Applied Mechanics and Materials
ISSN: 1662-7482, Vol. 859, pp 129-143
doi:10.4028/www.scientific.net/AMM.859.129
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Submitted: 2016-07-21
Revised: 2016-08-22
Accepted: 2016-08-23
Online: 2016-12-01

Multicriteria Decision Making Using Double Refined Indeterminacy Neutrosophic Cross Entropy and Indeterminacy Based Cross Entropy

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Keywords: Neutrosophic set, Double Refined Indeterminacy Neutrosophic Set (DRINS), Single Valued Neutrosophic Set (SVNS), Cross entropy, Multi criteria decision making.

Abstract. Double Refined Indeterminacy Neutrosophic Set (DRINS) is an inclusive case of the refined neutrosophic set, defined by Smarandache [1], which provides the additional possibility to represent with sensitivity and accuracy the uncertain, imprecise, incomplete, and inconsistent information which are available in real world. More precision is provided in handling indeterminacy; by classifying indeterminacy (I) into two, based on membership; as indeterminacy leaning towards truth membership (I_T) and indeterminacy leaning towards false membership (I_F). This kind of classification of indeterminacy is not feasible with the existing Single Valued Neutrosophic Set (SVNS), but it is a particular case of the refined neutrosophic set (where each T, I, F can be refined into T_1, T_2, …; I_1, I_2, …; F_1, F_2, …). DRINS is better equipped at dealing indeterminate and inconsistent information, with more accuracy than SVNS, which fuzzy sets and Intuitionistic Fuzzy Sets (IFS) are incapable of. Based on the cross entropy of neutrosophic sets, the cross entropy of DRINSs, known as double refined Indeterminacy neutrosophic cross entropy, is proposed in this paper. This proposed cross entropy is used for a multicriteria decision-making problem, where the criteria values for alternatives are considered under a DRINS environment. Similarly, an indeterminacy based cross entropy using DRINS is also proposed. The double refined Indeterminacy neutrosophic weighted cross entropy and indeterminacy based cross entropy between the ideal alternative and an alternative is obtained and utilized to rank the alternatives corresponding to the cross entropy values. The most desirable one(s) in decision making process is selected. An illustrative example is provided to demonstrate the application of the proposed method. A brief comparison of the proposed method with the existing methods is carried out.

Introduction

Fuzzy set theory introduced by Zadeh (1965) [2] provides a constructive analytic tool for soft division of sets. Zadeh's fuzzy set theory [2] was extended to intuitionistic fuzzy set (A-IFS), in which each element is assigned a membership degree and a non-membership degree by Atanassov (1986) [3]. A-IFS is more suitable in dealing with data that has fuzziness and uncertainty than fuzzy set. A-IFS was further generalized into the notion of interval valued intuitionistic fuzzy set (IVIFS) by Atanassov and Gargov (1989) [4].

Entropy is an essential concept for measuring uncertain information. Zadeh introduced the concept of fuzzy entropy [5]. The beginning for the cross entropy approach was founded in information theory by Shannon [6]. A measure of the cross entropy distance between two probability distributions was put forward by Kullback-Leibler [7], later a modified cross entropy measure was proposed by Lin [8]. A fuzzy cross entropy measure and a symmetric discrimination information measure between fuzzy sets was proposed by Shang and Jiang [9]. Since intuitionistic fuzzy set is a generalization of a fuzzy set, an extension of the De-Luca-Termini non probabilistic entropy [10] known as intuitionistic fuzzy cross-entropy was proposed by Vlachos and Sergiadis [11] and it was applied to pattern recognition, image segmentation and also to medical diagnosis. Vague cross-entropy between Vague Sets (VSs) by

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equivalence with the cross entropy of probability distributions was defined by Zhang and Jiang [12] and its application to the pattern recognition and medical diagnosis was carried out.

The fault diagnosis problem of turbine using the cross entropy of Vague Sets was investigated by Ye [13]. Intuitionistic fuzzy cross entropy was applied to multicriteria fuzzy decision-making problems by Ye [14]. An interval-valued intuitionistic fuzzy cross-entropy based on the generalization of the vague cross-entropy was proposed and applied to multicriteria decision-making problems by Ye [15].

To represent uncertain, imprecise, incomplete, and inconsistent information that are present in real world, the concept of a neutrosophic set from philosophical point of view was proposed by Smarandache [16]. The neutrosophic set is a prevailing framework that generalizes the concept of the classic set, fuzzy set, intuitionistic fuzzy set, interval valued fuzzy set, interval valued intuitionistic fuzzy set, paraconsistent set, paradoxist set, and tautological set. Truth membership, indeterminacy membership, and falsity membership are represented independently in the neutrosophic set. However, the neutrosophic set generalizes the above mentioned sets from the philosophical point of view, and its functions $T_A(x), I_A(x)$, and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$, that is, $T_A(x) : X \rightarrow]^-0, 1^+[$, $I_A(x) : X \rightarrow]^-0, 1^+[$, and $F_A(x) : X \rightarrow]^-0, 1^+[$, respectively with the condition $^-0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^+$.

It is difficult to apply neutrosophic set in this form in real scientific and engineering areas. To overcome this difficulty, Wang et al. [17] introduced a Single Valued Neutrosophic Set (SVNS), which is an instance of a neutrosophic set. SVNS can deal with indeterminate and inconsistent information, which fuzzy sets and intuitionistic fuzzy sets are incapable of. Ye [18, 19, 20] presented the correlation coefficient of SVNSs and its cross-entropy measure and applied them to single-valued neutrosophic decision-making problems. Recently, Ye [21] had proposed a Single Valued Neutrosophic cross entropy to do decision making in multicriteria decision making problems with the data represented by SVNSs.

Owing to the fuzziness, uncertainty and indeterminate nature of many practical problems in the real world, neutrosophy has found application in many fields including Social Network Analysis (Salama et al [22]), Image Processing (Cheng and Guo[23], Sengur and Guo[24], Zhang et al [25]), Social problems (Vasantha and Smarandache [26], [27]) etc.

