# Neutrosophic Physics: More Problems, More Solutions 

Collection of papers

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North-European Scientific Publishers
Hanko, Finland

North-European Scientific Publishers (NESP) is a publishing house registered with the National Library of Finland, Helsinki.

Postal address for correspondence: Köydenpunojankatu 2 D 11, 10940, Hanko, Finland Publisher's identifier: 978-952-67349

On the front cover there is a 'quantum art', by Prof. Florentin Smarandache.
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Peer-Reviewers: Dmitri Rabounski and Larissa Borissova (editors of Progress in Physics, an international journal of physics), and Matti Pitkänen

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ISBN: 978-952-67349-0-3

# Neutrosophic Physics: More Problems, More Solutions 

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Positive, Neutral, and Negative Mass-Charges in General Relativity, by Larissa Borissova and Florentin Smarandache, Progress in Physics, 51-54, Vol. 3, 2006.

Extension of the Big Bang Theory, by Florentin Smarandache, Bulletin of Pure and Applied Sciences, 139-140, Vol. 23D, No. 2, 2004.

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## Academic Freedom

Declaraţie asupra Libertăţii Academice (Drepturile Omului în Domeniul Ştiinţific), de Dmitri Rabounski, tradusă din engleză de Florentin Smarandache, Progress in Physics, 94-97, Vol. 3, 2007; and in Curierul de Fizică, Horia Hulubei Publ. Hse., Bucharest, Romania, No. 59, 2007.

Déclaration de la Liberté Académique (Les Droits de l'Homme dans le Domaine Scientifique), par Dmitri Rabounski, traduit de l'anglais par Florentin Smarandache, edité par Stéphanie Robitaille, Progress in Physics, 98-101, Vol. 3, 2007.

# Neutrosophic Physics: More Problems, More Solutions Collected papers 

## Preface

When considering the laws of theoretical physics, one of the physicists says that these laws - the actual expressions of the laws of mathematics and logics being applied to physical phenomena should be limited according to the physical meaning we attribute to the phenomena. In other word, there is an opinion that a theoretical physicist should put some limitations onto mathematics, in order to "reduce" it to the observed reality.

No doubt, we can do it. However, if following this way, we would arrive at only mathematical models of already known physical phenomena. Of course, this might be useful in applied physics or industry, but nothing could be found new in physics itself: no new physical laws or discovered phenomena unknown before, just only more detailed description of that was already known before.

We can, however, follow in another way. When applying the laws of mathematics and logics to physical phenomena, do not cancel any solutions, even if they seem to be inapplicable to reality. Contrary, we can study the "inadequate" solutions, and look what physical phenomena may be predicted on the basis. Many examples manifested the success of this research approach in the history of physics. Most powerful results were obtained by this method in the theory of relativity and quantum theory - the most "impossible" sections of physics.

In this concern, neutrosophic logics and neutrosophy in general, established by Prof. Smarandache, is one of the promising research instruments, which could be successfully applied by a theoretical physicist.

Naturally, neutrosophic logics, being a part of modern logics, states that neutralities may be between any physical states, or states of space-time. In particular, this leads, sometimes, to paradoxist situations, when two opposite states are known in physics, while the neutral state between them seems absolutely impossible from a physical viewpoint! Meanwhile, when considering the theoretically possible neutralities in detail, we see that these neutral states indicate new phenomena which were just discovered by the experimentalists in the last decade, or shows a new field for further experimental studies, as for example unmatter which is a state between matter and antimatter.

Research papers presented in this collection manifest only a few of many possible applications of neutrosophic logics to theoretical physics. Most of these applications target the theory of relativity and quantum physics, but other sections of physics are also possible to be considered. One may say that these are no many. However this is only the first small step along the long path. We just opened the gate at. I believe that, after years, neutrosophic logics will yield a new section of physics neutrosophic physics - whose motto will be "more problems, more solutions".

General Relativity, Gravitation, and Cosmology

# S-Denying of the Signature Conditions Expands General Relativity's Space 

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#### Abstract

We apply the S-denying procedure to signature conditions in a four-dimensional pseudo-Riemannian space - i.e. we change one (or even all) of the conditions to be partially true and partially false. We obtain five kinds of expanded space-time for General Relativity. Kind I permits the space-time to be in collapse. Kind II permits the space-time to change its own signature. Kind III has peculiarities, linked to the third signature condition. Kind IV permits regions where the metric fully degenerates: there may be non-quantum teleportation, and a home for virtual photons. Kind V is common for kinds I, II, III, and IV.


## 1 Einstein's basic space-time

Euclidean geometry is set up by Euclid's axioms: (1) given two points there is an interval that joins them; (2) an interval can be prolonged indefinitely; (3) a circle can be constructed when its centre, and a point on it, are given; (4) all right angles are equal; (5) if a straight line falling on two straight lines makes the interior angles on one side less than two right angles, the two straight lines, if produced indefinitely, meet on that side. Non-Euclidean geometries are derived from making assumptions which deny some of the Euclidean axioms. Three main kinds of non-Euclidean geometry are conceivable - Lobachevsky-Bolyai-Gauss geometry, Riemann geometry, and Smarandache geometry.

In Lobachevsky-Bolyai-Gauss (hyperbolic) geometry the fifth axiom is denied in the sense that there are infinitely many lines passing through a given point and parallel to a given line. In Riemann (elliptic) geometry*, the axiom is satisfied formally, because there is no line passing through a given point and parallel to a given line. But if we state the axiom in a broader form, such as "through a point not on a given line there is only one line parallel to the given line", the axiom is also denied in Riemann geometry. Besides that, the second axiom is also denied in Riemann geometry, because herein the straight lines are closed: an infinitely long straight line is possible but then all other straight lines are of the same infinite length.

In Smarandache geometry one (or even all) of the axioms is false in at least two different ways, or is false and also true $[1,2]$. This axiom is said to be Smarandachely denied (S-denied). Such geometries have mixed properties of Euclidean, Lobachevsky-Bolyai-Gauss, and Riemann geometry. Manifolds that support such geometries were introduced by Iseri [3].

Riemannian geometry is the generalization of Riemann geometry, so that in a space of Riemannian geometry:
(1) The differentiable field of a 2 nd rank non-degenerate

[^0]symmetric tensor $g_{\alpha \beta}$ is given so that the distance $d s$ between any two infinitesimally close points is given by the quadratic form
$$
d s^{2}=\sum_{0 \leqslant \alpha, \beta \leqslant n} g_{\alpha \beta}(x) d x^{\alpha} d x^{\beta}=g_{\alpha \beta} d x^{\alpha} d x^{\beta}
$$
known as the Riemann metric ${ }^{\dagger}$. The tensor $g_{\alpha \beta}$ is called the fundamental metric tensor, and its components define the geometrical structure of the space;
(2) The space curvature may take different numerical values at different points in the space.
Actually, a Riemann geometry space is the space of the Riemannian geometry family, where the curvature is constant and has positive numerical value.

In the particular case where $g_{\alpha \beta}$ takes the diagonal form

$$
g_{\alpha \beta}=\left(\begin{array}{cccc}
1 & 0 & \ldots & 0 \\
0 & 1 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & 1
\end{array}\right)
$$

the Riemannian space becomes Euclidean.
Pseudo-Riemannian spaces consist of specific kinds of Riemannian spaces, where $g_{\alpha \beta}$ (and the Riemannian metric $d s^{2}$ ) has sign-alternating form so that its diagonal components bear numerical values of opposite sign.

Einstein's basic space-time of General Relativity is a four-dimensional pseudo-Riemannian space having the signalternating signature ( +--- ) or ( -+++ ), which reserves one dimension for time $x^{0}=c t$ whilst the remaining three are reserved for three-dimensional space, so that the space metric is ${ }^{\ddagger}$
$d s^{2}=g_{\alpha \beta} d x^{\alpha} d x^{\beta}=g_{00} c^{2} d t^{2}+2 g_{0 i} c d t d x^{i}+g_{i k} d x^{i} d x^{k}$.

[^1]In general the four-dimensional pseudo-Riemannian space is curved, inhomogeneous, gravitating, rotating, and deforming (any or all of the properties may be anisotropic). In the particular case where the fundamental metric tensor $g_{\alpha \beta}$ takes the strictly diagonal form

$$
g_{\alpha \beta}=\left(\begin{array}{rrrr}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{array}\right)
$$

the space becomes four-dimensional pseudo-Euclidean

$$
d s^{2}=g_{\alpha \beta} d x^{\alpha} d x^{\beta}=c^{2} d t^{2}-d x^{2}-d y^{2}-d z^{2}
$$

which is known as Minkowski's space (he had introduced it first). It is the basic space-time of Special Relativity.

## 2 S-denying the signature conditions

In a four-dimensional pseudo-Riemannian space of signature (+---) or ( -+++ ), the basic space-time of General Relativity, there are four signature conditions which define this space as pseudo-Riemannian.
Question: What happens if we S-deny one (or even all) of the four signature conditions in the basic space-time of General Relativity? What happens if we postulate that one (or all) of the signature conditions is to be denied in two ways, or, alternatively, to be true and false?

Answer: If we S-deny one or all of the four signature conditions in the basic space-time, we obtain a new expanded basic space-time for General Relativity. There are five main kinds of such expanded spaces, due to four possible signature conditions there.

Here we are going to consider each of the five kinds of expanded spaces.

Starting from a purely mathematical viewpoint, the signature conditions are derived from sign-alternation in the diagonal terms $g_{00}, g_{11}, g_{22}, g_{33}$ in the matrix $g_{\alpha \beta}$. From a physical perspective, see $\S 84$ in [4], the signature conditions are derived from the requirement that the three-dimensional observable interval

$$
d \sigma^{2}=h_{i k} d x^{i} d x^{k}=\left(-g_{i k}+\frac{g_{0 i} g_{0 k}}{g_{00}}\right) d x^{i} d x^{k}
$$

must be positive. Hence the three-dimensional observable metric tensor $h_{i k}=-g_{i k}+\frac{g_{0 i} g_{0 k}}{g_{00}}$, being a $3 \times 3$ matrix defined in an observer's reference frame accompanying its references, must satisfy three obvious conditions

$$
\begin{gathered}
\operatorname{det}\left\|h_{11}\right\|=h_{11}>0 \\
\operatorname{det}\left\|\begin{array}{ll}
h_{11} & h_{12} \\
h_{21} & h_{22}
\end{array}\right\|=h_{11} h_{22}-h_{12}^{2}>0
\end{gathered}
$$

$$
h=\operatorname{det}\left\|h_{i k}\right\|=\operatorname{det}\left\|\begin{array}{lll}
h_{11} & h_{12} & h_{13} \\
h_{21} & h_{22} & h_{23} \\
h_{31} & h_{32} & h_{33}
\end{array}\right\|>0 .
$$

From here we obtain the signature conditions in the fundamental metric tensor's matrix $g_{\alpha \beta}$. In a space of signature (+---), the signature conditions are

$$
\begin{gather*}
\operatorname{det}\left\|g_{00}\right\|=g_{00}>0  \tag{I}\\
\operatorname{det}\left\|\begin{array}{ll}
g_{00} & g_{01} \\
g_{10} & g_{11}
\end{array}\right\|=g_{00} g_{11}-g_{01}^{2}<0  \tag{II}\\
\operatorname{det}\left\|\begin{array}{lll}
g_{00} & g_{01} & g_{02} \\
g_{10} & g_{11} & g_{12} \\
g_{20} & g_{21} & g_{22}
\end{array}\right\|>0  \tag{III}\\
g=\operatorname{det}\left\|g_{\alpha \beta}\right\|=\operatorname{det}\left\|\begin{array}{llll}
g_{00} & g_{01} & g_{02} & g_{03} \\
g_{10} & g_{11} & g_{12} & g_{13} \\
g_{20} & g_{21} & g_{22} & g_{23} \\
g_{30} & g_{31} & g_{32} & g_{33}
\end{array}\right\|<0 \tag{IV}
\end{gather*}
$$

An expanded space-time of kind I: In such a spacetime the first signature condition $g_{00}>0$ is S-denied, while the other signature conditions remain unchanged. Given the expanded space-time of kind I, the first signature condition is S -denied in the following form

$$
\operatorname{det}\left\|g_{00}\right\|=g_{00} \geqslant 0
$$

which includes two particular cases, $g_{00}>0$ and $g_{00}=0$, so $g_{00}>0$ is partially true and partially false.

Gravitational potential is $\mathrm{w}=c^{2}\left(1-\sqrt{g_{00}}\right)$ [6, 7], so the S-denied first signature condition $g_{00} \geqslant 0$ means that in such a space-time $\mathrm{w} \leqslant c^{2}$, i. e. two different states occur

$$
\mathrm{w}<c^{2}, \quad \mathrm{w}=c^{2}
$$

The first one corresponds to the regular space-time, where $g_{00}>0$. The second corresponds to a special space-time state, where the first signature condition is simply denied $g_{00}=0$. This is the well-known condition of gravitational collapse.

Landau and Lifshitz wrote, "nonfulfilling of the condition $g_{00}>0$ would only mean that the corresponding system of reference cannot be accomplished with real bodies" [4].

Conclusion on the kind I: An expanded space-time of kind $\mathrm{I}\left(g_{00} \geqslant 0\right)$ is the generalization of the basic space-time of General Relativity ( $g_{00}>0$ ), including regions where this space-time is in a state of collapse, $\left(g_{00}=0\right)$.

An expanded space-time of kind II: In such a space-time the second signature condition $g_{00} g_{11}-g_{01}^{2}<0$ is S-denied, the other signature conditions remain unchanged. Thus, given the expanded space-time of kind II, the second signature condition is S-denied in the following form

$$
\operatorname{det}\left\|\begin{array}{ll}
g_{00} & g_{01} \\
g_{10} & g_{11}
\end{array}\right\|=g_{00} g_{11}-g_{01}^{2} \leqslant 0
$$

which includes two different cases

$$
g_{00} g_{11}-g_{01}^{2}<0, \quad g_{00} g_{11}-g_{01}^{2}=0
$$

whence the second signature condition $g_{00} g_{11}-g_{01}^{2}<0$ is partially true and partially false.

The component $g_{00}$ is defined by the gravitational potential $\mathrm{w}=c^{2}\left(1-\sqrt{g_{00}}\right)$. The component $g_{0 i}$ is defined by the space rotation linear velocity (see $[6,7]$ for details)

$$
v_{i}=-c \frac{g_{0 i}}{\sqrt{g_{00}}}, \quad v^{i}=-c g^{0 i} \sqrt{g_{00}}, \quad v_{i}=h_{i k} v^{k}
$$

Then we obtain the S -denied second signature condition $g_{00} g_{11}-g_{01}^{2} \leqslant 0$ (meaning the first signature condition is not denied $g_{00}>0$ ) as follows

$$
g_{11}-\frac{1}{c^{2}} v_{1}^{2} \leqslant 0,
$$

having two particular cases

$$
g_{11}-\frac{1}{c^{2}} v_{1}^{2}<0, \quad g_{11}-\frac{1}{c^{2}} v_{1}^{2}=0
$$

To better see the physical sense, take a case where $g_{11}$ is close to -1 .* Then, denoting $v^{1}=v$, we obtain

$$
v^{2}>-c^{2}, \quad v^{2}=-c^{2}
$$

The first condition $v^{2}>-c^{2}$ is true in the regular basic space-time. Because the velocities $v$ and $c$ take positive numerical values, this condition uses the well-known fact that positive numbers are greater than negative ones.

The second condition $v^{2}=-c^{2}$ has no place in the basic space-time; it is true as a particular case of the common condition $v^{2} \geqslant-c^{2}$ in the expanded spaces of kind II. This condition means that as soon as the linear velocity of the space rotation reaches light velocity, the space signature changes from (+---) to $(-+++)$. That is, given an expanded space-time of kind II, the transit from a non-isotropic sublight region into an isotropic light-like region implies change of signs in the space signature.

