596550$ .

$11^{0.599} - 7^{0.599} \approx 0.997426$ .  
 $17^{0.599} - 13^{0.599} \approx 0.810218$ .  
 $37^{0.599} - 31^{0.599} \approx 0.874526$ .  
 $127^{0.599} - 113^{0.599} \approx 1.230100$ .

$997^{0.599} - 991^{0.599} \approx 0.225749$

$127^{0.5} - 113^{0.5} \approx 0.639282$

3)  $C_n = p_{n+1}^{1/k} - p_n^{1/k} < 2/k$ , where  $p_n$  is the n-th prime, and  $k \geq 2$  is an integer.

$11^{1/2} - 7^{1/2} \approx 0.670873$ .

$11^{1/4} - 7^{1/4} \approx 0.1945837251$ .

$$\begin{aligned}
11^{1/5} - 7^{1/5} &\approx 0.1396211046 . \\
127^{1/5} - 113^{1/5} &\approx 0.060837 . \\
3^{1/2} - 2^{1/2} &\approx 0.317837 . \\
3^{1/3} - 2^{1/3} &\approx 0.1823285204 . \\
5^{1/3} - 3^{1/3} &\approx 0.2677263764 . \\
7^{1/3} - 5^{1/3} &\approx 0.2029552361 . \\
11^{1/3} - 7^{1/3} &\approx 0.3110489078 . \\
13^{1/3} - 11^{1/3} &\approx 0.1273545972 . \\
17^{1/3} - 13^{1/3} &\approx 0.2199469029 . \\
37^{1/3} - 31^{1/3} &\approx 0.1908411993 . \\
127^{1/3} - 113^{1/3} &\approx 0.191938 .
\end{aligned}$$

4)  $D_n = p_{n+1}^a - p_n^a < 1/n$ , (4)  
where  $a < a_0$  and  $n$  big enough,  $n = n(a)$ , holds for infinitely many consecutive primes.

- a) Is this still available for  $a < 1$ ?
- b) Is there any rank  $n_0$  depending on  $a$  and  $n$  such that (4) is verified for all  $n \geq n_0$ ?

A few examples:

$$\begin{aligned}
5^{0.8} - 3^{0.8} &\approx 0.21567 . \\
7^{0.8} - 5^{0.8} &\approx 1.11938 . \\
11^{0.8} - 7^{0.8} &\approx 2.06621 . \\
127^{0.8} - 113^{0.8} &\approx 4.29973 . \\
307^{0.8} - 293^{0.8} &\approx 3.57934 . \\
997^{0.8} - 991^{0.8} &\approx 1.20716 .
\end{aligned}$$

5)  $p_{n+1} / p_n \leq 5/3$ , (5)  
the maximum occurs at  $n = 2$ .

{The ratio of two consecutive primes is limited,  
while the difference  $p_{n+1} - p_n$  can be as big as we want!}

6) However,  $1/p_n - 1/p_{n+1} \leq 1/6$ , and the maximum occurs for  $n = 1$ .

### REFERENCE

[1] Sloane, N.J.A. – Sequence A001223/M0296 in “An On-Line Version of the Encyclopedia of Integer Sequences”.