To provide more accuracy and precision to indeterminacy, the indeterminacy value present in the neutrosophic set has been classified into two; based on membership; as indeterminacy leaning towards truth membership and as indeterminacy leaning towards false membership. When the indeterminacy I can be identified as indeterminacy which is more of truth value than false value, but it cannot be classified as truth it is considered to be indeterminacy leaning towards truth (I_T). When the indeterminacy can be identified to be indeterminacy which is more of the false value than the truth value, but it cannot be classified as false it is considered to be indeterminacy leaning towards false (I_F). Indeterminacy leaning towards truth and indeterminacy leaning towards falsity makes the indeterminacy involved in the scenario to be more accurate and precise. This modified refined neutrosophic set was defined as Double Refined Indeterminacy Neutrosophic Set (DRINS) alias double refined Indeterminacy Neutrosophic Set (DVNS) by Kandasamy [28].

To provide a illustration of real world problem where DRINS can be used to represent the problem; the following scenarios are given: Consider the scenario where the expert's opinion is requested about a particular statement, he/she may state that the possibility in which the statement is true is 0.6 and the statement is false is 0.5, the degree in which he/she is not sure but thinks it is true is 0.2 and the degree in which he/she is not sure but thinks it is false is 0.1. Using a double refined Indeterminacy neutrosophic notation or double refined Indeterminacy neutrosophic representation it can be expressed as x(0.6, 0.2, 0.1, 0.5).

Assume another example, suppose there are 10 voters during a voting process. Two people vote yes, two people vote no, three people are for yes but still undecided and two people are favouring towards a no but still undecided. Using a double refined Indeterminacy neutrosophic notation, it can be expressed as x(0.2, 0.3, 0.3, 0.2). However, these expressions are beyond the scope of representation using the existing SVNS. Therefore, the notion of a Double Refined Indeterminacy neutrosophic set is more general and it overcomes the aforementioned issues.

This paper is organised into seven sections: Section one is introductory in nature. The basic concepts related to the paper is given in section two. Section three of the paper introduces and defines the cross entropy of Double Refined Indeterminacy Neutrosophic Set (DRINS). Section four deals with the solving multi criteria decision making problems using the cross entropy of DRINS under a DRINS based environment. Illustrative examples are provided to demonstrate the proposed approach in section five. Section six provides a brief comparison of the proposed approach with the existing approach. Conclusions and future direction of work is given in the last section.

Preliminaries / Basic Concepts

Neutrosophy and Single Valued Neutrosophic Set (SVNS). Neutrosophy is a branch of philosophy, introduced by Smarandache [16], which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. It considers a proposition, concept, theory, event, or entity, "A" in relation to its opposite, "Anti-A" and that which is not A, "Non-A", and that which is neither "A" nor "Anti-A", denoted by "Neut-A". Neutrosophy is the basis of neutrosophic logic, neutrosophic probability, neutrosophic set, and neutrosophic statistics.

The concept of a neutrosophic set from philosophical point of view, introduced by Smarandache [16], is as follows.

Definition 1. [16] Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and a falsity membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$, that is, $T_A(x) : X \to]^-0, 1^+[$, $I_A(x) : X \to]^-0, 1^+[$, and $F_A(x) : X \to]^-0, 1^+[$, with the condition $^-0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^+$.

This definition of neutrosophic set is difficult to apply in real world application of scientific and engineering fields. Therefore, the concept of Single Valued Neutrosophic Set (SVNS), which is an instance of a neutrosophic set was introduced by Wang et al. [17].

Definition 2. [17] Let X be a space of points (objects) with generic elements in X denoted by x. An Single Valued Neutrosophic Set (SVNS) A in X is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$, and falsity membership function $F_A(x)$. For each point x in X, there are $T_A(x), I_A(x), F_A(x) \in [0, 1]$, and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. Therefore, an SVNS A can be represented by $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$. The following expressions are defined in [17] for SVNSs A, B:

- $A \in B$ if and only if $T_A(x) \leq T_B(x), I_A(x) \geq I_B(x), F_A(x) \geq F_B(x)$ for any x in X.
- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
- $A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle | x \in X\}$.

The refined neutrosophic logic defined by [1] is as follows:

Definition 3. T can be split into many types of truths: $T_1, T_2, \ldots, T_p$, and I into many types of indeterminacies: I_1, I_2, $\ldots$, I_r, and F into many types of falsities: $F_1, F_2, \ldots, F_s$, where all $p, r, s \geq 1$ are integers, and $p + r + s = n$. In the same way, but all subcomponents T_j, I_k, F_l are not symbols, but subsets of [0, 1], for all $j \in \{1, 2, \ldots, p\}$ all $k \in \{1, 2, \ldots, r\}$ and all $l \in \{1, 2, \ldots, s\}$. If all sources of information that separately provide neutrosophic values for a specific subcomponent are independent sources, then in the general case we consider that each of the subcomponents T_j, I_k, F_l is independent with respect to the others and it is in the non-standard set $]^-0, 1^+[$.

Cross Entropy of SVNSs and Multicriteria Decision Making.

The concepts of cross-entropy and symmetric discrimination information measures between two fuzzy sets proposed by Shang and Jiang [9] and between two SVNSs was proposed by Ye [20].

Definition 4. Assume that $A = (A(x_1), A(x_2), \ldots, A(x_n))$ and $B = (B(x_1), B(x_2), \ldots, B(x_n))$ are two fuzzy sets in the universe of discourse $X = x_1, x_2, \ldots, x_n$. The fuzzy cross entropy of A from B is defined as follows:

$$H(A,B) = \sum_{i=1}^{n} \left\{ A(x_i) log_2 \frac{A(x_i)}{\frac{1}{2}(A(x_i) + B(x_i))} + (1 - A(x_i)) log_2 \frac{(1 - A(x_i))}{1 - \frac{1}{2}(A(x_i) + B(x_i))} \right\} \quad (1)$$

which indicates the degree of discrimination of A from B.

Shang and Jiang [9] proposed a symmetric discrimination information measure $I(A,B) = H(A,B) + H(B,A)$ since $H(A,B)$ is not symmetric with respect to its arguments. Moreover, there are $I(A,B) \geq 0$ and $I(A,B) = 0$ if and only if $A = B$. The cross entropy and symmetric discrimination information measures between two fuzzy sets was extended to SVNSs by Ye [20].