Conclusion on the kind II: An expanded space-time of kind II ( $v^{2} \geqslant-c^{2}$ ) is the generalization of the basic spacetime of General Relativity ( $v^{2}>-c^{2}$ ) which permits the peculiarity that the space-time changes signs in its own signature as soon as we, reaching the light velocity of the space rotation, encounter a light-like isotropic region.

An expanded space-time of kind III: In this space-time the third signature condition is S-denied, the other signature conditions remain unchanged. So, given the expanded space-time of kind III, the third signature condition is

$$
\operatorname{det}\left\|\begin{array}{lll}
g_{00} & g_{01} & g_{02} \\
g_{10} & g_{11} & g_{12} \\
g_{20} & g_{21} & g_{22}
\end{array}\right\| \geqslant 0
$$

[^2]which, taking the other form of the third signature condition into account, can be transformed into the formula
\[

\operatorname{det}\left\|$$
\begin{array}{ll}
h_{11} & h_{12} \\
h_{21} & h_{22}
\end{array}
$$\right\|=h_{11} h_{22}-h_{12}^{2} \geqslant 0
\]

that includes two different cases

$$
h_{11} h_{22}-h_{12}^{2}>0, \quad h_{11} h_{22}-h_{12}^{2}=0
$$

so that the third initial signature condition $h_{11} h_{22}-h_{12}^{2}>0$ is partially true and partially false. This condition is not clear. Future research is required.

An expanded space-time of kind IV: In this space-time the fourth signature condition $g=\operatorname{det}\left\|g_{\alpha \beta}\right\|<0$ is S-denied, the other signature conditions remain unchanged. So, given the expanded space-time of kind IV, the fourth signature condition is

$$
g=\operatorname{det}\left\|g_{\alpha \beta}\right\|=\operatorname{det}\left\|\begin{array}{llll}
g_{00} & g_{01} & g_{02} & g_{03} \\
g_{10} & g_{11} & g_{12} & g_{13} \\
g_{20} & g_{21} & g_{22} & g_{23} \\
g_{30} & g_{31} & g_{32} & g_{33}
\end{array}\right\| \leqslant 0,
$$

that includes two different cases

$$
g=\operatorname{det}\left\|g_{\alpha \beta}\right\|<0, \quad g=\operatorname{det}\left\|g_{\alpha \beta}\right\|=0
$$

so that the fourth signature condition $g<0$ is partially true and partially false: $g<0$ is true in the basic space-time, $g=0$ could be true in only he expanded spaces of kind IV.

Because the determinants of the fundamental metric tensor $g_{\alpha \beta}$ and the observable metric tensor $h_{i k}$ are connected as follows $\sqrt{-g}=\sqrt{h} \sqrt{g_{00}}[6,7]$, degeneration of the fundamental metric tensor ( $g=0$ ) implies that the observable metric tensor is also degenerate $(h=0)$. In such fully degenerate areas the space-time interval $d s^{2}$, the observable spatial interval $d \sigma^{2}=h_{i k} d x^{i} d x^{k}$ and the observable time interval $d \tau$ become zero ${ }^{\dagger}$

$$
d s^{2}=c^{2} d \tau^{2}-d \sigma^{2}=0, \quad c^{2} d \tau^{2}=d \sigma^{2}=0
$$

Taking formulae for $d \tau$ and $d \sigma$ into account, and also the fact that in the accompanying reference frame we have $h_{00}=h_{0 i}=0$, we write $d \tau^{2}=0$ and $d \sigma^{2}=0$ as

$$
\begin{gathered}
d \tau=\left[1-\frac{1}{c^{2}}\left(\mathrm{w}+v_{i} u^{i}\right)\right] d t=0, \quad d t \neq 0 \\
d \sigma^{2}=h_{i k} d x^{i} d x^{k}=0
\end{gathered}
$$

where the three-dimensional coordinate velocity $u^{i}=d x^{i} / d t$ is different to the observable velocity $\mathrm{v}^{i}=d x^{i} / d \tau$.

[^3]With $h_{i k}=-g_{i k}+\frac{1}{c^{2}} v_{i} v_{k}$, we obtain aforementioned physical conditions of degeneration in the final form

$$
\mathrm{w}+v_{i} u^{i}=c^{2}, \quad g_{i k} u^{i} u^{k}=c^{2}\left(1-\frac{\mathrm{w}}{c^{2}}\right)^{2}
$$

As recently shown [8, 9], the degenerate conditions permit non-quantum teleportation and also virtual photons in General Relativity. Therefore we expect that, employing an expanded space of kind IV, one may join General Relativity and Quantum Electrodynamics.

Conclusion on the kind IV: An expanded space-time of kind IV $(g \leqslant 0)$ is the generalization of the basic space-time of General Relativity $(g<0)$ including regions where this space-time is in a fully degenerate state $(g=0)$. From the viewpoint of a regular observer, in a fully degenerate area time intervals between any events are zero, and spatial intervals are zero. Thus, such a region is observable as a point.

An expanded space-time of kind V : In this space-time all four signature conditions are S -denied, therefore given the expanded space-time of kind V the signature conditions are

$$
\begin{gathered}
\operatorname{det}\left\|g_{00}\right\|=g_{00} \geqslant 0 \\
\operatorname{det}\left\|\begin{array}{ll}
g_{00} & g_{01} \\
g_{10} & g_{11}
\end{array}\right\|=g_{00} g_{11}-g_{01}^{2} \leqslant 0 \\
\operatorname{det}\left\|\begin{array}{lll}
g_{00} & g_{01} & g_{02} \\
g_{10} & g_{11} & g_{12} \\
g_{20} & g_{21} & g_{22}
\end{array}\right\| \geqslant 0 \\
g=\operatorname{det}\left\|g_{\alpha \beta}\right\|=\operatorname{det}\left\|\begin{array}{llll}
g_{00} & g_{01} & g_{02} & g_{03} \\
g_{10} & g_{11} & g_{12} & g_{13} \\
g_{20} & g_{21} & g_{22} & g_{23} \\
g_{30} & g_{31} & g_{32} & g_{33}
\end{array}\right\| \leqslant 0
\end{gathered}
$$

so all four signature conditions are partially true and partially false. It is obvious that an expanded space of kind V contains expanded spaces of kind I, II, III, and IV as particular cases, it being a common space for all of them.

Negative S-denying expanded spaces: We could also Sdeny the signatures with the possibility that say $g_{00}>0$ for kind I, but this means that the gravitational potential would be imaginary, or, even take into account the "negative" cases for kind II, III, etc. But most of them are senseless from the geometrical viewpoint. Hence we have only included five main kinds in our discussion.

## 3 Classification of the expanded spaces for General Relativity

In closing this paper we repeat, in brief, the main results.
There are currently three main kinds of non-Euclidean geometry conceivable - Lobachevsky-Bolyai-Gauss geometry, Riemann geometry, and Smarandache geometries.

A four-dimensional pseudo-Riemannian space, a space of the Riemannian geometry family, is the basic space-time of General Relativity. We employed S-denying of the signature conditions in the basic four-dimensional pseudo-Riemannian space, when a signature condition is partially true and partially false. S-denying each of the signature conditions (or even all the conditions at once) gave an expanded space for General Relativity, which, being an instance of the family of Smarandache spaces, include the pseudo-Riemannian space as a particular case. There are four signature conditions. So, we obtained five kinds of the expanded spaces for General Relativity:
Kind I Permits the space-time to be in collapse;
Kind II Permits the space-time to change its own signature as reaching the light speed of the space rotation in a light-like isotropic region;
Kind III Has some specific peculiarities (not clear yet), linked to the third signature condition;
Kind IV Permits full degeneration of the metric, when all degenerate regions become points. Such fully degenerate regions provide trajectories for non-quantum teleportation, and are also a home space for virtual photons.
Kind V Provides an expanded space, which has common properties of all spaces of kinds I, II, III, and IV, and includes the spaces as particular cases.

The foregoing results are represented in detail in the book [10], which is currently in print.

## 4 Extending this classification: mixed kinds of the expanded spaces

We can S-deny one axiom only, or two axioms, or three axioms, or even four axioms simultaneously. Hence we may have: $C_{4}^{1}+C_{4}^{2}+C_{4}^{3}+C_{4}^{4}=2^{4}-1=15$ kinds of expanded spaces for General Relativity, where $C_{n}^{i}$ denotes combinations of $n$ elements taken in groups of $i$ elements, $0 \leqslant i \leqslant n$. And considering the fact that each axiom can be S-denied in three different ways, we obtain $15 \times 3=45$ kinds of expanded spaces for General Relativity. Which expanded space would be most interesting?

We collect all such "mixed" spaces into a table. Specific properties of the mixed spaces follow below.
1.1.1: $g_{00} \geqslant 0, h_{11}>0, h_{11} h_{22}-h_{12}^{2}>0, h>0$. At $g_{00}=0$, we have the usual space-time permitting collapse.
1.1.2: $g_{00}>0, h_{11} \geqslant 0, h_{11} h_{22}-h_{12}^{2}>0, h>0$. At $h_{11}=0$ we have $h_{12}^{2}<0$ that is permitted for imaginary values of $h_{12}$ : we obtain a complex Riemannian space.
1.1.3: $g_{00}>0, h_{11}>0, h_{11} h_{22}-h_{12}^{2} \geqslant 0, h>0$. At $h_{11} h_{22}-h_{12}^{2}=0$, the spatially observable metric $d \sigma^{2}$ permits purely spatial isotropic lines.
1.1.4: $g_{00}>0, h_{11}>0, h_{11} h_{22}-h_{12}^{2}>0, h \geqslant 0$. At $h=0$, we have the spatially observed metric $d \sigma^{2}$ completely degenerate. An example - zero-space [9], obtained as a completely degenerate Riemannian space. Because $h=-\frac{g}{g_{00}}$, the

| Positive S-denying spaces, $N \geqslant 0$ |  | Negative S-denying spaces, $N \leqslant 0$ |  | S-denying spaces, where $N>0 \cup N<0$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Kind | Signature conditions | Kind | Signature conditions | Kind | Signature conditions |
| One of the signature conditions is S-denied |  |  |  |  |  |
| 1.1.1 | I $\geqslant 0$, II $>0$, III $>0$, IV $>0$ | 1.2.1 | $\mathrm{I} \leqslant 0, \mathrm{II}>0$, III $>0$, IV $>0$ | 1.3.1 | $\mathrm{I} \gtrless 0$, $\mathrm{II}>0$, $\mathrm{III}>0$, IV $>0$ |
| 1.1.2 | I $>0$, II $\geqslant 0$, III $>0$, IV $>0$ | 1.2.2 | $\mathrm{I}>0, \mathrm{II} \leqslant 0, \mathrm{III}>0, \mathrm{IV}>0$ | 1.3.2 | $\mathrm{I}>0, \mathrm{II} \gtrless 0, \mathrm{III}>0, \mathrm{IV}>0$ |
| 1.1.3 | I $>0$, II $>0$, III $\geqslant 0$, IV $>0$ | 1.2.3 | $\mathrm{I}>0, \mathrm{II}>0, \mathrm{III} \leqslant 0$, IV $>0$ | 1.3.3 | $\mathrm{I}>0, \mathrm{II}>0, \mathrm{III} \gtrless 0, \mathrm{IV}>0$ |
| 1.1.4 | I $>0$, II $>0$, III $>0$, IV $\geqslant 0$ | 1.2.4 | I $>0$, II $>0$, III $>0$, IV $\leqslant 0$ | 1.3.4 | $\mathrm{I}>0$, $\mathrm{II}>0$, $\mathrm{III}>0$, $\mathrm{IV} \gtrless 0$ |
| Two of the signature conditions are S-denied |  |  |  |  |  |
| 2.1.1 | $\mathrm{I} \geqslant 0, \mathrm{II} \geqslant 0, \mathrm{III}>0$, IV $>0$ | 2.2.1 | $\mathrm{I} \leqslant 0, \mathrm{II} \leqslant 0$, III $>0$, IV $>0$ | 2.3.1 | $\mathrm{I} \gtrless 0$, $\mathrm{II} \gtrless 0$, $\mathrm{III}>0$, IV $>0$ |
| 2.1.2 | $\mathrm{I} \geqslant 0, \mathrm{II}>0, \mathrm{III} \geqslant 0, \mathrm{IV}>0$ | 2.2.2 | $\mathrm{I} \leqslant 0, \mathrm{II}>0, \mathrm{III} \leqslant 0, \mathrm{IV}>0$ | 2.3.2 | $\mathrm{I} \gtrless 0, \mathrm{II}>0, \mathrm{III} \gtrless 0, \mathrm{IV}>0$ |
| 2.1.3 | $\mathrm{I} \geqslant 0, \mathrm{II}>0, \mathrm{III}>0, \mathrm{IV} \geqslant 0$ | 2.2.3 | $\mathrm{I} \leqslant 0, \mathrm{II}>0, \mathrm{III}>0, \mathrm{IV} \leqslant 0$ | 2.3.3 | $\mathrm{I} \gtrless 0, \mathrm{II}>0, \mathrm{III}>0, \mathrm{IV} \gtrless 0$ |
| 2.1.4 | I $>0$, II $\geqslant 0$, III $>0$, IV $\geqslant 0$ | 2.2.4 | $\mathrm{I}>0, \mathrm{II} \leqslant 0, \mathrm{III}>0, \mathrm{IV} \leqslant 0$ | 2.3.4 | $\mathrm{I}>0, \mathrm{II} \gtrless 0, \mathrm{III}>0, \mathrm{IV} \gtrless 0$ |
| 2.1.5 | $\mathrm{I}>0, \mathrm{II} \geqslant 0, \mathrm{III} \geqslant 0, \mathrm{IV}>0$ | 2.2.5 | $\mathrm{I}>0, \mathrm{II} \leqslant 0, \mathrm{III} \leqslant 0, \mathrm{IV}>0$ | 2.3.5 | $\mathrm{I}>0, \mathrm{II} \gtrless 0, \mathrm{III} \gtrless 0$, IV $>0$ |
| 2.1.6 | I $>0$, II $>0, \mathrm{III} \geqslant 0, \mathrm{IV} \geqslant 0$ | 2.2.6 | $\mathrm{I}>0, \mathrm{II}>0, \mathrm{III} \leqslant 0, \mathrm{IV} \leqslant 0$ | 2.3.6 | I $>0$, II $>0$, III $\gtrless 0$, IV $\gtrless 0$ |
| Three of the signature conditions are S-denied |  |  |  |  |  |
| 3.1.1 | $\mathrm{I}>0, \mathrm{II} \geqslant 0, \mathrm{III} \geqslant 0, \mathrm{IV} \geqslant 0$ | 3.2.1 | $\mathrm{I}>0, \mathrm{II} \leqslant 0, \mathrm{III} \leqslant 0, \mathrm{IV} \leqslant 0$ | 3.3.1 | $\mathrm{I}>0$, $\mathrm{II} \gtrless 0$, $\mathrm{III} \gtrless 0, \mathrm{IV} \gtrless 0$ |
| 3.1.2 | I $\geqslant 0$, II $>0$, III $\geqslant 0$, IV $\geqslant 0$ | 3.2.2 | $\mathrm{I} \leqslant 0, \mathrm{II}>0, \mathrm{III} \leqslant 0, \mathrm{IV} \leqslant 0$ | 3.3.2 | $\mathrm{I} \gtrless 0, \mathrm{II}>0, \mathrm{III} \gtrless 0, \mathrm{IV} \gtrless 0$ |
| 3.1.3 | $\mathrm{I} \geqslant 0, \mathrm{II} \geqslant 0, \mathrm{III}>0, \mathrm{IV} \geqslant 0$ | 3.2.3 | $\mathrm{I} \leqslant 0, \mathrm{II} \leqslant 0, \mathrm{III}>0, \mathrm{IV} \leqslant 0$ | 3.3.3 | $\mathrm{I} \gtrless 0, \mathrm{II} \gtrless 0, \mathrm{III}>0, \mathrm{IV} \gtrless 0$ |
| 3.1.4 | $\mathrm{I} \geqslant 0, \mathrm{II} \geqslant 0, \mathrm{III} \geqslant 0, \mathrm{IV}>0$ | 3.2.4 | $\mathrm{I} \leqslant 0, \mathrm{II} \leqslant 0, \mathrm{III} \leqslant 0, \mathrm{IV}>0$ | 3.3.4 | $\mathrm{I} \gtrless 0, \mathrm{II} \gtrless 0, \mathrm{III} \gtrless 0, \mathrm{IV}>0$ |
| All the signature conditions are S-denied |  |  |  |  |  |
| 4.1.1 | $\mathrm{I} \geqslant 0, \mathrm{II} \geqslant 0, \mathrm{III} \geqslant 0, \mathrm{IV} \geqslant 0$ | 4.2.1 | $\mathrm{I} \leqslant 0, \mathrm{II} \leqslant 0, \mathrm{III} \leqslant 0, \mathrm{IV} \leqslant 0$ | 4.3.1 | $\mathrm{I} \gtrless 0$, $\mathrm{II} \gtrless 0, \mathrm{III} \gtrless 0, \mathrm{IV} \gtrless 0$ |

