

Modeling virus as elastic sphere in newtonian fluid based on 3d Navier-Stokes equations

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Although virus is widely known to significantly affect many biological form of life, its physical model is quite rare. In this regard, experiments on the acoustic vibrations of elastic nanostructures in fluid media have been used to study the mechanical properties of materials, as well as for

mechanical and biological sensing. The medium surrounding the nanostructure is typically modeled as a Newtonian fluid. In this paper, we present a mathematical model of virus as an elastic sphere in a Newtonian fluid, i.e. via 3D Navier-Stokes equations. We also obtain a numerical solution by the help of Mathematica 11.

KEYWORDS: Newtonian fluid; virus model; nonlinear physics; 3D Navier-Stokes equations; computational physics

INTRODUCTION

Although virus is widely known to significantly affect many biological form of life, its physical model is quite rare. In a paper, L.H. Ford wrote:

“Two simple models for the particle are treated, a liquid drop model and an elastic sphere model. Some estimates for the lowest vibrational frequency are given for each model. It is concluded that this frequency is likely to be of the order of a few GHz for particles with a radius of the order of 50nm” [1-4].

Such an investigation on acoustic vibration of virus particles may resonate with other reports by Prof. Luc Montagnier [8,9] and also our own hypothesis [10,11] on wave character of biological entities such as DNA, virus, water, etc.

In this regard, there are studies on the mechanical properties of (biology) materials based on experiments on the acoustic vibrations of elastic nanostructures in fluid media, where the medium surrounding the nanostructure is typically modeled as a Newtonian fluid.

In this paper we will also discuss a Newtonian fluid, i.e. 3D Navier-Stokes equations.

It is our hope that the new proposed method can be verified with experiments.

A MODEL OF LINEARIZED NAVIER-STOKES EQUATIONS

In 2015, Vahe Galstyan, On Shun Pak and Howard A. Stone published a paper where they discuss breathing mode of an elastic sphere in Newtonian and complex fluids [2]. They consider the radial vibration of an elastic sphere in a compressible viscous fluid, where the displacement field of the elastic fluid medium is governed by the Navier equation in elasticity. This spherically symmetric motion is also called the breathing mode.

They use a linearized version of Navier-Stokes equations, as follows: [2]

$$\rho_v \frac{\partial v}{\partial t} = -\nabla p + \eta \nabla^2 v + \left(\kappa + \frac{\eta}{3}\right) \nabla, \quad (1)$$

where ρ_v is the density of the fluid, η is the shear viscosity, κ is the bulk viscosity, and p is the thermodynamic pressure.

There are other authors who work on linearized NS problem; here we mention a few of them: Foias and Saut [12], Thomann and Guenther [13], A. Leonard [14].

NUMERICAL SOLUTION OF 3D NAVIER-STOKES EQUATIONS

In fluid mechanics, there is an essential deficiency of the analytical solutions of non-stationary 3D Navier–Stokes equations. Now, instead of using linearized NS equations as above, we will discuss a numerical solution of 3D Navier-Stokes equations based on Sergey Ershkov’s papers [4,5].

The Navier-Stokes system of equations for incompressible flow of Newtonian fluids can be written in the Cartesian coordinates as below (under the proper initial conditions): [4]

$$\nabla \cdot \vec{u} = 0, \quad (2)$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} = -\frac{\nabla p}{\rho} + \nu \cdot \nabla^2 \vec{u} + \vec{F}. \quad (3)$$

Where u is the flow velocity, a vector field, ρ is the fluid density, p is the pressure, ν is the kinematic viscosity, and F represents external force (per unit mass of volume) acting on the fluid. [4]

In ref. [4], Ershkov explores new ansatz of derivation of non-stationary solution for the Navier–Stokes equations in the case of incompressible flow, where his results can be written in general case as a mixed system of 2 coupled-Riccati ODEs (in regard to the time-parameter t). But instead of solving the problem analytically, we will try to find a numerical solution with the help of computer algebra package of Mathematica 11 (Figure 1).

The coupled Riccati ODEs read as follows: [4]

$$a' = \frac{w_y}{2} \cdot a^2 - (w_x \cdot b) \cdot a - \frac{w_y}{2}(b^2 - 1) + w_z \cdot b. \quad (4)$$

$$b' = -\frac{w_x}{2} \cdot b^2 + (w_y \cdot a) \cdot b + \frac{w_x}{2}(a^2 - 1) - w_z \cdot a. \quad (5)$$

First, equations (4) and (5) can be rewritten in the form as follows:

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$$x(t)' = \frac{v}{2} \cdot x(t)^2 - (u \cdot y(t)) \cdot x(t) - \frac{v}{2}(y(t)^2 - 1) + w \cdot y(t). \quad (6)$$

$$y(t)' = -\frac{u}{2} \cdot y(t)^2 + (v \cdot x(t)) \cdot y(t) + \frac{u}{2}(x(t)^2 - 1) - w \cdot x(t). \quad (7)$$

Then we can put the above equations into Mathematica expression: [3]

```
v=1;
u=1;
w=1;
{xans6[t_],vans6[t_]}={x[t],y[t]}/.Flatten[NDSolve[{x'[t]==(v/2)*x[t]^2-
(u*y[t])*x[t]-v/2*(y[t]^2-1)+w*y[t], y'[t]==-(u/2)*y[t]^2+(v*x[t])*y[t]
+(u/2)*(x[t]^2-1)-w*x[t], x[0]==1,y[0]==0}, {x[t],y[t]}, {t,0,10}]]
graphx6 = Plot [xans6[t],{t,0,10}, Axes Label->{"t","x"}, Plot Style-
>Dashing[{0.02,0.02}]];
Show [graphx6,graphx6]
```

The result is as shown below: [3]

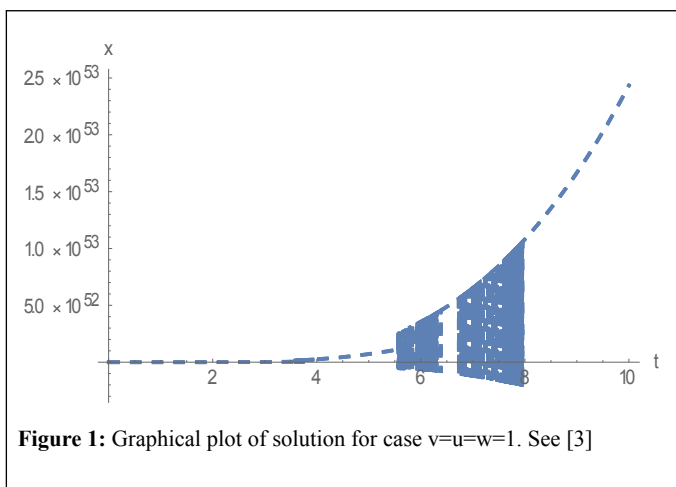


Figure 1: Graphical plot of solution for case v=u=w=1. See [3]

CONCLUDING REMARKS

In this paper we review 3D Navier-stokes equations obtained by Ershkov, as a model of virus as an elastic sphere in Newtonian fluid, and we solve the equations numerically with the help of Mathematica 11.

It is our hope that the above numerical solution of 3D Navier-Stokes equations can be found useful in mathematical modeling of virus [6-9].

All in all, here we would like to emphasize that such an investigation on acoustic vibration of virus particles may resonate with other reports by Prof. Luc Montagnier and also our own hypothesis, on wave character of biological entities such as DNA, virus, water, etc. [8-10].

We are quite optimistic that this novel approach can lead to new kinds of nanomedicine of virus based on acoustic vibration in Newtonian fluid medium [11-14].

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