Let A and B be two SVNSs in a universe of discourse $X = \{x_l, x_2, \ldots, x_n\}$, which are denoted by $A = \{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle \mid x_i \in X\}$ and $B = \{\langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\rangle \mid x_i \in X\}$, where $T_A(x_i), I_A(x_i), F_A(x_i), T_B(x_i), I_B(x_i), F_B(x_i) \in [0,1]$ for every $x_i \in X$.

The information carried by the truth, indeterminacy and falsity memberships in SVNSs, A and B is considered as fuzzy spaces with three elements. Based on Equation 1, the amount of information for discrimination of $T_A(x_i)$ from $T_B(x_i)$ ($i = 1, 2, \ldots, n$) is given as

$$E^T(A,B;x_i) = T_A(x_i) log_2 \frac{T_A(x_i)}{T_B(x_i)} + (1 - T_A(x_i)) log_2 \frac{1 - T_A(x_i)}{1 - \frac{1}{2}(T_A(x_i) + T_B(x_i))}.$$

The expected information based on the single membership for discrimination of A against B is

$$E^T(A,B) = \sum_{i=1}^{n} \left\{ T_A(x_i) log_2 \frac{T_A(x_i)}{T_B(x_i)} + \quad (1 - T_A(x_i)) log_2 \frac{1 - T_A(x_i)}{1 - \frac{1}{2}(T_A(x_i) + T_B(x_i))} \right\}.$$

Similarly, the indeterminacy and the falsity membership function, have the following amounts of information:

$$E^I(A,B) = \sum_{i=1}^{n} \left\{ I_A(x_i) log_2 \frac{I_A(x_i)}{I_B(x_i)} + (1 - I_A(x_i)) log_2 \frac{1 - I_A(x_i)}{1 - \frac{1}{2}(I_A(x_i) + I_B(x_i))} \right\}$$

$$E^F(A,B) = \sum_{i=1}^{n} \left\{ F_A(x_i) log_2 \frac{F_A(x_i)}{F_B(x_i)} + (1 - F_A(x_i)) log_2 \frac{1 - F_A(x_i)}{1 - \frac{1}{2}(F_A(x_i) + F_B(x_i))} \right\}.$$

The single valued neutrosophic cross entropy measure between A and B is obtained as the sum of the three measures:

$$E(A,B) = E^T(A,B) + E^I(A,B) + E^F(A,B)$$

$E(A,B)$ also indicates discrimination degree of A from B.

According to Shannon's inequality [6], it is seen that $E(A,B) \geq 0$, and $E(A,B) = 0$ if and only if $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$ for any $x_i \in X$. Moreover, it is seen that $E(A^c, B^c) = E(A,B)$, where A^c and B^c are the complement of SVNSs of A and B, respectively. Then, $E(A,B)$ is not symmetric, i.e., $E(B,A) \neq E(A,B)$, so it is modified to a symmetric discrimination information measure for SVNSs as

$$D(A,B) = E(A,B) + E(B,A).$$

The larger the difference between A and B is, the larger $D(A,B)$ is. The cross entropy of SVNS was used to handle the multicriteria decision making problem under single valued neutrosophic environment by means of the cross entropy measure of SVNSs.

The weighted cross entropy between an alternative A_i and the ideal alternative A^* is calculated as

$$D(A^*,A_i)=\sum_{i=1}^{n}w_j\left\{log_2\frac{1}{\frac{1}{2}(1+T_{ij})}+log_2\frac{1}{1+\frac{1}{2}(I_{ij})}+log_2\frac{1}{1+\frac{1}{2}(F_{ij})}\right\}$$
$$+\sum_{i=1}^{n}w_j\left\{T_{ij}log_2\frac{T_{ij}}{\frac{1}{2}(1+T_{ij})}+(1-T_{ij})log_2\frac{1-T_{ij}}{1-\frac{1}{2}(1+T_{ij})}\right\}$$
$$+\sum_{i=1}^{n}w_j\left\{I_{ij}+(1-I_{ij})log_2\frac{1-I_{ij}}{1-\frac{1}{2}(I_{ij})}\right\}$$
$$+\sum_{i=1}^{n}w_j\left\{F_{ij}+(1-F_{ij})log_2\frac{1-F_{ij}}{1-\frac{1}{2}(F_{ij})}\right\}.$$

Based on the cross entropy value the ranking is carried out. The best alternative is selected based in the ranking of the cross entropy values.

Double Refined Indeterminacy Neutrosophic Sets (DRINSs) and Their Properties.

Indeterminacy deals with uncertainty that is faced in every sphere of life by everyone. It makes research/science more realistic and sensitive by introducing the indeterminate aspect of life as a concept. There are times in real world where the indeterminacy I can be identified to be indeterminacy which has more of truth value than false value, but it cannot be classified as truth. Similarly in some cases the indeterminacy can be identified to be indeterminacy which has more of false value than truth value, but it cannot be classified as false. To provide more sensitivity to indeterminacy, this kind of indeterminacy is classified into two. When the indeterminacy I can be identified as indeterminacy which is more of truth value than false value, but it cannot be classified as truth, it is considered to be indeterminacy leaning towards truth (I_T). Whereas in case the indeterminacy can be identified to be indeterminacy which is more of false value than truth value, but it cannot be classified as false, it is considered to be indeterminacy leaning towards false (I_F).

Indeterminacy leaning towards truth and indeterminacy leaning towards falsity make the handling of the indeterminacy involved in the scenario to be more meaningful, logical, accurate and precise. It provides a better and detailed view of the existing indeterminacy.