Table 1: The expanded spaces for General Relativity (all 45 mixed kinds of S-denying). The signature conditions are denoted by Roman numerals
metric $d s^{2}$ is also degenerate.
1.2.1: $g_{00} \leqslant 0, h_{11}>0, h_{11} h_{22}-h_{12}^{2}>0, h>0$. At $g_{00}=0$, we have kind 1.1.1. At $g_{00}<0$ physically observable time becomes imaginary $d \tau=\frac{g_{0 i} d x^{i}}{c \sqrt{g_{00}}}$.
1.2.2: $g_{00}>0, h_{11} \leqslant 0, h_{11} h_{22}-h_{12}^{2}>0, h>0$. At $h_{11}=0$, we have kind 1.1.2. At $h_{11}<0$, distances along the axis $x^{1}$ (i.e. the values $\sqrt{h_{11}} d x^{1}$ ) becomes imaginary, contradicting the initial conditions in General Relativity.
1.2.3: $g_{00}>0, h_{11}>0, h_{11} h_{22}-h_{12}^{2} \leqslant 0, h>0$. This is a common space built on a particular case of kind 1.1.3 where $h_{11} h_{22}-h_{12}^{2}=0$ and a subspace where $h_{11} h_{22}-h_{12}^{2}<0$. In the latter subspace the spatially observable metric $d \sigma^{2}$ becomes sign-alternating so that the space-time metric has the signature (+-+-) (this case is outside the initial statement of General Relativity).
1.2.4: $g_{00}>0, h_{11}>0, h_{11} h_{22}-h_{12}^{2}>0, h \leqslant 0$. This space is built on a particular case of kind 1.1.2 where $h=0$ and a subspace where $h<0$. At $h<0$ we have the spatial metric $d \sigma^{2}$ sign-alternating so that the space-time metric has the signature $(+--+)$ (this case is outside the initial statement of General Relativity).
1.3.1: $g_{00} \gtrless 0, h_{11}>0, h_{11} h_{22}-h_{12}^{2}>0, h>0$. Here we have the usual space-time area ( $g_{00}>0$ ) with the signature ( +--- ), and a sign-definite space-time $\left(g_{00}<0\right)$ where the signature is (----). There are no intersections of the
areas in the common space-time; they exist severally.
1.3.2: $g_{00}>0, h_{11} \gtrless 0, h_{11} h_{22}-h_{12}^{2}>0, h>0$. Here we have a common space built on two separated areas where (+---) (usual space-time) and a subspace where (++--). The areas have no intersections.
1.3.3: $g_{00}>0, h_{11}>0, h_{11} h_{22}-h_{12}^{2} \gtrless 0, h>0$. This is a common space built on the usual space-time and a particular space-time of kind 1.2.3, where the signature is (+-+-). The areas have no intersections.
1.3.4: $g_{00}>0, h_{11}>0, h_{11} h_{22}-h_{12}^{2}>0, h \gtrless 0$. This is a common space built on the usual space-time and a particular space-time of kind 1.2.4, where the signature is $(+--+)$. The areas have no intersections.
2.1.1: $g_{00} \geqslant 0, h_{11} \geqslant 0, h_{11} h_{22}-h_{12}^{2}>0, h>0$. This is a complex Riemannian space with a complex metric $d \sigma^{2}$, permitting collapse.
2.1.2: $g_{00} \geqslant 0, h_{11}>0, h_{11} h_{22}-h_{12}^{2} \geqslant 0, h>0$. This space permits collapse, and purely spatial isotropic directions.
2.1.3: $g_{00} \geqslant 0, h_{11}>0, h_{11} h_{22}-h_{12}^{2}>0, h \geqslant 0$. This space permits complete degeneracy and collapse. At $g_{00}=0$ and $h=0$, we have a collapsed zero-space.
2.1.4: $g_{00}>0, h_{11} \geqslant 0, h_{11} h_{22}-h_{12}^{2}>0, h \geqslant 0$. Here we have a complex Riemannian space permitting complete degeneracy.
2.1.5: $g_{00}>0, h_{11} \geqslant 0, h_{11} h_{22}-h_{12}^{2} \geqslant 0, h>0$. At
$h_{11}=0$, we have $h_{12}^{2}=0$ : a partial degeneration of the spatially observable metric $d \sigma^{2}$.
2.1.6: $g_{00}>0, h_{11}>0, h_{11} h_{22}-h_{12}^{2} \geqslant 0, h \geqslant 0$. This space permits the spatially observable metric $d \sigma^{2}$ to completely degenerate: $h=0$.
2.2.1: $g_{00} \leqslant 0, h_{11} \leqslant 0, h_{11} h_{22}-h_{12}^{2}>0, h>0$. At $g_{00}=0$ and $h_{11}=0$, we have a particular space-time of kind 2.1.1. At $g_{00}<0, h_{11}<0$ we have a space with the signature (----) where time is like a spatial coordinate (this case is outside the initial statement of General Relativity).
2.2.2: $g_{00} \leqslant 0, h_{11}>0, h_{11} h_{22}-h_{12}^{2} \leqslant 0, h>0$. At $g_{00}=0$ and $h_{11} h_{22}-h_{12}^{2}=0$, we have a particular spacetime of kind 2.1.2. At $g_{00}<0$ and $h_{11} h_{22}-h_{12}^{2}<0$, we have a space with the signature $(-+-+$ ) (it is outside the initial statement of General Relativity).
2.2.3: $g_{00} \leqslant 0, h_{11}>0, h_{11} h_{22}-h_{12}^{2}>0, h \leqslant 0$. At $g_{00}=0$ and $h=0$, we have a particular space-time of kind 2.1.3. At $g_{00}<0$ and $h_{11} h_{22}-h_{12}^{2}<0$, we have a space-time with the signature $(---+$ ) (it is outside the initial statement of General Relativity).
2.2.4: $g_{00}>0, h_{11} \leqslant 0, h_{11} h_{22}-h_{12}^{2} \leqslant 0, h>0$. At $h_{11}=0$ and $h_{11} h_{22}-h_{12}^{2}=0$, we have a particular spacetime of kind 2.1.5. At $h_{11}<0$ and $h_{11} h_{22}-h_{12}^{2}<0$, we have a space-time with the signature $(++-+$ ) (outside the initial statement of General Relativity).
2.2.5: $g_{00}>0, h_{11} \leqslant 0, h_{11} h_{22}-h_{12}^{2} \geqslant 0, h \leqslant 0$. At $h_{11}=0$ and $h=0$, we have a particular space-time of kind 2.1.4. At $h_{11}<0$ and $h<0$, a space-time with the signature $(+--+)$ (outside the initial statement of General Relativity).
2.2.6: $g_{00}>0, h_{11}>0, h_{11} h_{22}-h_{12}^{2} \leqslant 0, h \leqslant 0$. At $h_{11} h_{22}-h_{12}^{2}=0$ and $h=0$, we have a particular spacetime of kind 2.1.6. At $h_{11} h_{22}-h_{12}^{2}<0$ and $h<0$, we have a space-time with the signature $(+-++$ ) (outside the initial statement of General Relativity).
2.3.1: $g_{00} \gtrless 0, h_{11} \gtrless 0, h_{11} h_{22}-h_{12}^{2}>0, h>0$. This is a space built on two areas. At $g_{00}>0$ and $h_{11}>0$, we have the usual space-time. At $g_{00}<0$ and $h_{11}<0$, we have a particular space-time of kind 2.2.1. The areas have no intersections: the common space is actually built on nonintersecting areas.
2.3.2: $g_{00} \gtrless 0, h_{11}>0, h_{11} h_{22}-h_{12}^{2} \gtrless 0, h>0$. This space is built on two areas. At $g_{00}>0$ and $h_{11} h_{22}-h_{12}^{2}>0$, we have the usual space-time. At $g_{00}<0$ and $h_{11} h_{22}-h_{12}^{2}<0$, we have a particular space-time of kind 2.2.2. The areas, building a common space, have no intersections.
2.3.3: $g_{00} \gtrless 0, h_{11}>0, h_{11} h_{22}-h_{12}^{2}>0, h \gtrless 0$. This space is built on two areas. At $g_{00}>0$ and $h_{11}>0$, we have the usual space-time. At $g_{00}<0$ and $h_{11}<0$, a particular space-time of kind 2.2.3. The areas, building a common space, have no intersections.
2.3.4: $g_{00}>0, h_{11} \gtrless 0, h_{11} h_{22}-h_{12}^{2}>0, h \gtrless 0$. This space is built on two areas. At $h_{11}>0$ and $h>0$, we have the usual space-time. At $h_{11}<0$ and $h<0$, a particular spacetime of kind 2.2.4. The areas, building a common space, have
no intersections.
2.3.5: $g_{00}>0, h_{11} \gtrless 0, h_{11} h_{22}-h_{12}^{2} \gtrless 0, h>0$. This space is built on two areas. At $h_{11}>0$ and $h_{11} h_{22}-h_{12}^{2}>0$, we have the usual space-time. At $h_{11}<0$ and $h_{11} h_{22}-h_{12}^{2}<0$, a particular space-time of kind 2.2.5. The areas, building a common space, have no intersections.
2.3.6: $g_{00}>0, h_{11}>0, h_{11} h_{22}-h_{12}^{2} \gtrless 0, h \gtrless 0$. This space is built on two areas. At $h_{11} h_{22}-h_{12}^{2}>0$ and $h>0$, we have the usual space-time. At $h_{11} h_{22}-h_{12}^{2}<0$ and $h<0$, a particular space-time of kind 2.2.6. The areas, building a common space, have no intersections.
3.1.1: $g_{00}>0, h_{11} \geqslant 0, h_{11} h_{22}-h_{12}^{2} \geqslant 0, h \geqslant 0$. This space permits complete degeneracy. At $h_{11}>0, h_{11} h_{22}-$ $-h_{12}^{2}>0, h>0$, we have the usual space-time. At $h_{11}=0$, $h_{11} h_{22}-h_{12}^{2}=0, h=0$, we have a particular case of a zerospace.
3.1.2: $g_{00} \geqslant 0, h_{11}>0, h_{11} h_{22}-h_{12}^{2} \geqslant 0, h \geqslant 0$. At $g_{00}>0, h_{11} h_{22}-h_{12}^{2}>0, h>0$, we have the usual spacetime. At $g_{00}=0, h_{11} h_{22}-h_{12}^{2}=0, h=0$, we have a particular case of a collapsed zero-space.
3.1.3: $g_{00} \geqslant 0, h_{11} \geqslant 0, h_{11} h_{22}-h_{12}^{2}>0, h \geqslant 0$. At $g_{00}>0, h_{11}>0, h>0$, we have the usual space-time. At $g_{00}=0, h_{11}=0, h=0$, we have a collapsed zero-space, derived from a complex Riemannian space.
3.1.4: $g_{00} \geqslant 0, h_{11} \geqslant 0, h_{11} h_{22}-h_{12}^{2} \geqslant 0, h>0$. At $g_{00}>0, h_{11}>0, h_{11} h_{22}-h_{12}^{2}>0$, we have the usual spacetime. At $g_{00}=0, h_{11}=0, h_{11} h_{22}-h_{12}^{2}=0$, we have the usual space-time in a collapsed state, while there are permitted purely spatial isotropic directions $\sqrt{h_{11}} d x^{1}$.
3.2.1: $g_{00}>0, h_{11} \leqslant 0, h_{11} h_{22}-h_{12}^{2} \leqslant 0, h \leqslant 0$. At $h_{11}=0, h_{11} h_{22}-h_{12}^{2}=0$ and $h=0$, we have a particular space-time of kind 3.1.1. At $h_{11}<0, h_{11} h_{22}-h_{12}^{2}<0$ and $h<0$, we have a space-time with the signature (++++) (outside the initial statement of General Relativity).
3.2.2: $g_{00} \leqslant 0, h_{11}>0, h_{11} h_{22}-h_{12}^{2} \leqslant 0, h \leqslant 0$. At $g_{00}=0, h_{11} h_{22}-h_{12}^{2}=0$ and $h=0$, we have a particular space-time of kind 3.1.2. At $h_{11}<0, h_{11} h_{22}-h_{12}^{2}<0$ and $h<0$, we have a space-time with the signature ( --++ ) (outside the initial statement of General Relativity).
3.2.3: $g_{00} \leqslant 0, \quad h_{11} \leqslant 0, h_{11} h_{22}-h_{12}^{2}>0, h \leqslant 0$. At $g_{00}=0, h_{11}=0$ and $h=0$, we have a particular space-time of kind 3.1.3. At $h_{11}<0, h_{11} h_{22}-h_{12}^{2}<0$ and $h<0$, we have a space-time with the signature $(-+-+$ ) (outside the initial statement of General Relativity).
3.2.4: $g_{00} \leqslant 0, h_{11} \leqslant 0, h_{11} h_{22}-h_{12}^{2} \leqslant 0, h>0$. At $g_{00}=0, h_{11}=0$ and $h_{11} h_{22}-h_{12}^{2}=0$, we have a particular space-time of kind 3.1.4. At $g_{00}<0, h_{11}<0$ and $h_{11} h_{22}-$ $-h_{12}^{2}<0$, we have a space-time with the signature ( -++- ) (outside the initial statement of General Relativity).
3.3.1: $g_{00}>0, h_{11} \gtrless 0, h_{11} h_{22}-h_{12}^{2} \gtrless 0, h \gtrless 0$. This is a space built on two areas. At $h_{11}>0, h_{11} h_{22}-h_{12}^{2}>0$ and $h_{11}>0$, we have the usual space-time. At $h_{11}<0$, $h_{11} h_{22}-h_{12}^{2}<0$ and $h_{11}<0$, we have a particular spacetime of kind 3.2.1. The areas have no intersections: the
common space is actually built on non-intersecting areas.
3.3.2: $g_{00} \gtrless 0, h_{11}>0, h_{11} h_{22}-h_{12}^{2} \gtrless 0, h \gtrless 0$. This space is built on two areas. At $g_{00}>0, h_{11} h_{22}-h_{12}^{2}>0$ and $h>0$, we have the usual space-time. At $g_{00}<0, h_{11} h_{22}-$ $-h_{12}^{2}<0$ and $h<0$, we have a particular space-time of kind 3.2.2. The areas, building a common space, have no intersections.
3.3.3: $g_{00} \gtrless 0, h_{11} \gtrless 0, h_{11} h_{22}-h_{12}^{2}>0, h \gtrless 0$. This space is built on two areas. At $g_{00}>0, h_{11}>0$ and $h>0$, we have the usual space-time. At $g_{00}<0, h_{11}<0$ and $h<0$, we have a particular space-time of kind 3.2.3. The areas, building a common space, have no intersections.
3.3.4: $g_{00} \gtrless 0, h_{11} \gtrless 0, h_{11} h_{22}-h_{12}^{2} \gtrless 0, h>0$. This space is built on two areas. At $g_{00}>0, h_{11}>0$ and $h_{11} h_{22}-$ $-h_{12}^{2}>0$, we have the usual space-time. At $g_{00}<0, h_{11}<0$ and $h_{11} h_{22}-h_{12}^{2}<0$, a particular space-time of kind 3.2.4. The areas, building a common space, have no intersections.
4.4.1: $g_{00} \geqslant 0, h_{11} \geqslant 0, h_{11} h_{22}-h_{12}^{2} \geqslant 0, h \geqslant 0$. At $g_{00}>0, h_{11}>0, h_{11} h_{22}-h_{12}^{2} \geqslant 0$ and $h \geqslant 0$, we have the usual space-time. At $g_{00}=0, h_{11}=0, h_{11} h_{22}-h_{12}^{2}=0$ and $h=0$, we have a particular case of collapsed zero-space.
4.4.2: $g_{00} \leqslant 0, \quad h_{11} \leqslant 0, \quad h_{11} h_{22}-h_{12}^{2} \leqslant 0, h \leqslant 0$. At $g_{00}=0, h_{11}=0, h_{11} h_{22}-h_{12}^{2}=0$ and $h=0$, we have a particular case of space-time of kind 4.4.1. At $g_{00}<0, h_{11}<0$, $h_{11} h_{22}-h_{12}^{2}<0$ and $h<0$, we have a space-time with the signature (----) (outside the initial statement of General Relativity). The areas have no intersections.
4.4.3: $g_{00} \gtrless 0, \quad h_{11} \gtrless 0, \quad h_{11} h_{22}-h_{12}^{2} \gtrless 0, \quad h \gtrless 0$. At $g_{00}>0, h_{11}>0, h_{11} h_{22}-h_{12}^{2}>0$ and $h>0$, we have the usual space-time. At $g_{00}<0, h_{11}<0, h_{11} h_{22}-h_{12}^{2}<0$ and $h<0$, we have a space-time with the signature (----) (outside the initial statement of General Relativity). The areas have no intersections.