The definition of Double Refined Indeterminacy Neutrosophic Set (DRINS) [28] is as follows:

Definition 5. Let X be a space of points (objects) with generic elements in X denoted by x. A Double Refined Indeterminacy Neutrosophic Set (DRINS) A in X is characterized by truth membership function $T_A(x)$, indeterminacy leaning towards truth membership function $I_{TA}(x)$, indeterminacy leaning towards falsity membership function $I_{FA}(x)$, and falsity membership function $F_A(x)$. For each generic element $x\in X$, there are $T_A(x)$, $I_{TA}(x)$, $I_{FA}(x)$, $F_A(x)\in[0,1]$, and $0\le T_A(x)+I_{TA}(x)+I_{FA}(x)+F_A(x)\le 4$.

Therefore, a DRINS A can be represented by

$$A=\{\langle x,T_A(x),I_{TA}(x),I_{FA}(x),F_A(x)\rangle\mid x\in X\}.$$

A DRINS A is represented as

$$A=\int_X\{\langle T(x),I_T(x),I_F(x),F(x)\rangle/dx,x\in X\}$$

when X is continuous. It is represented as

$$A = \sum_{i=1}^{n} \{\langle T(x_i), I_T(x_i), I_F(x_i), F(x_i)\rangle \mid x_i, x_i \in X\}$$

when X is discrete.

To illustrate the application of DRINS in the real world condsider parameters that are commonly used to define quality of service of semantic web services like capability, trustworthiness and price for illustrative purpose. The evaluation of quality of service of semantic web services [29] is used to illustrate set theoretic operation on Double Refined Indeterminacy Neutrosophic Sets (DRINSs).

Definition 6. The complement of a DRINS A denoted by $c(A)$ is defined as $T_{c(A)}(x) = F_A(x)$, $I_{Tc(A)}(x) = 1 - I_{TA}(x)$, $I_{Fc(A)}(x) = 1 - I_{FA}(x)$ and $F_{c(A)}(x) = T_A(x)$ for all x in X.

Definition 7. A DRINS A is contained in the other DRINS B, that is $A \subseteq B$, if and only if $T_A(x) \leq T_B(x)$, $I_{TA}(x) \leq I_{TB}(x)$, $I_{FA}(x) \leq I_{FB}(x)$ and $F_A(x) \geq F_B(x)$ $\forall x$ in X.

Note that by the definition of containment relation, X is a partially ordered set and not a totally ordered set.

Definition 8. Two DRINSs A and B are equal, denoted as $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.

The union of two DRINSs A and B is a DRINS C, denoted as $C = A \cup B$, , the intersection of two DRINSs A and B is a DRINS C, denoted as $C = A \cap B$, and the difference of two DRINSs A and B is D, written as $D = A \setminus B$, was defined in [28]. Three operators called as truth favourite ($\triangle$), falsity favourite ($\bigtriangledown$) and indeterminacy neutral (∇) are defined over DRINSs. Two operators truth favourite ($\triangle$) and falsity favourite ($\bigtriangledown$) are defined to remove the indeterminacy in the DRINSs and transform it into intuitionistic fuzzy sets or paraconsistent sets. Similarly the DRINS can be transformed into a SVNS by applying indeterminacy neutral (∇) operator that combines the indeterminacy values of the DRINS. These three operators are unique on DRINSs.

Definition 9. The truth favourite of a DRINS A, written as $B = \triangle A$, whose truth membership and falsity membership functions are related to those of A by $T_B(x) = min(T_A(x)+I_{TA}(x), 1)$, $I_{TB}(x) = 0$, $I_{FB}(x) = 0$ and $F_B(x) = F_A(x)$ for all x in X.

Definition 10. The falsity favourite of a DRINS A, written as $B = \bigtriangledown A$, whose truth membership and falsity membership functions are related to those of A by $T_B(x) = T_A(x)$, $I_{TB}(x) = 0$, $I_{FB}(x) = 0$ and $F_B(x) = min(F_A(x) + I_{FA}(x), 1)$ for all x in X.

Definition 11. The indeterminacy neutral of a DRINS A, written as $B = \nabla A$, whose truth membership, indeterminate membership and falsity membership functions are related to those of A by $T_B(x) = T_A(x)$, $I_{TB}(x) = min(I_{TA}(x) + I_{TB}(x), 1)$, $I_{FB}(x) = 0$ and $F_B(x) = F_A(x)$ for all x in X.

All set theoretic operators like commutativity, Associativity, Distributivity, Idempotency, Absorption and the De Morgan's Laws were defined over DRINSs [28]. The definition of complement, union and intersection of DRINSs and DRINSs itself satisfies most properties of the classical set, fuzzy set, intuitionistic fuzzy set and SNVS. Similar to fuzzy set, intuitionistic fuzzy set and SNVS, it does not satisfy the principle of middle exclude.

Cross Entropy of Double Refined Indeterminacy Neutrosophic Sets (DRINSs)

Consider two DRINSs A and B in a universe of discourse $X = x_l, x_2, \ldots, x_n$, which are denoted by $A = \{\langle x_i, T_A(x_i), I_{TA}(x_i), I_{FA}(x_i), F_A(x_i)\rangle \mid x_i \in X\}$
and $B = \{\langle x_i, T_B(x_i), I_{TB}(x_i), I_{FB}(x_i), F_B(x_i)\rangle \mid x_i \in X\}$,
where $T_A(x_i), I_{TA}(x_i), I_{FA}(x_i), F_A(x_i), T_B(x_i), I_{TB}(x_i), I_{FB}(x_i), F_B(x_i) \in [0, 1]$ for every $x_i \in X$.

The information carried by the truth membership, indeterminacy leaning towards truth membership, indeterminacy leaning towards falsity membership, and the falsity membership in DRINSs A and B are considered as fuzzy spaces with four elements. Thus based on Equation 1, the amount of information for discrimination of $T_A(x_i)$ from $T_B(x_i)$ (i = 1, 2, . . . , n) can be given by

$$E^T(A, B; x_i) = T_A(x_i)log_2\frac{T_A(x_i)}{T_B(x_i)} + (1 - T_A(x_i))log_2\frac{1 - T_A(x_i)}{1 - \frac{1}{2}(T_A(x_i) + T_B(x_i))}$$

Therefore, the expected information based on the single membership for discrimination of A against B is expressed by

$$E^T(A, B) = \sum_{i=1}^{n}\left\{T_A(x_i)log_2\frac{T_A(x_i)}{T_B(x_i)} + + (1 - T_A(x_i))log_2\frac{1 - T_A(x_i)}{1 - \frac{1}{2}(T_A(x_i) + T_B(x_i))}\right\}$$

Similarly, considering the indeterminacy leaning towards truth membership function, indeterminacy leaning towards falsity membership function and falsity membership function the following amounts of information is given:

$$E^{I_T}(A, B) = \sum_{i=1}^{n}\left\{I_{TA}(x_i)log_2\frac{I_{TA}(x_i)}{I_{TB}(x_i)} + (1 - I_{TA}(x_i))log_2\frac{1 - I_{TA}(x_i)}{1 - \frac{1}{2}(I_{TA}(x_i) + I_{TB}(x_i))}\right\},$$

$$E^{I_F}(A, B) = \sum_{i=1}^{n}\left\{I_{FA}(x_i)log_2\frac{I_{FA}(x_i)}{I_{FB}(x_i)} + (1 - I_{FA}(x_i))log_2\frac{1 - I_{FA}(x_i)}{1 - \frac{1}{2}(I_{FA}(x_i) + I_{FB}(x_i))}\right\},$$

$$E^F(A, B) = \sum_{i=1}^{n}\left\{F_A(x_i)log_2\frac{F_A(x_i)}{F_B(x_i)} + (1 - F_A(x_i))log_2\frac{1 - F_A(x_i)}{1 - \frac{1}{2}(F_A(x_i) + F_B(x_i))}\right\}.$$

The Double Refined Indeterminacy neutrosophic cross entropy measure between A and B is obtained as the sum of the four measures:

$$E(A, B) = E^T(A, B) + E^{I_T}(A, B) + E^{I_F}(A, B) + E^F(A, B)$$

$E(A, B)$ also indicates discrimination degree of A from B. According to Shannon's inequality [5], it can be easily proved that $E(A, B) \geq 0$, and $E(A, B) = 0$ if and only if $T_A(x_i) = T_B(x_i)$, $I_{TA}(x_i) = I_{TB}(x_i)$, $I_{FA}(x_i) = I_{FB}(x_i)$, and $F_A(x_i) = F_B(x_i)$ for any $x_i \in X$. It easily seen that $E(A^c, B^c) = E(A, B)$, where A^c and B^c are the complement of DRINSs A and B, respectively. Since $E(A, B)$ is not symmetric it is modified to a symmetric discrimination information measure for DRINSs as

$$D(A, B) = E(A, B) + E(B, A) \tag{2}$$

The larger $D(A, B)$ is, the larger the difference between A and B is.

A cross entropy measure based only on the indeterminacy involved in the scenario is introduced in this paper. Indeterminacy based cross entropy is defined as the sum of information of indeterminacy leaning towards falsity membership and information of indeterminacy leaning towards truth membership. The indeterminacy based cross entropy $IE(A,B)$ is the Double Refined Indeterminacy neutrosophic cross entropy measure based on indeterminacy between A and B; is obtained as the sum of the two measures:

$$IE(A,B) = \sum_{i=1}^{n} \left\{ I_{TA}(x_i) log_2 \frac{I_{TA}(x_i)}{I_{TB}(x_i)} + (1 - I_{TA}(x_i)) log_2 \frac{1 - I_{TA}(x_i)}{1 - \frac{1}{2}(I_{TA}(x_i) + I_{TB}(x_i))} \right\}$$
$$+ \sum_{i=1}^{n} \left\{ I_{FA}(x_i) log_2 \frac{I_{FA}(x_i)}{I_{FB}(x_i)} + (1 - I_{FA}(x_i)) log_2 \frac{1 - I_{FA}(x_i)}{1 - \frac{1}{2}(I_{FA}(x_i) + I_{FB}(x_i))} \right\}$$

It indicates the discrimination degree of indeterminacy of A from B. According to Shannon's inequality [5], it can be easily proved that $IE(A,B) \geq 0$, and $IE(A,B) = 0$ if and only if $T_A(x_i) = T_B(x_i), I_{TA}(x_i) = I_{TB}(x_i), I_{FA}(x_i) = I_{FB}(x_i)$, and $F_A(x_i) = F_B(x_i)$ for any $x_i \in X$. It is easily seen that $IE(A^c, B^c) = IE(A,B)$, where A^c and B^c are the complement of DRINSs A and B, respectively. Since $IE(A,B)$ is not symmetric, it is modified to a symmetric discrimination information measure for DRINSs as

$$ID(A,B) = IE(A,B) + IE(B,A). \tag{3}$$

The larger the difference in indeterminacy between A and B is, the larger $ID(A,B)$ is.

Multicriteria Decision Making Method Based on the Cross Entropy of DRINS

In a multicriteria decision making problem all the alternatives are evaluated depending on a number of criteria or some attributes, and the best alternative is selected from all the possible alternatives. Mostly multicriteria decision making problem have to be inclusive of uncertain, imprecise, incomplete, and inconsistent information that are present in real world to make it more realistic. DRINS can be used to represent this information with accuracy and precision. In this section, by means of utilizing the cross entropy measure of DRINSs and indeterminacy based cross entropy a method for solving the multi-criteria decision making problem when considered in a Double Refined Indeterminacy neutrosophic environment, is proposed.

Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of feasible alternatives and $C = \{C_1, C_2, \ldots, C_n\}$ be the set of criteria under consideration. The weight of the criterion $C_j (j = 1, 2, \ldots, n)$, provided by the decision maker, is w_j, $w_j \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$. The characteristic of the alternative $A_i (i = 1, 2, \ldots, m)$ is given by DRINS $A_i = \{\langle C_j, T_{A_i}(C_j), I_{TA_i}(C_j), I_{FA_i}(C_j), F_{A_i}(C_j)\rangle \mid C_j \in C\}$ where $T_{A_i}(C_j), I_{TA_i}(C_j), I_{FA_i}(C_j), F_{A_i}(C_j) \in [0,1]$, $j = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, m$.