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# Positive, Neutral and Negative Mass-Charges in General Relativity 

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#### Abstract

As shown, any four-dimensional proper vector has two observable projections onto time line, attributed to our world and the mirror world (for a mass-bearing particle, the projections posses are attributed to positive and negative mass-charges). As predicted, there should be a class of neutrally mass-charged particles that inhabit neither our world nor the mirror world. Inside the space-time area (membrane) the space rotates at the light speed, and all particles move at as well the light speed. So, the predicted particles of the neutrally mass-charged class should seem as light-like vortices.


## 1 Problem statement

As known, neutrosophy is a new branch of philosophy which extends the current dialectics by the inclusion of neutralities. According to neutrosophy [1, 2, 3], any two opposite entities <A> and <Anti-A> exist together with a whole class of neutralities $<$ Neut-A $>$.

Neutrosophy was created by Florentin Smarandache and then applied to mathematics, statistics, logic, linguistic, and other branches of science. As for geometry, the neutrosophic method expanded the Euclidean set of axioms by denying one or more of them in at least two distinct ways, or, alternatively, by accepting one or more axioms true and false in the same space. As a result, it was developed a class of Smarandache geometries [4], that includes Euclidean, Riemann, and Loba-chevski-Gauss-Bolyai geometries as partial cases.

In nuclear physics the neutrosophic method theoretically predicted "unmatter", built on particles and anti-particles, that was recently observed in CERN and Brookhaven experiments (see [5, 6] and References there). In General Relativity, the method permits the introduction of entangled states of particles, teleportation of particles, and also virtual particles [7], altogether known before in solely quantum physics. Aside for these, the method permits to expand the basic space-time of General Relativity (the four-dimensional pseudo-Riemannian space) by a family of spaces where one or more space signature conditions is permitted to be both true and false [8].

In this research we consider another problem: masscharges of particles. Rest-mass is a primordial property of particles. Its numerical value remains unchanged. On the contrary, relativistic mass has "charges" dependent from relative velocity of particles. Relativistic mass displays itself in only particles having interaction. Therefore theory considers relativistic mass as mass-charge.

Experimental physics knows two kinds of regular particles. Regular mass-bearing particles possessing non-zero restmasses and relativistic masses (masses-in-motion). Massless
light-like particles (photons) possess zero rest-masses, while their relativistic masses are non-zeroes. Particles of other classes (as virtual photons, for instance) can be considered as changed states of mass-bearing or massless particles.

Therefore, following neutrosophy, we do claim:
Aside for observed positively mass-charged (i. e. massbearing) particles and neutrally mass-charged (light-like) particles, there should be a third class of "negatively" masscharged particles unknown in today's experimental physics.

We aim to establish such a class of particles by the methods of General Relativity.

## 2 Two entangled states of a mass-charge

As known, each particle located in General Relativity's spacetime is characterized by its own four-dimensional impulse vector. For instance, for a mass-bearing particle the proper impulse vector $P^{\alpha}$ is

$$
\begin{equation*}
P^{\alpha}=m_{0} \frac{d x^{\alpha}}{d s}, \quad P_{\alpha} P^{\alpha}=1, \quad \alpha=0,1,2,3 \tag{1}
\end{equation*}
$$

where $m_{0}$ is the rest-mass of this particle. Any vector or tensor quantity can be projected onto an observer's time line and spatial section. Namely the projections are physically observable quantities for the observer [9]. As recently shown [10, 11], the four-dimensional impulse vector (1) has two projections onto the time line*

$$
\begin{equation*}
\frac{P_{0}}{\sqrt{g_{00}}}= \pm m, \quad \text { where } m=\frac{m_{0}}{\sqrt{1-\mathrm{v}^{2} / \mathrm{c}^{2}}} \tag{2}
\end{equation*}
$$

and solely the projection onto the spatial section

$$
\begin{equation*}
P^{i}=\frac{m}{c} \mathrm{v}^{i}=\frac{1}{c} p^{i}, \quad \text { where } \quad \mathrm{v}^{i}=\frac{d x^{i}}{d \tau}, \quad i=1,2,3 \tag{3}
\end{equation*}
$$

where $p_{i}$ is the three-dimensional observable impulse. Therefore, we conclude:
${ }^{*}$ Where $d \tau=\sqrt{g_{00}} d t+\frac{g_{0 i}}{c \sqrt{g_{00}}} d x^{i}$ is the properly observed time interval [9, 12].

Any mass-bearing particle, having two time projections, exists in two observable states, entangled to each other: the positively mass-charged state is observed in our world, while the negatively masscharged state is observed in the mirror world.

The mirror world is almost the same that ours with the following differences:

1. The particles bear negative mass-charges and energies;
2. "Left" and "right" have meanings opposite to ours;
3. Time flows oppositely to that in our world.

From the viewpoint of an observer located in the mirror world, our world will seem the same that his world for us.

Because both states are attributed to the same particle, and entangled, both our world and the mirror world are two entangled states of the same world-object.

To understand why the states remain entangled and cannot be joined into one, we consider the third difference between them - the time flow.

Terms "direct" and "opposite" time flows have a solid mathematical ground in General Relativity. They are connected to the sign of the derivative of the coordinate time interval by the proper time interval. The derivative arrives from the purely geometrical law that the square of a unit fourdimensional vector remains unchanged in a four-dimensional space. For instance, the four-dimensional velocity vector

$$
\begin{equation*}
U^{\alpha} U_{\alpha}=g_{\alpha \beta} U^{\alpha} U^{\beta}=1, \quad U^{\alpha}=\frac{d x^{\alpha}}{d s} \tag{4}
\end{equation*}
$$

Proceeding from by-component notation of this formula, and using $\mathrm{w}=c^{2}\left(1-\sqrt{g_{00}}\right)$ and $v_{i}=-c \frac{g_{0 i}}{\sqrt{g_{00}}}$, we arrive to a square equation

$$
\begin{align*}
& \left(\frac{d t}{d \tau}\right)^{2}-\frac{2 v_{i} \mathrm{v}^{i}}{c^{2}\left(1-\frac{\mathrm{w}}{c^{2}}\right)} \frac{d t}{d \tau}+ \\
& \quad+\frac{1}{\left(1-\frac{\mathrm{w}}{c^{2}}\right)^{2}}\left(\frac{1}{c^{4}} v_{i} v_{k} \mathrm{v}^{i} \mathrm{v}^{k}-1\right)=0 \tag{5}
\end{align*}
$$

which solves with two roots

$$
\begin{equation*}
\left(\frac{d t}{d \tau}\right)_{1,2}=\frac{1}{1-\frac{\mathrm{w}}{c^{2}}}\left(\frac{1}{c^{2}} v_{i} \mathrm{v}^{i} \pm 1\right) \tag{6}
\end{equation*}
$$

Observer's proper time lows anyhow directly $d \tau>0$, because this is a relative effect connected to the his viewpoint at clocks. Coordinate time $t$ flows independently from his views. Accordingly, the direct flow of time is characterized by the time function $d t / d \tau>0$, while the opposite flow of time is $d t / d \tau<0$.

If $d t / d \tau=0$ happens, the time flow stops. This is a boundary state between two entangled states of a masscharged particle, one of which is located in our world (the positively directed time flow $d t / d \tau>0$ ), while another - in
the mirror world (where the time flow is negatively directed $d t / d \tau<0$ ).

From purely geometric standpoints, the state $d t / d \tau=0$ describes a space-time area, which, having special properties, is the boundary space-time membrane between our world and the mirror world (or the mirror membrane, in other word). Substituting $d t / d \tau=0$ into the main formula of the spacetime interval $d s^{2}=g_{\alpha \beta} d x^{\alpha} d x^{\beta}$

$$
\begin{equation*}
d s^{2}=c^{2} d t^{2}+2 g_{0 i} c d t d x^{i}+g_{i k} d x^{i} d x^{k}, \tag{7}
\end{equation*}
$$

we obtain the metric of the space within the area

$$
\begin{equation*}
d s^{2}=g_{i k} d x^{i} d x^{k} . \tag{8}
\end{equation*}
$$

So, the mirror membrane between our world and the mirror world has a purely spatial metric which is also stationary.

As Kotton showed [13], any three-dimensional Riemannian space permits a holonomic orthogonal reference frame, in respect to which the three-dimensional metric can be reduced to the sum of Pythagorean squares. Because our initially four-dimensional metric $d s^{2}$ is sign-alternating with the signature (+---), the three-dimensional metric of the mirror membrane between our world and the mirror world is negatively defined and has the form

$$
\begin{equation*}
d s^{2}=-H_{1}^{2}\left(d x^{1}\right)^{2}-H_{2}^{2}\left(d x^{2}\right)^{2}-H_{3}^{2}\left(d x^{3}\right)^{2} \tag{9}
\end{equation*}
$$

where $H_{i}\left(x^{1}, x^{2}, x^{3}\right)$ are Lamé coefficients (see for Lamé coefficients and the tetrad formalism in [14]). Determination of this metric is connected to the proper time of observer, because we mean therein.

Substituting $d t=0$ into the time function (6), we obtain the physical conditions inside the area (mirror membrane)

$$
\begin{equation*}
v_{i} d x^{i}= \pm c^{2} d \tau \tag{10}
\end{equation*}
$$

Owning the definition of the observer's proper time

$$
\begin{equation*}
d \tau=\sqrt{g_{00}} d t+\frac{g_{0 i d x^{i}}}{\sqrt{g_{00}}}=\left(1-\frac{\mathrm{w}}{c^{2}}\right) d t-\frac{1}{c^{2}} v_{i} d x^{i} \tag{11}
\end{equation*}
$$

and using $d x^{i}=v^{i} d \tau$ therein, we obtain: the observer's proper state $d \tau>0$ can be satisfied commonly with the state $d t=0$ inside the membrane only if there is*

$$
\begin{equation*}
v_{i} \mathrm{v}^{i}=-c^{2} \tag{12}
\end{equation*}
$$

thus we conclude:

> The space inside the mirror membrane between our world and the mirror world seems as the rotating at the light speed, while all particles located there move at as well the light speed. So, particles that inhabit the space inside the membrane seem as light-like vortices.
${ }^{*}$ Here is a vector product of two vectors $v_{i}$ and $\mathrm{v}^{i}$, dependent on the cosine between them (which can be both positive and negative). Therefore the modules may not be necessarily imaginary quantities.

| Class of mass-charge | Particles | Energies | Class of motion | Area |
| :--- | :---: | :---: | :--- | :---: |
| Positive mass-charges, $m>0$ | mass-bearing particles | $E>0$ | move at sub-light speeds | our world |
| Neutral mass-charges, $m=0$ | massless (light-like) particles | $E>0$ | move at the light speed |  |
|  | light-like vortices | $E=0$ | move at the light speed <br> within the area, rotating <br> at the light speed | the membrane |
|  |  | $E<0$ | move at sub-light speeds |  |

This membrane area is the "barrier", which prohibits the annihilation between positively mass-charged particles and negatively mass-charged particles - the barrier between our world and the mirror world. In order to find its mirror twin, a particle should be put in an area rotating at the light speed, and accelerated to the light speed as well. Then the particle penetrates into the space inside the membrane, where annihilates with its mirror twin.

As a matter of fact, no mass-bearing particle moved at the light speed: this is the priority of massless (light-like) particles only. Therefore:

Particles that inhabit the space inside the membrane seem as light-like vortices.
Their relativistic masses are zeroes $m=0$ as those of massless light-like particles moving at the light speed. However, in contrast to light-like particles whose energies are non-zeroes, the particles inside the membrane possess zero energies $E=0$ because the space metric inside the membrane (8) has no time term.

The connexion between our world and the mirror world can be reached by matter only filled in the light-like vortical state.

## 3 Two entangled states of a light-like matter

As known, each massless (light-like) particle located in General Relativity's space-time is characterized by its own fourdimensional wave vector

$$
\begin{equation*}
K^{\alpha}=\frac{\omega}{c} \frac{d x^{\alpha}}{d \sigma}, \quad K_{\alpha} K^{\alpha}=0 \tag{13}
\end{equation*}
$$

where $\omega$ is the proper frequency of this particle linked to its energy $E=\hbar \omega$, and $d \sigma=\left(-g_{i k}+\frac{g_{0 i} g_{0 k}}{g_{00}}\right) d x^{i} d x^{k}$ is the measured spatial interval. (Because massless particles move along isotropic trajectories, the trajectories of light, one has $d s^{2}=0$, however the measured spatial interval and the proper interval time are not zeroes.)

As recently shown [10, 11], the four-dimensional wave vector has as well two projections onto the time line

$$
\begin{equation*}
\frac{K_{0}}{\sqrt{g_{00}}}= \pm \omega \tag{14}
\end{equation*}
$$

and solely the projection onto the spatial section

$$
\begin{equation*}
K^{i}=\frac{\omega}{c} c^{i}=\frac{1}{c} p^{i}, \quad \text { where } \quad c^{i}=\frac{d x^{i}}{d \tau} \tag{15}
\end{equation*}
$$

while $c^{i}$ is the three-dimensional observable vector of the light velocity (its square is the world-invariant $c^{2}$, while the vector's components $c^{i}$ can possess different values). Therefore, we conclude:

Any massless (light-like) particle, having two time projections, exists in two observable states, entangled to each other: the positively energy-charged state is observed in our world, while the negatively energycharged state is observed in the mirror world.
Because along massless particles' trajectories $d s^{2}=0$, the mirror membrane between the positively energy-charged massless states and their entangled mirror twins is characterized by the metric

$$
\begin{equation*}
d s^{2}=g_{i k} d x^{i} d x^{k}=0 \tag{16}
\end{equation*}
$$

or, expressed with Lamé coefficients $H_{i}\left(x^{1}, x^{2}, x^{3}\right)$,

$$
\begin{equation*}
d s^{2}=-H_{1}^{2}\left(d x^{1}\right)^{2}-H_{2}^{2}\left(d x^{2}\right)^{2}-H_{3}^{2}\left(d x^{3}\right)^{2}=0 \tag{17}
\end{equation*}
$$

As seen, this is a particular case, just considered, the membrane between the positively mass-charged and negatively mass-charge states.

## 4 Neutrosophic picture of General Relativity's world

As a result we arrive to the whole picture of the world provided by the purely mathematical methods of General Relativity, as shown in Table.

It should be noted that matter inside the membrane is not the same as the so-called zero-particles that inhabit fully degenerated space-time areas (see [15] and [8]), despite the fact they posses zero relativistic masses and energies too. Fully degenerate areas are characterized by the state $\mathrm{w}+$ $+v_{i} u^{i}=c^{2}$ as well as particles that inhabit them*. At first, inside the membrane the space is regular, non-degenerate. Second. Even in the absence of gravitational fields, the zerospace state becomes $v_{i} u^{i}=c^{2}$ that cannot be trivially reduced to $v_{i} u^{i}=-c^{2}$ as inside the membrane.

[^4]Particles inside the membrane between our world and the mirror world are filled into a special state of light-like vortices, unknown before.

This is one more illustration to that, between the opposite states of positively mass-charge and negatively mass-charge, there are many neutral states characterized by "neutral" masscharge. Probably, further studying light-like vortices, we'd find more classes of neutrally mass-charged states (even, probably, an infinite number of classes).

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# Extension of the Big Bang Theory ${ }^{1}$ 

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#### Abstract

:


In this note we propose the extension of the Big Bang Theory of the origin of the Universe to the model that there are cycles of beginning and ending.

## Questions about the Big Bang Theory:

Considering the Big Bang Theory, promulgated by the Belgian priest Georges Lemaître [2] in 1927 who said that the universe has begun through an explosion of a primeval atom, which is based on the Christianity believe that the universe was created, the following questions will naturally occur:
a) where did this primeval atom come from?
b) what was before this big bang?

The term "big bang" was derogatorily coined by Fred Hoyle [4] in a BBC interview and it is supposed that the universe, according to this theory, was created between 10-20 billion years ago.

## Extension of the Bing Bang Theory to Cycles of Beginning and Ending:

In order to overcome these questions and provide some answers, we should rather suppose that there is no beginning or ending but cycles of beginning and ending, inspired by Hinduism. Cosmology should be looked at as a periodical beginning/development/ending cycles.

## Hindu support of this extension:

As part of the ancient Indian knowledge, coming from the early Vedic times, the concept of cycles of birth and death was used in Hinduism (Sanatan Dharma).

It is neither ending or beginning but only cycles - a philosophy that is also reflected in the Hindu belief on cycles of birth and re-birth. Time in Hindu philosophy is depicted as a "wheel" which corroborates its cyclical nature as opposed to the thermodynamic concept of time as a one-way linear progression from a state of order to a state of disorder (entropy). [1]

In the chapter Theory of Creation, Vivekananda [3] asserts that "Maya is infinite, without beginning" (p. 17), "Maya" being the illusory world of the senses, personified as the

[^5]goddess Devi, or Shakti, consort of Siva. "The creative energy is still going on. God is eternally creating - is never at rest."

## Scientific facts in support of this extension:

The red shift (Hubble, 1929) that galaxies are moving further from the Milky Way at great speeds, and the existence of cosmic background radiation (A. Penzias - R. Wilson, 1964) can still be explained in this model of beginning-ending cycles since they manifest in our cycle of beginning-ending.
The universe in each of its cycles should be characterized by homogeneity and isotropy. Each cycle is a temporal sub-universe of the whole universe.

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# What Gravity Is. Some Recent Considerations 

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#### Abstract

It is well-known, that when it comes to discussions among physicists concerning the meaning and nature of gravitation, the room temperature can be so hot. Therefore, for the sake of clarity, it seems worth that all choices were put on a table, and we consider each choice's features and problems. The present article describes a nonexhaustive list of such gravitation theories for the purpose of inviting further and more clear discussions.


## 1 Introduction

The present article summarizes a non-exhaustive list of gravitation theories for the purpose of inviting further and more clear discussions. It is well-known, that when it comes to discussions among physicists concerning the meaning and nature of gravitation, the room temperature can be so hot. Therefore, for the sake of clarity, it seems worth that all choices were put on a table, and we consider each choice's features and problems. Of course, our purpose here is not to say the last word on this interesting issue.

## 2 Newtonian and non-relativistic approaches

Since the days after Newton physicists argued what is the meaning of "action at a distance" (Newton term) or "spooky action" (Einstein term). Is it really possible to imagine how an apple can move down to Earth without a medium whatsoever?

Because of this difficulty, from the viewpoint of natural philosophy, some physicists maintained (for instance Euler with his impulsion gravity), that there should be "pervasive medium" which can make the attraction force possible. They call this medium "ether" though some would prefer this medium more like "fluid" instead of "solid". Euler himself seems to suggest that gravitation is some kind of "external force" acting on a body, instead of intrinsic force:
"gravity of weight: It is a power by which all bodies are forced towards the centre of the Earth" [3].
But the Michelson-Morley experiment [37] opened the way for Einstein to postulate that ether hypothesis is not required at all in order to explain Lorentz's theorem, which was the beginning of Special Relativity. But of course, one can ask whether the Michelson-Morley experiment really excludes the so-called ether hypothesis. Some experiments after Michelson seem to indicate that "ether" is not excluded in the experiment setup, which means that there is Earth absolute motion [4, 5].

To accept that gravitation is external force instead of intrinsic force implies that there is distinction between gravitation and inertial forces, which also seem to indicate that inertial force can be modified externally via electromagnetic field [6].

The latter notion brings us to long-time discussions in various physics journals concerning the electromagnetic nature of gravitation, i.e. whether gravitation pulling force have the same properties just as electromagnetic field is described by Maxwell equations. Proponents of this view include Tajmar and de Matos [7, 8], Sweetser [9]. And recently Rabounski [10] also suggests similar approach.

Another version of Euler's hypothesis has emerged in modern way in the form of recognition that gravitation was carried by a boson field, and therefore gravitation is somehow related to low-temperature physics (superfluid as boson gas, superconductivity etc.). The obvious advantage of superfluidity is of course that it remains frictionless and invisible; these are main features required for true ether medium - i.e. no resistance will be felt by objects surrounded by the ether, just like the passenger will not feel anything inside the falling elevator. No wonder it is difficult to measure or detect the ether, as shown in Michelson-Morley experiment. The superfluid Bose gas view of gravitation has been discussed in a series of paper by Consoli et al. [11], and also Volovik [12].

Similarly, gravitation can also be associated to superconductivity, as shown by de Matos and Beck [29], and also in Podkletnov's rotating disc experiment. A few words on Podkletnov's experiment. Descartes conjectured that there is no gravitation without rotation motion [30]. And since rotation can be viewed as solution of Maxwell equations, one can say that there is no gravitation separated from electromagnetic field. But if we consider that equations describing superconductivity can be viewed as mere generalization of Maxwell equations (London field), then it seems we can find a modern version of Descartes' conjecture, i.e. there is no gravitation without superconductivity rotation. This seems to suggest the significance of Podkletnov's experiments [31, 32].

## 3 Relativistic gravitation theories

Now we will consider some alternative theories which agree with both Newton theory and Special Relativity, but differ either slightly or strongly to General Relativity. First of all, Einstein's own attempt to describe gravitation despite earlier gravitation theories (such as by Nordstrom [1]) has been inspired by his thought-experiment, called the "falling elevator" experiment. Subsequently he came up with conjecture that there is proper metric such that a passenger inside the elevator will not feel any pulling gravitation force. Therefore gravitation can be replaced by certain specific-chosen metric.

Now the questions are twofold: (a) whether the propermetric to replace gravitation shall have non-zero curvature or it can be flat-Minkowskian; (b) whether the formulation of General relativity is consistent enough with Mach principle from where GTR was inspired. These questions inspired heated debates for several decades, and Einstein himself (with colleagues) worked on to generalize his own gravitation theories, which implies that he did find that his theory is not complete. His work with Strauss, Bergmann, Pauli, etc. (Princeton School) aimed toward such a unified theory of gravitation and electromagnetism.

There are of course other proposals for relativistic gravitation theories, such as by Weyl, Whitehead etc. [1]. Meanwhile, R. Feynman and some of his disciples seem to be more flexible on whether gravitation shall be presented in the General-Relativity "language" or not.

Recently, there is also discussion in online forum over the question: (a) above, i.e. whether curvature of the metric surface is identical to the gravitation. While most physicists seem to agree with this proposition, there is other argument suggesting that it is also possible to conceive General Relativity even with zero curvature [13, 14].

Of course, discussion concerning relativistic gravitation theories will not be complete without mentioning the PVgravitation theory (Puthoff et al. [15]) and also Yilmaz theory [16], though Misner has discussed weaknesses of Yilmaz theory [17], and Yilmaz et al. have replied back [18]. Perhaps it would be worth to note here that General Relativity itself is also not without limitations, for instance it shall be modified to include galaxies' rotation curve, and also it is actually theory for one-body problem only [2], therefore it may be difficult to describe interaction between bodies in GTR.

Other possible approaches on relativistic gravitation theories are using the fact that the "falling-elevator" seems to suggest that it is possible to replace gravitation force with certain-chosen metric. And if we consider that one can find simplified representation of Maxwell equations with Special Relativity (Minkowski metric), then the next logical step of this "metrical" (some physicists prefer to call it "geometrodynamics") approach is to represent gravitation with yet another special relativistic but with extra-dimension(s). This was first conjectured in Kaluza-Klein theory [19]. Einstein
himself considered this theory extensively with Strauss etc. [20]. There are also higher-dimensional gravitation theories with $6 \mathrm{D}, 8 \mathrm{D}$ and so forth.

In the same direction, recently these authors put forth a new proposition using Carmeli metric [21], which is essentially a "phase-space" relativity theory in 5-dimensions.

Another method to describe gravitation is using "torsion", which is essentially to introduce torsion into Einstein field equations. See also torsional theory developed by Hehl, Kiehn, Rapoport etc. cited in [21].

It seems worth to remark here, that relativistic gravitation does not necessarily exclude the possibility of "aether" hypothesis. B. Riemann extended this hypothesis by assuming (in 1853) that the gravitational aether is an incompressible fluid and normal matter represents "sinks" in this aether [34], while Einstein discussed this aether in his Leiden lecture Ether and Relativity.

A summary of contemporary developments in gravitation theories will not be complete without mentioning Quantum Gravity and Superstring theories. Both are still major topics of research in theoretical physics and consist of a wealth of exotic ideas, some or most of which are considered controversial or objectionable. The lack of experimental evidence in support of these proposals continues to stir a great deal of debate among physicists and makes it difficult to draw definite conclusions regarding their validity [38]. It is generally alleged that signals of quantum gravity and superstring theories may occur at energies ranging from the mid or far TeV scale all the way up to the Planck scale.

Loop Quantum Gravity (LQG) is the leading candidate for a quantum theory of gravitation. Its goal is to combine the principles of General Relativity and Quantum Field Theory in a consistent non-perturbative framework [39]. The features that distinguish LQG from other quantum gravity theories are: (a) background independence and (b) minimality of structures. Background independence means that the theory is free from having to choose an apriori background metric. In LQG one does not perturb around any given classical background geometry, rather arbitrary fluctuations are allowed, thus enabling the quantum "replica" of Einstein's viewpoint that gravity is geometry. Minimality means that the general covariance of General Relativity and the principles of canonical quantization are brought together without new concepts such as extra dimensions or extra symmetries. It is believed that LQG can unify all presently known interactions by implementing their common symmetry group, the four-dimensional diffeomorphism group, which is almost completely broken in perturbative approaches.

The fundamental building blocks of String Theory (ST) are one-dimensional extended objects called strings [40, 41]. Unlike the "point particles" of Quantum Field Theories, strings interact in a way that is almost uniquely specified by mathematical self-consistency, forming an allegedly valid quantum theory of gravity. Since its launch as a dual res-
onance model (describing strongly interacting hadrons), ST has changed over the years to include a group of related superstring theories (SST) and a unifying picture known as the M-theory. SST is an attempt to bring all the particles and their fundamental interactions under one umbrella by modeling them as vibrations of super-symmetric strings.

In the early 1990s, it was shown that the various superstring theories were related by dualities, allowing physicists to map the description of an object in one superstring theory to the description of a different object in another superstring theory. These relationships imply that each of SST represents a different aspect of a single underlying theory, proposed by E. Witten and named M-theory. In a nut-shell, M-theory combines the five consistent ten-dimensional superstring theories with eleven-dimensional supergravity. A shared property of all these theories is the holographic principle, that is, the idea that a quantum theory of gravity has to be able to describe physics occurring within a volume by degrees of freedom that exist on the surface of that volume. Like any other quantum theory of gravity, the prevalent belief is that true testing of SST may be prohibitively expensive, requiring unprecedented engineering efforts on a large-system scale. Although SST is falsifiable in principle, many critics argue that it is un-testable for the foreseeable future, and so it should not be called science [38].

One needs to draw a distinction in terminology between string theories (ST) and alternative models that use the word "string". For example, Volovik talks about "cosmic strings" from the standpoint of condensed matter physics (topological defects, superfluidity, superconductivity, quantum fluids). Beck refers to "random strings" from the standpoint of statistical field theory and associated analytic methods (spacetime fluctuations, stochastic quantization, coupled map lattices). These are not quite the same as ST, which are based on "brane" structures that live on higher dimensional spacetime.

There are other contemporary methods to treat gravity, i.e. by using some advanced concepts such as group(s), topology and symmetries. The basic idea is that Nature seems to prefer symmetry, which lead to higher-dimensional gravitation theories, Yang-Mills gravity etc.

Furthermore, for the sake of clarity we have omitted here more advanced issues (sometimes they are called "fringe research"), such as faster-than-light (FTL) travel possibility, warpdrive, wormhole, cloaking theory (Greenleaf et al. [35]), antigravity (see for instance Naudin's experiment) etc. [36].

## 4 Wave mechanical method and diffraction hypothesis

The idea of linking gravitation with wave mechanics of Quantum Mechanics reminds us to the formal connection between Helmholtz equation and Schrödinger equation [22].

The use of (modified) Schrödinger equation has become so extensive since 1970s, started by Wheeler-DeWitt (despite
the fact that the WDW equation lacks observation support). And recently Nottale uses his scale relativistic approach based on stochastic mechanics theory in order to generalize Schrödinger equation to describe wave mechanics of celestial bodies [23]. His scale-relativity method finds support from observations both in Solar system and also in exo-planets.