Now, $T_{A_i}(C_j)$ specifies the degree to which the alternative A_i fulfils the criterion C_j, $I_{TA_i}(C_j)$ specifies the indeterminacy leaning towards truth degree to which the alternative A_i fulfils or does not fulfil the criterion C_j. Similarly $I_{FA_i}(C_j)$ specifies the indeterminacy leaning towards false degree (or false leaning indeterminacy) to which the alternative A_i fulfils or does not fulfil the criterion C_j, and $F_{A_i}(C_j)$ specifies the degree to which the alternative A_i does not fulfil the criterion C_j.

A criterion value is generally obtained from the calculation of an alternative A_i with respect to a criteria C_j by means of a score law and data processing in practice [12, 17]. It is represented as $\langle C_j, T_{A_i}(C_j), I_{TA_i}(C_j), I_{FA_i}(C_j), F_{A_i}(C_j)\rangle$ in A_i, is denoted by the symbol $a_{ij} = \langle T_{ij}, I_{Tij}, I_{Fij}, F_{ij}\rangle$ $(j = 1, 2, \ldots, n$, and $i = 1, 2, \ldots, m)$,

Therefore, a Double Refined Indeterminacy neutrosophic decision matrix $A = (a_{ij})_{m\times n}$ is obtained.

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix}$$

$$= \begin{pmatrix} \langle T_{11}, I_{T11}, I_{F11}, F_{11}\rangle & \langle T_{12}, I_{T12}, I_{F12}, F_{12}\rangle & \dots & \langle T_{1n}, I_{T1n}, I_{F1n}, F_{1n}\rangle \\ \langle T_{21}, I_{T21}, I_{F21}, F_{21}\rangle & \langle T_{22}, I_{T22}, I_{F22}, F_{22}\rangle & \dots & \langle T_{2n}, I_{T2n}, I_{F2n}, F_{2n}\rangle \\ \vdots & \vdots & & \vdots \\ \langle T_{m1}, I_{Tm1}, I_{Fm1}, F_{m1}\rangle & \langle T_{m2}, I_{Tm2}, I_{Fm2}, F_{m2}\rangle & \dots & \langle T_{mn}, I_{Tmn}, I_{Fmn}, F_{mn}\rangle \end{pmatrix}.$$

The concept of ideal point is utilized to aid the identification of the best alternative in the decision set, in multicriteria decision making environments. It is known that an ideal alternative cannot exist in the real world; but it does serves as a useful theoretical construct against which alternatives can be evaluated [12].

Therefore an ideal criterion value $a_j^* = \langle T_j^*, I_{Tj}^*, I_{Fj}^*, F_j^*\rangle = \langle 1,0,0,0\rangle (j = 1,2,\dots,n)$ is defined in the ideal alternative A^*. By applying Equation 2 the weighted cross entropy between an alternative A_i and ideal alternative A^* is obtained to be

$$D(A^*, A_i) = \sum_{j=1}^{n} w_j \left\{ log_2 \frac{1}{\frac{1}{2}(1+T_{ij})} + log_2 \frac{1}{1-\frac{1}{2}(I_{Tij})} + log_2 \frac{1}{1-\frac{1}{2}(I_{Fij})} + log_2 \frac{1}{1-\frac{1}{2}(F_{ij})} \right\} +$$
$$\sum_{j=1}^{n} w_j \left\{ T_{ij} log_2 \frac{T_{ij}}{\frac{1}{2}(1+T_{ij})} + (1-T_{ij}) log_2 \frac{1-T_{ij}}{1-\frac{1}{2}(1+T_{ij})} \right\}$$
$$+ \sum_{j=1}^{n} w_j \left\{ I_{Tij} + (1-I_{Tij}) log_2 \frac{1-I_{Tij}}{1-\frac{1}{2}(I_{Tij})} \right\} + \sum_{j=1}^{n} w_j \left\{ I_{Fij} + (1-I_{Fij}) log_2 \frac{1-I_{Fij}}{1-\frac{1}{2}(I_{Fij})} \right\}$$
$$+ \sum_{j=1}^{n} w_j \left\{ F_{ij} + (1-F_{ij}) log_2 \frac{1-F_{ij}}{1-\frac{1}{2}(F_{ij})} \right\}. \quad (4)$$

The smaller the value of $D_i(A^*, A_i)$ is, the better the alternative A_i is, it implies that the alternative A_i is close to the ideal alternative A^*. The ranking order of all alternatives is determined and the best one is identified, through the calculation of the weighted cross entropy $D_i(A^*, A_i)$ $(i = 1,2,\dots,m)$ between each alternative and the ideal alternative.

For calculating the indeterminate based cross entropy measure $I_T D$ between alternative A and the indeterminate ideal alternative $A_{I_T}^*$ the indeterminate ideal alternative $A_{I_T}^*$ is defined as an ideal criterion value

$$a_j^* = \langle T_j^*, I_{Tj}^*, I_{Fj}^*, F_j^*\rangle = \langle 0,1,0,0\rangle (j = 1,2,\dots,n).$$

By applying Equation 3 the weighted indeterminacy based cross entropy between an alternative A_i and the ideal alternative $A_{I_T}^*$ is obtained to be

$$I_T D(A_{I_T}^*, A_i) = \sum_{j=1}^{n} w_j \left\{ log_2 \frac{1}{\frac{1}{2}(1+I_{Tij})} + log_2 \frac{1}{1-\frac{1}{2}(I_{Fij})} \right\} +$$
$$\sum_{j=1}^{n} w_j \left\{ I_{Tij} log_2 \frac{I_{Tij}}{\frac{1}{2}(1+I_{Tij})} + (1-I_{Tij}) log_2 \frac{1-I_{Tij}}{1-\frac{1}{2}(1+I_{Tij})} \right\}$$
$$+ \sum_{j=1}^{n} w_j \left\{ I_{Fij} + (1-I_{Fij}) log_2 \frac{1-I_{Fij}}{1-\frac{1}{2}(I_{Fij})} \right\}. \quad (5)$$

To study the indeterminate ideal alternative $A_{I_F}^*$, which based on indeterminacy leaning towards falsity, it is defined using the ideal criterion value $a_j^* = \langle T_j^*, I_{Tj}^*, I_{Fj}^*, F_j^*\rangle = \langle 0,0,1,0\rangle (j = 1,2,\dots,n)$.