Interestingly, one can also find vortex solution of Schrödinger equation, and therefore it is worth to argue that the use of wave mechanics to describe celestial systems implies that there are vortex structure in the Solar system and beyond. This conjecture has also been explored by these authors in the preceding paper. [24] Furthermore, considering formal connection between Helmholtz equation and Schrödinger equation, then it seems also possible to find out vortex solutions of Maxwell equations [25, 26, 27]. Interestingly, experiments on plasmoid by Bostick et al. seem to vindicate the existence of these vortex structures [28].

What's more interesting in this method, perhaps, is that one can expect to to consider gravitation and wave mechanics (i.e. Quantum Mechanics) in equal footing. In other words, the quantum concepts such as ground state, excitation, and zero-point energy now can also find their relevance in gravitation too. This "classical" implications of Wave Mechanics has been considered by Ehrenfest and also Schrödinger himself.

In this regards, there is a recent theory proposed by Gulko [33], suggesting that matter absorbs from the background small amounts of energy and thus creates a zone of reduced energy, and in such way it attracts objects from zones of higher energy.

Another one, by Glenn E. Perry, says that gravity is diffraction (due to the changing energy density gradient) of matter or light as it travels through the aether [33].

We can remark here that Perry's Diffraction hypothesis reminds us to possible production of energy from physical vacuum via a small fluctuation in it due to a quantum indeterminancy (such a small oscillation of the background can be suggested in any case because the indeterminancy principle). On the average the background vacuum does not radiate its energy is constant. On the other hand, it experiences small oscillation. If an engine built on particles or field interacts with the small oscillation of the vacuum, or at least "senses the oscillation, there is a chance to get energy from them. Because the physical vacuum is eternal capacity of energy, it is easy to imagine some possible techniques to be discovered in the future to extract this energy.

Nonetheless, diffraction of gravity is not a "new hot topic" at all. Such ideas were already proposed in the 1920's by the founders of relativity. They however left those ideas, even unpublished but only mentioned in memoirs and letters. The main reason was that (perhaps) almost infinitely small energy which can be extracted from such background per second. (In the mean time, there are other vaious proposals suggesting that it is possible to 'extract' energy from gravitation field).

About Glenn Perry and his theory. There is a drawback that that matter he called "aether" was not properly determined by him. In such a way like that, everything can be "proven". To produce any calculation for practical purpose, we should have exact data on the subject of this calculation, and compare it with actual experiments.

On the other hand, such an idea could be put into another field - the field of Quantum Mechanics. That is, to study diffraction not gravitational radiation (gravitational waves which is so weak that not discovered yet), but waves of the field of the gravitational force - in particular those can be seismic-like waves travelling in the cork of the Earth (we mean not the earthquakes) but in the gravitational field of the planet. These seismic-like oscillations (waves) of the gravitational force are known to science, and they aren't weak: everyone who experienced an earthquake knows this fact.

Other hint from wave aspect of this planet is known in the form of Schumann resonance, that the Earth produces vibration at very-low frequency, which seems to support the idea that planetary mass vibrates too, just as hypothesized in Wave Mechanics (de Broglie's hypothesis). Nonetheless, there are plenty of things to study on the large-scale implications of the Wave Mechanics.

## 5 Concluding remarks

The present article summarizes a non-exhaustive list of gravitation theories for the purpose of inviting further and more clear discussions. Of course, our purpose here is not to say the last word on this interesting issue. For the sake of clarity, some advanced subjects have been omitted, such as faster-than-light (FTL) travel possibility, warpdrive, wormhole, cloaking theory (Greenleaf et al.), antigravity etc. As to the gravitation research in the near future, it seems that there are multiple directions which one can pursue, with which we're not so sure. The only thing that we can be sure is that everything changes (Heraclitus of Ephesus), including how we define "what the question is" (Wheeler's phrase), and also what we mean with "metric", "time", and "space". Einstein himself once remarked that 'distance' itself is merely an illusion.

## Acknowledgment

The first writer wishes to thank to Prof. T. Van Flandern for sending his Meta Research Bulletin around 8 years ago. Both writers wish to thank to Dr. J. Sarfatti and P. Zielinski and others for stimulating discussions concerning the meaning of curvature in General Relativity. Special thanks to D. Rabounski for his remarks on Glenn Perry diffraction theory; and to E. Goldfain for summary of Loop Quantum gravity and Superstring. Previous discussions with Profs. M. Pitkanen, D. Rapoport, A. Povolotsky, Yoshio Kishi, and others are ac-
knowledged. At the time of writing this paper, P. LaViolette has just released a new book discussing antigravity research.

Submitted on May 02, 2008 / Accepted on June 19, 2008

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# A Few Remarks on "The Length of Day: A Cosmological Perspective" 

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#### Abstract

An interesting hypothesis concerning the varying length of day has been formulated in this edition, proposed by A.I. Arbab, based on a proposition of varying gravitational constant, $G$. The main ideas are pointed out, and alternative frameworks are also discussed in particular with respect to the present common beliefs in astrophysics. Further observation is of course recommended in order to refute or verify this proposition.


## 1 Introduction

An interesting hypothesis has been formulated in this edition, proposed by A. I. Arbab [1,2], based on a proposition of varying gravitational constant, $G$. The main ideas are pointed out, and alternative frameworks are also discussed in particular because the idea presents a quite different approach compared to the present common beliefs in astrophysics and cosmology, i.e. that the Earth is not expanding because the so-called Cosmological expansion does not take place at the Solar system scale.

## 2 Basic ideas of Arbab's hypothesis

Arbab's hypothesis is mainly an empirical model based on a set of observational data corresponding to cosmological expansion [1]. According to this model, the day increases at a present rate of $0.002 \mathrm{sec} /$ century. His model started with a hypothesis of changing gravitational constant as follows [1]:

$$
\begin{equation*}
G_{\mathrm{eff}}=G_{0}\left(\frac{t}{t_{0}}\right)^{\beta} \tag{1}
\end{equation*}
$$

We shall note, however, that such a model of varying constants in nature (such as $G$, etc.) has been discussed by numerous authors. The idea itself can be traced back to Dirac, see for instance [3].

What seems interesting here is that he is able to explain the Well's data $[4,5]$. In a sense, one can say that even the coral reef data can be considered as "cosmological benchmark". Furthermore, from this viewpoint one could expect to describe the "mechanism" behind Wegener's idea of tectonic plate movement between continents [6]. It can be noted that Wegener's hypothesis has not been described before in present cosmological theories. Moreover, it is also quite safe to say that: "There has been no consensus on the main driving mechanism for the plate tectonics since its introduction" [7].

It is worth noting here that the idea presented in [1,2] can be considered as quite different compared to the present common beliefs in astrophysics and cosmology, i.e. that the Earth is not expanding because the so-called Cosmological expansion does not take place at the Solar system scale. Apparently in [1] the author doesn't offer any explanation of such a discrepancy with the present beliefs in astrophysics; nor the author offers the "physics" of the causal relation of such an expansion at the Solar system scale. Nonetheless, the empirical finding seems interesting to discuss further.

In the subsequent section we discuss other alternative models which may yield more-or-less similar prediction.

## 3 A review of other solutions for cosmological expansion

In this regards it seems worth noting here that there are other theories which may yield similar prediction concerning the expansion of Earth. For instance one can begin with the inhomogeneous scalar field cosmologies with exponential potential [8], where the scalar field component of Einstein-KleinGordon equation can be represented in terms of:

$$
\begin{equation*}
\phi=-\frac{k}{2}+\log (G)+\psi \tag{2}
\end{equation*}
$$

Alternatively, considering the fact that Klein-Gordon equation is neatly related to Proca equation, and then one can think that the right terms of Proca equation cannot be neglected, therefore the scalar field model may be expressed better as follows [9]:

$$
\begin{equation*}
(\square+1) A_{\mu}=j_{\mu}+\partial_{\mu}\left(\partial_{\nu} j^{\nu}\right) \tag{3}
\end{equation*}
$$

Another approach has been discussed in a preceding paper [10], where we argue that it is possible to explain the lengthening of the day via the phase-space relativity as implication of Kaluza-Klein-Carmeli metric. A simpler way to predict the effect described by Arbab can be done by including
equation (1) into the time-dependent gravitational Schrödinger equation, see for instance [11].

Another recent hypothesis by M. Pitkanen [12] is worth noting too, and it will be outlined here, for the purpose of stimulating further discussion. Pitkanen's explanation is based on his TGD theory, which can be regarded as generalization of General Relativity theory.

The interpretation is that cosmological expansion does not take place smoothly as in classical cosmology but by quantum jumps in which Planck constant increases at particular level of many-sheeted space-time and induces the expansion of space-time sheets. The accelerating periods in cosmic expansion would correspond to these periods. This would allow also avoiding the predicted tearing up of the space-time predicted by alternative scenarios explaining accelerated expansion.

The increase of Earth's radius by a factor of two is required to explain the finding of Adams that all continents fit nicely together. Increases of Planck constant by a factor of two are indeed favoured because $p$-adic lengths scales come in powers of two and because scaling by a factor two are fundamental in quantum TGD. The basic structure is causal diamond (CD), a pair of past and future directed light cones forming diamond like structure. Because two copies of same structure are involved, also the time scale $T / 2$ besides the temporal distance $T$ between the tips of CD emerges naturally. CD's would form a hierarchy with temporal distances $T / 2^{n}$ between the tips.

After the expansion the geological evolution is consistent with the tectonic theory so that the hypothesis only extends this theory to earlier times. The hypothesis explains why the continents fit together not only along their other sides as Wegener observed but also along other sides: the whole Earth would have been covered by crust just like other planets.

The recent radius would indeed be twice the radius that it was before the expansion. Gravitational force was 4 time stronger and Earth rotated 4 times faster so that day-night was only 6 hours. This might be visible in the biorhythms of simple bacteria unless they have evolved after that to the new rhythm. The emergence of gigantic creatures like dinosaur and even crabs and trees can be seen as a consequence of the sudden weakling of the gravitational force. Later smaller animals with more brain than muscles took the power.

Amusingly, the recent radius of Mars is one half of the recent radius of Earth (same Schumann frequency) and Mars is now known to have underground water: perhaps Mars contains complex life in underground seas waiting to the time to get to the surface as Mars expands to the size of Earth.

Nonetheless what appears to us as a more interesting question is whether it is possible to find out a proper metric, where both cosmological expansion and other observed expansion phenomena at Solar-system scale can be derived from the same theory (from a Greek word, theoros - "to look on or to contemplate" [13]). Unlike the present beliefs
in astrophysics and cosmological theories, this seems to be a continuing journey. An interesting discussion of such a possibility of "generalized" conformal map can be found in [14]. Of course, further theoretical and experiments are therefore recommended to verify or refute these propositions with observed data in Nature.*

Submitted on November 01, 2008 / Accepted on November 10, 2008

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[^6]Quantum Physics and Statistics

# A Note on Unified Statistics Including Fermi-Dirac, Bose-Einstein, and Tsallis Statistics, and Plausible Extension to Anisotropic Effect 

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#### Abstract

In the light of some recent hypotheses suggesting plausible unification of thermostatistics where Fermi-Dirac, Bose-Einstein and Tsallis statistics become its special subsets, we consider further plausible extension to include non-integer Hausdorff dimension, which becomes realization of fractal entropy concept. In the subsequent section, we also discuss plausible extension of this unified statistics to include anisotropic effect by using quaternion oscillator, which may be observed in the context of Cosmic Microwave Background Radiation. Further observation is of course recommended in order to refute or verify this proposition.


## 1 Introduction

In recent years, there have been some hypotheses suggesting that the spectrum and statistics of Cosmic Microwave Background Radiation has a kind of scale invariant character [1], which may be related to non-integer Hausdorff dimension. Interestingly, in this regard there is also proposition sometime ago suggesting that Cantorian spacetime may have deep link with Bose condensate with non-integer Hausdorff dimension [2]. All of these seem to indicate that it is worth to investigate further the non-integer dimension effect of BoseEinstein statistics, which in turn may be related to Cosmic Microwave Background Radiation spectrum.

In the meantime, some authors also consider a plausible generalization of known statistics, i.e. Fermi-Dirac, BoseEinstein, and Tsallis statistics, to become more unified statistics [3, 4]. This attempt can be considered as one step forward from what is already known, i.e. to consider anyons as a generalization of bosons and fermions in two-dimensional systems [5, p. 2] Furthermore, it is known that superfluidity phenomena can also be observed in Fermi liquid [6].

First we will review the existing procedure to generalize Fermi-Dirac, Bose-Einstein, and Tsallis statistics, to become more unified statistics [3, 4]. And then we explore its plausible generalization to include fractality of Tsallis' nonextensive entropy parameter.

In the subsequent section, we also discuss plausible extension of this proposed unified statistics to include anisotropic effect, which may be observed in the context of Cosmic Microwave Background Radiation. In particular we consider possibility to introduce quaternionic momentum. To our knowledge this proposition has never been considered before elsewhere.

Further observation is of course recommended in order to verify or refute the propositions outlined herein.

2 Unified statistics including Fermi-Dirac, BoseEinstein, and Tsallis statistics

In this section we consider a different theoretical framework to generalize Fermi-Dirac and Bose-Einstein statistics, from conventional method using anyons, [5] in particular because this conventional method cannot be generalized further to include Tsallis statistics which has attracted some attention in recent years.

First we write down the standard expression of Bose distribution [9, p. 7]:

$$
\begin{equation*}
\bar{n}\left(\epsilon_{i}\right)=\frac{1}{\exp \left(\beta\left(\epsilon_{i}-\mu\right)\right)-1}, \tag{1}
\end{equation*}
$$

where the harmonic energy levels are given by [9, p. 7]:

$$
\begin{equation*}
\epsilon_{i}=\left(n_{x}+n_{y}+n_{z}+\frac{3}{2}\right) \hbar \omega_{0} . \tag{2}
\end{equation*}
$$

When we assume that bosons and fermions are $g$-ons obeying fractional exclusion statistics, then we get a very different picture. In accordance with [3], we consider the spectrum of fractal dimension (also called generalized Renyi dimension [11]):

$$
\begin{equation*}
D_{q}=\lim _{\delta \rightarrow 0} \frac{1}{q-1} \frac{\ln \Omega_{q}}{\ln \delta}, \tag{3}
\end{equation*}
$$

(therefore the spectrum of fractal dimension is equivalent with Hausdorff dimension of the set A [11]).

Then the relation between the entropy and the spectrum of fractal dimension is given by: [3]

$$
\begin{equation*}
S_{q}=-K_{B} \lim _{\delta \rightarrow 0} \ln \delta D_{q} \tag{4}
\end{equation*}
$$

where $K_{B}$ is the Boltzmann constant.
The spectrum of fractal dimension may be expressed in terms of $p$ :

$$
\begin{equation*}
D_{q} \approx \frac{1}{q-1} \frac{\sum_{i=1}^{k} p_{i}^{q}-1}{\ln \delta} \tag{5}
\end{equation*}
$$

Then, substituting equation (5) into (4), we get the Tsallis non-extensive entropy [3]:

$$
\begin{equation*}
S_{q}=-K_{B} \frac{\sum_{i=1}^{k} p_{i}^{q}-1}{q-1} \tag{6}
\end{equation*}
$$

After a few more assumptions, and using $g$-on notation [3], i.e. $g=1$ for generalized Fermi-Dirac statistics and $g=0$ for generalised Bose-Einstein statistics, then one gets the most probable distribution for $g$-ons [3]:

$$
\begin{equation*}
\bar{n}_{k}\left(\epsilon_{i}, g, q\right)=\frac{1}{\left(1-(q-1) \beta\left(\epsilon_{i}-\mu\right)\right)^{\frac{1}{q-1}}+2 g-1} \tag{7}
\end{equation*}
$$

Which gives standard Planck distribution for $\mu=0, g=0$ and $q=1[3,9]$. In other words, we could expect that $g$ ons gas statistics could yield more generalized statistics than anyons'.

To introduce further generality of this expression (7), one may consider the parameter $q$ as function of another noninteger dimension, therefore:

$$
\begin{equation*}
\bar{n}_{k}\left(\epsilon_{i}, g, q, D\right)=\frac{1}{\left(1-\left(q^{D}-1\right) \beta\left(\epsilon_{i}-\mu\right)\right)^{\frac{1}{Q^{D}-1}}+2 g-1} \tag{8}
\end{equation*}
$$

where $D=1$ then equation (8) reduces to be (7).
Of course, the picture described above will be different if we introduce non-standard momentum [5, p. 7]:

$$
\begin{equation*}
p^{2}=-\frac{d^{2}}{d x^{2}}+\frac{\lambda}{x^{2}} \tag{9}
\end{equation*}
$$

In the context of Neutrosophic logic as conceived by one of these writers [8], one may derive a proposition from the arguments presented herein, i.e. apart from common use of anyons as a plausible generalization of fermion and boson, perhaps an alternative method for generalization of fermion and boson can be described as follows:

1. If we denote fermion with (f) and boson with (b), then it follows that there could be a mixture composed of both (f) and (b) $\rightarrow$ (f) $\cap$ (b), which may be called as "anyons";
2. If we denote fermion with (f) and boson with (b), and because $g=1$ for generalized Fermi-Dirac statistics and $g=0$ for generalised Bose-Einstein statistics, then it follows that the wholeness of both (f) and (b) $\rightarrow$ (f) $\cup($ b), which may be called as " $g$-on";
3. Taking into consideration of possibility of "neitherness", then if we denote non-fermion with ( $\neg \mathrm{f}$ ) and non-boson with ( $\neg \mathrm{b}$ ), then it follows that there shall be a mixture composed of both ( $\neg \mathrm{f})$ and also ( $\neg \mathrm{b}) \rightarrow$ $(\neg \mathrm{f}) \cap(\neg \mathrm{b})$, which may be called as "feynmion" (after physicist the late R. Feynman);
4. Taking into consideration of possibility of "neitherness", then it follows that the wholeness of both ( $\neg \mathrm{f}$ ) and $(\neg \mathrm{b}) \rightarrow(\neg \mathrm{f}) \cup(\neg \mathrm{b})$, which may be called as "anti-$g$-on".
Therefore, a conjecture which may follow from this proposition is that perhaps in the near future we can observe some new entities corresponding to $g$-on condensate or feynmion condensate.

## 3 Further extension to include anisotropic effect

At this section we consider the anisotropic effect which may be useful for analyzing the anisotropy of CMBR spectrum, see Fig. 1 [13].

For anisotropic case, one cannot use again equation (2), but shall instead use [7, p. 2]:

$$
\begin{equation*}
\epsilon_{i}=\left(n_{x}+\frac{1}{2}\right) \hbar \omega_{x}+\left(n_{y}+\frac{1}{2}\right) \hbar \omega_{y}+\left(n_{z}+\frac{1}{2}\right) \hbar \omega_{z} \tag{10}
\end{equation*}
$$

where $n_{x}, n_{y}, n_{z}$ are integers and $>0$. Or by neglecting the $1 / 2$ parts and assuming a common frequency, one can re-write (10) as [7a, p.1]:

$$
\begin{equation*}
\epsilon_{i}=\left(n_{x} r+n_{y} s+n_{z} t\right) \hbar \omega_{0} \tag{11}
\end{equation*}
$$

where $r, s, t$ is multiplying coefficient for each frequency:

$$
\begin{equation*}
r=\frac{\omega_{x}}{\omega_{0}}, \quad s=\frac{\omega_{y}}{\omega_{0}}, \quad t=\frac{\omega_{z}}{\omega_{0}} . \tag{12}
\end{equation*}
$$

This proposition will yield a different spectrum compared to isotropic spectrum by assuming isotropic harmonic oscillator (2). See Fig. 2 [7a]. It is interesting to note here that the spectrum produced by anisotropic frequencies yields number of peaks more than 1 (multiple-peaks), albeit this is not near yet to CMBR spectrum depicted in Fig. 1. Nonetheless, it seems clear here that one can expect to predict the anisotropy of CMBR spectrum by using of more anisotropic harmonic oscillators.

In this regard, it is interesting to note that some authors considered half quantum vortices in $p_{x}+i p_{y}$ superconductors [14], which indicates that energy of partition function may be generalized to include Cauchy plane, as follows:

$$
\begin{equation*}
E=p_{x} c+i p_{y} c \approx \hbar \omega_{x}+i \hbar \omega_{y} \tag{13}
\end{equation*}
$$

or by generalizing this Cauchy plane to quaternion number [12], one gets instead of (13):

$$
\begin{equation*}
E_{q k}=\hbar \omega+i \hbar \omega_{x}+j \hbar \omega_{y}+k \hbar \omega_{z} \tag{14}
\end{equation*}
$$

which is similar to standard definition of quaternion number:

$$
\begin{equation*}
Q \equiv a+b i+c j+d k \tag{15}
\end{equation*}
$$

Therefore the partition function with anisotropic harmon-


Fig. 1: Anisotropy of CMBR (after Tkachev [13]). Left panel: comparison of CMB power spectra in the models with adiabatic and isocurvature initial perturbations. Right panel: adiabatic power spectra in comparison with spectra appearing in models seeded by topological defects. In this panel some older, pre-WMAP, data are also shown.
ic potential can be written in quaternion form. Therefore instead of (11), we get:

$$
\begin{equation*}
\epsilon_{i}=\left(n_{x} r+n_{y} s+n_{z} t+i n_{x} r+j n_{y} s+k n_{z} t\right) \hbar \omega_{0} \tag{16}
\end{equation*}
$$

which can be written as:

$$
\begin{equation*}
\epsilon_{i}=\left(1+q_{k}\right)\left(n_{k} r_{k}\right) \hbar \omega_{0} \tag{17}
\end{equation*}
$$

where $k=1,2,3$ corresponding to index of quaternion number $i, j, k$. While we don't obtain numerical result here, it can be expected that this generalisation to anisotropic quaternion harmonic potential could yield better prediction, which perhaps may yield to exact CMBR spectrum. Numerical solution of this problem may be presented in another paper.

This proposition, however, may deserve further considerations. Further observation is also recommended in order to verify and also to explore various implications of.

## 4 Concluding remarks

In the present paper, we review an existing method to generalize Fermi-Dirac, Bose-Einstein, and Tsallis statistics, to become more unified statistics. And then we explore its plausible generalization to include fractality of Tsallis nonextensive entropy parameter .

Therefore, a conjecture which may follow this proposition is that perhaps in the near future we can observe some new entities corresponding to $g$-on condensate or feynmion condensate.

In the subsequent section, we also discuss plausible extension of this proposed unified statistics to include anisotropic effect, which may be observed in the context of Cosmic Microwave Background Radiation. In particular we consider possibility to introduce quaternionic harmonic oscillator. To our knowledge this proposition has never been considered before elsewhere.


It is recommended to conduct further observation in order to verify and also to explore various implications of our propositions as described herein.

## Acknowledgment

The writers would like to thank to Profs. C. Castro, A. Yefremov, D. L. Rapoport, D. Rabounski, S. Crothers and L. Borissova for valuable discussions.

Submitted on January 08, 2007
Accepted on January 30, 2007

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# A New Derivation of Biquaternion Schrödinger Equation and Plausible Implications 

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#### Abstract

In the preceding article we argue that biquaternionic extension of Klein-Gordon equation has solution containing imaginary part, which differs appreciably from known solution of KGE. In the present article we discuss some possible interpretation of this imaginary part of the solution of biquaternionic KGE (BQKGE); thereafter we offer a new derivation of biquaternion Schrödinger equation using this method. Further observation is of course recommended in order to refute or verify this proposition.


## 1 Introduction

There were some attempts in literature to generalise Schrödinger equation using quaternion and biquaternion numbers. Because quaternion number use in Quantum Mechanics has often been described $[1,2,3,4]$, we only mention in this paper the use of biquaternion number. Sapogin [5] was the first to introduce biquaternion to extend Schrödinger equation, while Kravchenko [4] use biquaternion number to describe neat link between Schrödinger equation and Riccati equation.

In the present article we discuss a new derivation of biquaternion Schrödinger equation using a method used in the preceding paper. Because the previous method has been used for Klein-Gordon equation [1], now it seems natural to extend it to Schrödinger equation. This biquaternion effect may be useful in particular to explore new effects in the context of low-energy reaction (LENR) [6]. Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

## 2 Some interpretations of preceding result of biquaternionic KGE

In our preceding paper [1], we argue that it is possible to write biquaternionic extension of Klein-Gordon equation as follows

$$
\begin{equation*}
\left[\left(\frac{\partial^{2}}{\partial t^{2}}-\nabla^{2}\right)+i\left(\frac{\partial^{2}}{\partial t^{2}}-\nabla^{2}\right)\right] \varphi(x, t)=-m^{2} \varphi(x, t) \tag{1}
\end{equation*}
$$

Or this equation can be rewritten as

$$
\begin{equation*}
\left(\diamond \bar{\diamond}+m^{2}\right) \varphi(x, t)=0 \tag{2}
\end{equation*}
$$

provided we use this definition

$$
\begin{align*}
& \diamond=\nabla^{q}+i \nabla^{q}=\left(-i \frac{\partial}{\partial t}+e_{1} \frac{\partial}{\partial x}+e_{2} \frac{\partial}{\partial y}+e_{3} \frac{\partial}{\partial z}\right)+  \tag{3}\\
& +i\left(-i \frac{\partial}{\partial T}+e_{1} \frac{\partial}{\partial X}+e_{2} \frac{\partial}{\partial Y}+e_{3} \frac{\partial}{\partial Z}\right)
\end{align*}
$$

where $e_{1}, e_{2}, e_{3}$ are quaternion imaginary units obeying (with ordinary quaternion symbols: $e_{1}=i, e_{2}=j, e_{3}=k$ )

$$
\begin{array}{ll}
i^{2}=j^{2}=k^{2}=-1, & i j=-j i=k \\
j k=-k j=i, & k i=-i k=j \tag{4}
\end{array}
$$

and quaternion Nabla operator is defined as [7]

$$
\begin{equation*}
\nabla^{q}=-i \frac{\partial}{\partial t}+e_{1} \frac{\partial}{\partial x}+e_{2} \frac{\partial}{\partial y}+e_{3} \frac{\partial}{\partial z} . \tag{5}
\end{equation*}
$$

Note that equation (3) and (5) included partial timedifferentiation.

It is worth nothing here that equation (2) yields solution containing imaginary part, which differs appreciably from known solution of KGE:

$$
\begin{equation*}
y(x, t)=\left(\frac{1}{4}-\frac{i}{4}\right) m^{2} t^{2}+\text { constant } . \tag{6}
\end{equation*}
$$

Some possible alternative interpretations of this imaginary part of the solution of biquaternionic KGE (BQKGE) are:
(a) The imaginary part implies that there is exponential term of the wave solution, which is quite similar to the Ginzburg-Landau extension of London phenomenology [8]

$$
\begin{equation*}
\psi(r)=|\psi(r)| e^{i \varphi(r)} \tag{7}
\end{equation*}
$$

because (6) can be rewritten (approximately) as:

$$
\begin{equation*}
y(x, t)=\frac{e^{i}}{4} m^{2} t^{2} \tag{8}
\end{equation*}
$$

(b) The aforementioned exponential term of the solution (8) can be interpreted as signature of vortices solution. Interestingly Navier-Stokes equation which implies vorticity equation can also be rewritten in terms of Yukawa equation [3];
(c) The imaginary part implies that there is spiral wave, which suggests spiralling motion of meson or other particles. Interestingly it has been argued that one can explain electron phenomena by assuming spiralling elec-
trons [9]. Alternatively this spiralling wave may already be known in the form of Bierkeland flow. For meson observation, this could be interpreted as another form of meson, which may be called "supersymmetricmeson" [1];
(d) The imaginary part of solution of BQKGE also implies that it consists of standard solution of KGE [1], and its alteration because of imaginary differential operator. That would mean the resulting wave is composed of two complementary waves;
(e) Considering some recent proposals suggesting that neutrino can have imaginary mass [10], the aforementioned imaginary part of solution of BQKGE can also imply that the (supersymmetric-) meson may be composed of neutrino(s). This new proposition may require new thinking both on the nature of neutrino and also supersymmetric-meson [11].
While some of these propositions remain to be seen, in deriving the preceding BQKGE we follow Dirac's phrase that "One can generalize his physics by generalizing his mathematics". More specifically, we focus on using a "theorem" from this principle, i.e.: "One can generalize his mathematics by generalizing his (differential) operator".

## 3 Extended biquaternion Schrödinger equation

One can expect to use the same method described above to generalize the standard Schrödinger equation [12]

$$
\begin{equation*}
\left[-\frac{\hbar^{2}}{2 m} \Delta u+V(x)\right] u=E u \tag{9}
\end{equation*}
$$

or, in simplified form, [12, p.11]:

$$
\begin{equation*}
\left(-\Delta+w_{k}\right) f_{k}=0, \quad k=0,1,2,3 \tag{10}
\end{equation*}
$$

In order to generalize equation (9) to biquaternion version (BQSE), we use first quaternion Nabla operator (5), and by noticing that $\Delta \equiv \nabla \nabla$, we get

$$
\begin{equation*}
-\frac{\hbar^{2}}{2 m}\left(\nabla^{q} \bar{\nabla}^{q}+\frac{\partial^{2}}{\partial t^{2}}\right) u+(V(x)-E) u=0 \tag{11}
\end{equation*}
$$

Note that we shall introduce the second term in order to 'neutralize' the partial time-differentiation of $\nabla^{q} \bar{\nabla}^{q}$ operator.

To get biquaternion form of equation (11) we can use our definition in equation (3) rather than (5), so we get

$$
\begin{equation*}
-\frac{\hbar^{2}}{2 m}\left(\diamond \bar{\diamond}+\frac{\partial^{2}}{\partial t^{2}}-i \frac{\partial^{2}}{\partial T^{2}}\right) u+(V(x)-E) u=0 \tag{12}
\end{equation*}
$$

This is an alternative version of biquaternionic Schrödinger equation, compared to Sapogin's [5] or Kravchenko's [4] method. We also note here that the route to quaternionize Schrödinger equation here is rather different from what is described by Horwitz [13, p. 6]

$$
\begin{equation*}
\tilde{H} \psi=\psi e_{1} E \tag{13}
\end{equation*}
$$

or

$$
\begin{equation*}
\tilde{H} \psi q=\psi q\left(q^{-1} e_{1} q\right) E \tag{14}
\end{equation*}
$$

where the quaternion number $q$, can be expressed as follows (see [13, p. 6] and [4])

$$
\begin{equation*}
q=q_{0}+\sum_{i=1}^{3} q_{i} e_{i} \tag{15}
\end{equation*}
$$

Nonetheless, further observation is of course recommended in order to refute or verify this proposition (12).

## 4 Numerical solution of biquaternion Schrödinger equation

It can be shown that numerical solution (using Maxima [14]) of biquaternionic extension of Schrödinger equation yields different result compared to the standard Schrödinger equation, as follows. For clarity, all solutions were computed in 1-D only.