Calculating the indeterminate based cross entropy measure $I_F D$ between alternative A and the indeterminate ideal alternative $A^*_{I_F}$

$$I_F D(A^*_{IF}, A_i) = \sum_{i=1}^{n} w_j \left\{ log_2 \frac{1}{\frac{1}{2}(1 + I_{Fij})} + log_2 \frac{1}{1 - \frac{1}{2}(I_{Tij})} \right\} +$$
$$\sum_{i=1}^{n} w_j \left\{ I_{Fij} log_2 \frac{I_{Fij}}{\frac{1}{2}(1 + I_{Fij})} + (1 - I_{Fij}) log_2 \frac{1 - I_{Fij}}{1 - \frac{1}{2}(1 + I_{Fij})} \right\}$$
$$+ \sum_{i=1}^{n} w_j \left\{ I_{Tij} + (1 - I_{Tij}) log_2 \frac{1 - I_{Tij}}{1 - \frac{1}{2}(I_{Tij})} \right\} \quad (6)$$

The average of $I_T D(A^*_{IT}, A_i)$ and $I_F D(A^*_{IF}, A_i)$ is taken as $ID(A^*_I, A_i)$.

$$ID(A^*, A_i) = \frac{I_T D(A^*_{IT}, A_i) + I_F D(A^*_{IF}, A_i)}{2}. \quad (7)$$

The larger the value of $ID(A^*, A_i)$ is, the better the alternative A_i is, it implies that the alternative A_i is farther to the ideal alternative A^*. The ranking order of all alternatives is determined and the best one is identified, through the calculation of the indeterminacy based weighted cross entropy $ID(A^*, A_i)$ $(i = 1, 2, \dots, m)$ between each alternative and the ideal alternative.

Illustrative Examples

To illustrate the application of the proposed method, the multicriteria decision making problem from Tan and Chen [30] and Ye[19] is adapted. It is related with a manufacturing company that wants to select the best global supplier according to the core competencies of suppliers. Suppose that there are four suppliers $A = A_1, A_2, A_3, A_4$ enlisted; whose core competencies are evaluated based of the following four criteria (C_1, C_2, C_3, C_4):

1. (C_1) the level of technology innovation,
2. (C_2) the control ability of flow,
3. (C_3) the ability of management, and
4. (C_4) the level of service.

The weight vector related to the four criteria is w = (0.3, 0.25, 0.25, 0.2).

The proposed multicriteria decision making approach is applied to select the best supplier. From the questionnaire of a domain expert, the evaluation of an alternative A_i (i = 1, 2, 3, 4) with respect to a criterion C_j(j = 1, 2, 3, 4), is obtained. For instance, when the opinion of an expert about an alternative A_1 with respect to a criterion C_1 is asked, he or she may say that the possibility in which the statement is true is 0.5, the degree in which he or she feels it true but is not sure is 0.07, the degree in which he or she feels it is false but is not sure is 0.03 and the possibility the statement is false is 0.3. It can be expressed as $a_{11} = \langle 0.5, 0.07, 0.03, 0.2 \rangle$, using the neutrosophic expression of DRINS. The possible alternatives with respect to the given four criteria is evaluated by the similar method from the expert, the following Double Refined Indeterminacy neutrosophic decision matrix A is obtained.

$$A = \begin{pmatrix} \langle 0.5, 0.07, 0.03, 0.2 \rangle & \langle 0.5, 0.08, 0.02, 0.4 \rangle & \langle 0.7, 0.06, 0.04, 0.2 \rangle & \langle 0.3, 0.4, 0.1, 0.1 \rangle \\ \langle 0.4, 0.12, 0.08, 0.3 \rangle & \langle 0.3, 0.04, 0.16, 0.4 \rangle & \langle 0.9, 0.06, 0.04, 0.1 \rangle & \langle 0.5, 0.1, 0.1, 0.2 \rangle \\ \langle 0.4, 0.17, 0.03, 0.1 \rangle & \langle 0.5, 0.18, 0.02, 0.3 \rangle & \langle 0.5, 0.06, 0.04, 0.4 \rangle & \langle 0.6, 0.14, 0.06, 0.1 \rangle \\ \langle 0.6, 0.07, 0.03, 0.2 \rangle & \langle 0.2, 0.15, 0.05, 0.5 \rangle & \langle 0.4, 0.16, 0.04, 0.2 \rangle & \langle 0.7, 0.11, 0.09, 0.1 \rangle \end{pmatrix}$$

The cross entropy values between an alternative A_i (i = 1, 2, 3, 4) and the ideal alternative A^* is obtained by applying Equation 4 is $D(A^*, A_1)$ = 1.5054, $D(A^*, A_2)$ = 1.1056, $D(A^*, A_3)$ = 1.0821 and $D(A^*, A_4)$ = 1.1849. The ranking order of the four suppliers according to the cross entropy values is

$$D(A^*, A_1) \leq D(A^*, A_3) \leq D(A^*, A_2) \leq D(A^*, A_4)$$

The truth-indeterminacy based cross entropy values between an alternative A_i (i = 1, 2, 3, 4) and the ideal alternative $A^*_{I_T}$ is obtained by applying Equation 5, are $I_T D(A^*, A_1)$ = 1.5054, $I_T D(A^*, A_2)$ = 1.6920, $I_T D(A^*, A_3)$ = 1.4392 and $I_T D(A^*, A_4)$ = 1.5067. The false-indeterminacy based cross entropy values between an alternative A_i (i = 1, 2, 3, 4) and the ideal alternative $A^*_{I_F}$ is obtained by applying Equation 6, are $I_F D(A^*, A_1)$ = 1.9000, $I_F D(A^*, A_2)$ = 1.6348, $I_F D(A^*, A_3)$ = 1.9256 and $I_F D(A^*, A_4)$ = 1.8447. The indeterminacy based cross entropy values based on Equation 7are $ID(A^*, A_1)$ = 1.7027, $ID(A^*, A_2)$ = 1.6634, $ID(A^*, A_3)$ = 1.6824 and $ID(A^*, A_4)$ = 1.6757.

The ranking order of the four suppliers according to the cross entropy values is

$$ID(A^*, A_1) \geq ID(A^*, A_3) \geq ID(A^*, A_4) \geq ID(A^*, A_2).$$

The DRINS cross entropy and indeterminacy based cross entropy results of the different alternatives and the ideal alternatives are tabulated in Table 1.

Table 1: DRIN Cross Entropy and indeterminacy based Cross Entropy Results

Cross Entropy Value A_i	DRIN Cross Entropy $D(A^*, A_i)$	Indeterminate based cross entropy $ID(A^*_I, A_i)$
A_1	1.0793	1.7027
A_2	1.1056	1.6634
A_3	1.0821	1.6824
A_4	1.1845	1.6757
Result	A_1	A_1

An alternative is considered to be best if it has the least DRIN cross entropy value and the maximum indeterminate based DRIN cross entropy. Therefore it is seen that A_1 is the best supplier.

It is clearly seen that the proposed Double Refined Indeterminacy neutrosophic multicriteria decision making method is more preferable and suitable for real scientific and engineering applications because it can handle not only incomplete information but also the indeterminate information and inconsistent information which exist commonly in real situations more logically with much more accuracy and precision that SVNS are incapable of dealing.

Comparison

This paper proposes a technique that extends existing SVNS and fuzzy decision making methods and provides an improvement in dealing indeterminate and inconsistent information with accuracy which is new for decision making problems. For comparative purpose, the results of cross entropy of SVNS [20] and the proposed method are given in Table 2.

From Table 2, it is seen that the results are quite different. The important reason can be obtained by the following comparative analysis of the methods and their capacity to deal indeterminate, inconsistent and incomplete information.

Double Refined Indeterminacy neutrosophic information is a generalization of neutrosophic information. It is observed that neutrosophic information / single valued neutrosophic information is

Table 2: Cross Entropy results of different cross entropy between ideal alternative and alternative

Cross Entropy Value	SVN cross Entropy $D_i(A^*, A_i)$	DRIN Cross Entropy $D(A^*, A_i)$	Indeterminate based cross entropy $ID(A_I^*, A_i)$
A_1	1.1101	1.0793	1.7027
A_2	1.1801	1.1056	1.6634
A_3	0.9962	1.0821	1.6824
A_4	1.2406	1.1850	1.6757
Result	A_3	A_1	A_1

generalization of intuitionistic fuzzy information, and intuitionistic fuzzy information is itself a generalization of fuzzy information.

DRINS is an instance of a neutrosophic set, which approaches the problem more logically with accuracy and precision to represent the existing uncertainty, imprecise, incomplete, and inconsistent information. It has the additional feature of being able to describe with more sensitivity the indeterminate and inconsistent information. While, the SVNS can handle indeterminate information and inconsistent information, it is cannot describe with accuracy about the existing indeterminacy.

It is known that the connector in fuzzy set is defined with respect to T (membership only) so the information of indeterminacy and non membership is lost. The connectors in intuitionistic fuzzy set are defined with respect to truth membership and false membership only; here the indeterminacy is taken as what is left after the truth and false membership.

The intuitionistic fuzzy set cannot deal with the indeterminate and inconsistent information but it has provisions to describe and deal with incomplete information. In SVNS, truth, indeterminacy and falsity membership are represented independently, and they can also be defined with respect to any of them (no restriction) and the approach is more logical. This makes SVNS equipped to deal information better than IFS, whereas in DRINS, more scope is given to describe and deal with the existing indeterminate and inconsistent information because the indeterminacy concept is classified as two distinct values. This provides more accuracy and precision to indeterminacy in DRINS, than SVNS.

It is clearly noted that in the case of the SVN cross entropy based multicriteria decision making method that was proposed in [20], that the indeterminacy concept/ value is not classified into two, but it is represented as a single valued neutrosophic data leading to a loss of accuracy of the indeterminacy. SVNS are incapable of giving this amount of logical approach with accuracy or precision about the indeterminacy concept. Similarly when the intuitionistic fuzzy cross entropy was considered, it was not possible to deal with the indeterminacy membership function independently as it is dealt in SVN or DRIN cross entropy based multicriteria decision making method, leading to a loss of information about the existing indeterminacy. In the fuzzy cross entropy, only the membership degree is considered, details of non membership and indeterminacy are completely lost. It is clearly observed that the DRINs representation and the DRIN-cross entropy based multicriteria decision-making method are better logically equipped to deal with indeterminate, inconsistent and incomplete information.

Conclusions

In this paper a special case of refined neutrosophic set, called as Double Refined Indeterminacy Neutrosophic Set (DRINS), with two distinct indeterminate values was utilized in multicriteria decision making problem. Better logical approach and precision is provided to indeterminacy since the indeterminate concept/value is classified into two based on membership: one as indeterminacy leaning towards truth membership and another as indeterminacy leaning towards false membership. This kind of classification of indeterminacy is not feasible with SVNS. DRINS is better equipped at dealing indeterminate and inconsistent information, with more accuracy than Single Valued Neutrosophic Set (SVNS), which fuzzy sets and Intuitionistic Fuzzy sets are incapable of.

In this paper the cross entropy of DRINS was defined and it was applied solve the multicriteria decision making problem, this approach is called as DRIN cross entropy based multicriteria decision-making method. Through the illustrative computational sample of the DRIN cross entropy based multicriteria decision-making method and other methods, the results have shown that the DRIN-cross entropy based multicriteria decision-making method is more general and more reasonable than the others. Furthermore, in situations that are represented by indeterminate information and inconsistent information, the DRIN cross entropy based multicriteria decision-making method exhibits its great superiority in clustering those Double Refined Indeterminacy neutrosophic data because the DRINSs are a powerful tool to deal with uncertain, imprecise, incomplete, and inconsistent information with accuracy. In the future, DRINS sets and the DRIN cross entropy based multicriteria decision-making method can be applied to many areas such as online social networks, information retrieval, investment decision making, and data mining where fuzzy theory has been used.

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