For standard Schrödinger equation [12], one can rewrite equation (9) as follows:
(a) For $V(x)>E$ :

$$
\begin{equation*}
-\frac{\hbar^{2}}{2 m} \Delta u+a \cdot u=0 \tag{16}
\end{equation*}
$$

(b) For $V(x)<E$ :

$$
\begin{equation*}
-\frac{\hbar^{2}}{2 m} \Delta u-a \cdot u=0 \tag{17}
\end{equation*}
$$

Numerical solution of equation (16) and (17) is given (by assuming $\hbar=1$ and $m=1 / 2$ for convenience)
$(\% \mathrm{i} 44)-{ }^{\prime} \mathrm{diff}^{(\mathrm{y}, \mathrm{x}, 2)+\mathrm{a} * \mathrm{y} ;}$
$(\% \mathrm{o} 44) a \cdot y-\frac{d^{2}}{d^{2} x} y$
(a) For $V(x)>E$ :

$$
\begin{aligned}
& \text { (\%i46) ode2 }(\% \mathrm{o} 44, \mathrm{y}, \mathrm{x}) \text {; } \\
& (\% \mathrm{o} 46) y=k_{1} \cdot \exp (\sqrt{a \cdot x})+k_{2} \cdot \exp (-\sqrt{a x})
\end{aligned}
$$

(b) For $V(x)<E$ :

$$
\begin{aligned}
& (\% \mathrm{i} 45) \text { ode2 }(\% \mathrm{o} 44, \mathrm{y}, \mathrm{x}) \\
& (\% \mathrm{o} 45) y=k_{1} \cdot \sinh (\sqrt{a \cdot x})+k_{2} \cdot \cosh (\sqrt{a \cdot x})
\end{aligned}
$$

In the meantime, numerical solution of equation (12), is given (by assuming $\hbar=1$ and $m=1 / 2$ for convenience)
(a) For $V(x)>E$ :

$$
\begin{aligned}
& \text { (\%i38) }(\% \mathrm{i}+1)^{\prime} \text { 'diff }(\mathrm{y}, \mathrm{x}, 2)+\mathrm{a}^{*} \mathrm{y} \text {; } \\
& (\% \mathrm{o} 38)(i+1) \frac{d^{2}}{d^{2} x} y+a \cdot y \\
& (\% \mathrm{i} 39) \text { ode2 }(\% \mathrm{o} 38, \mathrm{y}, \mathrm{x}) ; \\
& (\% \mathrm{o} 39) y=k_{1} \cdot \sin \left(\sqrt{\frac{a}{i+1} \cdot x}\right)+k_{2} \cdot \cos \left(\sqrt{\frac{a}{i+1} \cdot x}\right)
\end{aligned}
$$

(b) For $V(x)<E$ :

$$
\begin{aligned}
& (\% \mathrm{i} 40)(\% \mathrm{i}+1) * \text { 'diff }(\mathrm{y}, \mathrm{x}, 2)-\mathrm{a}{ }^{*} \mathrm{y} \text {; } \\
& (\% \mathrm{o} 40)(i+1) \frac{d^{2}}{d^{2} x} y-a \cdot y \\
& (\% \mathrm{i} 41) \text { ode2 }(\% \mathrm{o} 40, \mathrm{y}, \mathrm{x}) ; \\
& (\% \mathrm{o} 41) y=k_{1} \cdot \sin \left(\sqrt{-\frac{a}{i+1} \cdot x}\right)+k_{2} \cdot \cos \left(\sqrt{-\frac{a}{i+1} \cdot x}\right)
\end{aligned}
$$

Therefore, we conclude that numerical solution of biquaternionic extension of Schrödinger equation yields different result compared to the solution of standard Schrödinger equation. Nonetheless, we recommend further observation in order to refute or verify this proposition/numerical solution of biquaternion extension of spatial-differential operator of Schrödinger equation.

As side remark, it is interesting to note here that if we introduce imaginary number in equation (16) and equation (17), the numerical solutions will be quite different compared to solution of equation (16) and (17), as follows

$$
\begin{equation*}
-\frac{i \hbar^{2}}{2 m} \Delta u+a u=0 \tag{18}
\end{equation*}
$$

where $V(x)>E$, or

$$
\begin{equation*}
-\frac{i \hbar^{2}}{2 m} \Delta u-a u=0 \tag{19}
\end{equation*}
$$

where $V(x)<E$.
Numerical solution of equation (18) and (19) is given (by assuming $\hbar=1$ and $m=1 / 2$ for convenience)
(a) For $V(x)>E$ :

$$
\begin{aligned}
& \text { (\%i47) -\%i*'diff }(\mathrm{y}, \mathrm{x}, 2)+\mathrm{a}^{*} \mathrm{y} ; \\
& (\% \mathrm{o} 47) a \cdot y-i \frac{d^{2}}{d^{2} x} y \\
& \text { (\%i48) ode2 }(\% \mathrm{o} 47, \mathrm{y}, \mathrm{x}) ; \\
& (\% \mathrm{o} 48) y=k_{1} \cdot \sin (\sqrt{i a \cdot x})+k_{2} \cdot \cos (\sqrt{i a \cdot x})
\end{aligned}
$$

(b) For $V(x)<E$ :

$$
\begin{aligned}
& \text { (\%i50) -\%i*'diff }(\mathrm{y}, \mathrm{x}, 2)-\mathrm{a} * \mathrm{y} \\
& (\% \mathrm{o} 50)-a \cdot y-i \frac{d^{2}}{d^{2} x} y \\
& (\% \mathrm{i} 51) \text { ode2 }(\% \mathrm{o} 0, \mathrm{y}, \mathrm{x}) ; \\
& (\% \mathrm{o} 51) y=k_{1} \cdot \sin (-\sqrt{i a \cdot x})+k_{2} \cdot \cos (-\sqrt{i a \cdot x})
\end{aligned}
$$

It shall be clear therefore that using different sign for differential operator yields quite different results.

## Acknowledgement

Special thanks to Prof. Diego Rapoport who mentioned Sprössig's interesting paper [3]. VC would like to dedicate this article for RFF.

Submitted on September 24, 2007
Accepted on September 26, 2007

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# AN INTRODUCTION TO THE NEUTROSOPHIC PROBABILITY APPLIED IN QUANTUM PHYSICS 

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#### Abstract

. In this paper one generalizes the classical probability and imprecise probability to the notion of "neutrosophic probability" in order to be able to model Heisenberg's Uncertainty Principle of a particle's behavior, Schrödinger's Cat Theory, and the state of bosons which do not obey Pauli's Exclusion Principle (in quantum physics). Neutrosophic probability is close related to neutrosophic logic and neutrosophic set, and etymologically derived from "neutrosophy" [58, 59].


Keywords: imprecise probability, neutrosophic probability, neutrosophic logic, neutrosophic set, non-standard interval, quantum physics, Heisenberg's Uncertainty Principle, Schrödinger's Cat Theory, Pauli's Exclusion Principle, Chan doctrine

1991 MSC: 60A99, 81-05

## 1. Introduction.

One consequence of the Heisenberg's Uncertainty Principle says that it is impossible to fully predict the behavior of a particle, also the causality principle cannot apply at the atomic level.
For example the Schrödinger's Cat Theory says that the quantum state of a photon can basically be in more than one place in the same time which, translated to the neutrosophic set, means that an element (quantum state) belongs and does not belong to a set (a place) in the same time; or an element (quantum state) belongs to two different sets (two different places) in the same time. It is a question of "alternative worlds" theory very well represented by the neutrosophic set theory.
In Schrödinger's Equation on the behavior of electromagnetic waves and "matter waves" in quantum theory, the wave function $\psi$ which describes the superposition of possible states may be simulated by a neutrosophic function, i.e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points).

How to describe a particle $\zeta$ in the infinite micro-universe that belongs to two distinct places $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ in the same time? $\zeta \in \mathrm{P}_{1}$ and $\zeta \notin \mathrm{P}_{1}$ as a true contradiction, or $\zeta \in \mathrm{P}_{1}$ and $\zeta \in \neg \mathrm{P}_{1}$. Or, how to describe two distinct bosons $\mathrm{b}_{1}$ and $\mathrm{b}_{2}$, which do not obey Pauli's Exclusion Principle, i.e. they belong to the same quantum or energy state in the same time?

Or, how to calculate the truth-value of Zen (in Japanese) / Chan (in Chinese) doctrine philosophical proposition: the present is eternal and comprises in itself the past and the future?

In Eastern Philosophy the contradictory utterances form the core of the Taoism and Zen/Chan (which emerged from Buddhism and Taoism) doctrines.
How to judge the truth-value of a metaphor, or of an ambiguous statement, or of a social phenomenon which is positive from a standpoint and negative from another standpoint?

We better describe them, using the attribute "neutrosophic" than "fuzzy" or any other, a quantum particle that neither exists nor non-exists.

## 2. Non-Standard Real Numbers and Non-Standard Real Sets.

Let T, I, F be standard or non-standard real subsets of ] $0,1^{+}$[,
with $\sup T=t \_\sup , \inf T=\mathrm{t} \_$inf,
$\sup I=i \_\sup , \inf I=i \_i n f$,
$\sup F=\mathrm{f}_{-} \sup , \inf \mathrm{F}=\mathrm{f}$ _inf,
and $n_{-} \sup =\mathrm{t}_{-} \sup +\mathrm{i}_{-} \sup +\mathrm{f}_{-}$sup,
$\mathrm{n}_{-} \mathrm{inf}=\mathrm{t}$ _inf $+\mathrm{i}_{-} \mathrm{inf}+\mathrm{f}_{-}$inf.
Obviously: t_sup, i_sup, f_sup $\leq 1^{+}$, and t_inf, i_inf, f_inf $\geq 0$, whereas $n_{-}$sup $\leq 3^{+}$and $n_{-} i n f \geq{ }^{-} 0$.

The subsets T, I, F are not necessarily intervals, but may be any real subsets: discrete or continuous; single-element, finite, or (either countable or uncountable) infinite; union or intersection of various subsets; etc.
They may also overlap. These real subsets could represent the relative errors in determining t , $\mathrm{i}, \mathrm{f}$ (in the case when the subsets T, I, F are reduced to points).

This representation is closer to the human mind reasoning. It characterizes/catches the imprecision of knowledge or linguistic inexactitude received by various observers (that's why T, I, F are subsets - not necessarily single-elements), uncertainty due to incomplete knowledge or acquisition errors or stochasticity (that's why the subset I exists), and vagueness due to lack of clear contours or limits (that's why T, I, F are subsets and I exists; in particular for the appurtenance to the neutrosophic sets).
One has to specify the superior (x_sup) and inferior (x_inf) limits of the subsets because in many problems arises the necessity to compute them.

The real number x is said to be infinitesimal if and only if for all positive integers n one has $|\mathrm{x}|$ $<1 / \mathrm{n}$. Let $\varepsilon>0$ be a such infinitesimal number. The hyper-real number set is an extension of the real number set, which includes classes of infinite numbers and classes of infinitesimal numbers. Let's consider the non-standard finite numbers $1^{+}=1+\varepsilon$, where " 1 " is its standard part and " $\varepsilon$ " its non-standard part, and $0=0-\varepsilon$, where " 0 " is its standard part and " $\varepsilon$ " its nonstandard part.
Then, we call ] $0,1^{+}$[ a non-standard unit interval. Obviously, 0 and 1 , and analogously nonstandard numbers infinitely small but less than 0 or infinitely small but greater than 1 , belong to the non-standard unit interval. Actually, by "a" one signifies a monad, i.e. a set of hyper-real numbers in non-standard analysis:
$\mu(\bar{a})=\left\{a-x: x \in \mathbb{R}^{*}, x\right.$ is infinitesimal $\}$,
and similarly " b " is a monad:
$\mu\left(b^{+}\right)=\left\{b+x: x \in \mathbb{R}^{*}, \mathrm{x}\right.$ is infinitesimal $\}$.

Generally, the left and right borders of a non-standard interval ] ${ }^{-}$, $\mathrm{b}^{+}[$are vague, imprecise, themselves being non-standard (sub)sets $\mu(-a)$ and $\mu\left(\mathbf{b}^{+}\right)$as defined above.
Combining the two before mentioned definitions one gets, what we would call, a binad of " $c^{+}$":
$\mu\left(\mathrm{c}^{+}\right)=\left\{\mathrm{c}-\mathrm{x}: \mathrm{x} \in \mathbb{R}^{*}, \mathrm{x}\right.$ is infinitesimal $\} \cup\left\{\mathrm{c}+\mathrm{x}: \mathrm{x} \in \mathbb{R}^{*}, \mathrm{x}\right.$ is infinitesimal $\}$, which is a collection of open punctured neighborhoods (balls) of c .
Of course, ${ }^{-} \mathrm{a}<\mathrm{a}$ and $\mathrm{b}^{+}>\mathrm{b}$. No order between ${ }^{-} \mathrm{c}^{+}$and c .
Addition of non-standard finite numbers with themselves or with real numbers:
$-\mathrm{a}+\mathrm{b}={ }^{-}(\mathrm{a}+\mathrm{b})$
$a+b^{+}=(a+b)^{+}$
$-\mathrm{a}+\mathrm{b}^{+}={ }^{-}(\mathrm{a}+\mathrm{b})^{+}$
$-\mathrm{a}+\mathrm{b}={ }^{-}(\mathrm{a}+\mathrm{b})$ (the left monads absorb themselves)
$\mathrm{a}^{+}+\mathrm{b}^{+}=(\mathrm{a}+\mathrm{b})^{+}$(analogously, the right monads absorb themselves)
Similarly for subtraction, multiplication, division, roots, and powers of non-standard finite numbers with themselves or with real numbers.

By extension let inf $]^{-} \mathrm{a}, \mathrm{b}^{+}\left[={ }^{-} \mathrm{a} \text { and sup }\right]^{-} \mathrm{a}, \mathrm{b}^{+}\left[=\mathrm{b}^{+}\right.$.

## 3. A Logical Connection.

Łukasiewicz, together with Kotarbiński and Leśniewski from the Warsaw Polish Logic group (1919-1939), questioned the status of truth: eternal, sempiternal (everlasting, perpetual), or both?
Let's borrow from the modal logic the notion of "world", which is a semantic device of what the world might have been like. Then, one says that the neutrosophic truth-value of a statement $\mathrm{A}, \mathrm{NL}_{\mathrm{t}}(\mathrm{A})=1^{+}$if A is 'true in all possible worlds' (syntagme first used by Leibniz) and all conjunctures, that one may call "absolute truth" (in the modal logic it was named necessary truth, Dinulescu-Câmpina [9] names it 'intangible absolute truth' ), whereas $\mathrm{NL}_{\mathrm{t}}(\mathrm{A})=1$ if A is true in at least one world at some conjuncture, we call this "relative truth" because it is related to a 'specific' world and a specific conjuncture (in the modal logic it was named possible truth). Because each 'world' is dynamic, depending on an ensemble of parameters, we introduce the sub-category 'conjuncture' within it to reflect a particular state of the world.
How can we differentiate <the truth behind the truth>? What about the <metaphoric truth>, which frequently occurs in the humanistic field? Let's take the proposition " $99 \%$ of the politicians are crooked" (Sonnabend [60], Problem 29, p. 25). "No," somebody furiously comments, " $100 \%$ of the politicians are crooked, even more!" How do we interpret this "even more" (than $100 \%$ ), i. e. more than the truth?
One attempts to formalize. For $\mathrm{n} \mu 1$ one defines the " n -level relative truth" of the statement A if the statement is true in at least $n$ distinct worlds, and similarly "countable-" or "uncountablelevel relative truth" as gradual degrees between "first-level relative truth" (1) and "absolute truth" $\left(1^{+}\right)$in the monad $\mu\left(1^{+}\right)$. Analogue definitions one gets by substituting "truth" with "falsehood" or "indeterminacy" in the above.
In largo sensu the notion "world" depends on parameters, such as: space, time, continuity, movement, modality, (meta)language levels, interpretation, abstraction, (higher-order) quantification, predication, complement constructions, subjectivity, context, circumstances, etc. Pierre d'Ailly upholds that the truth-value of a proposition depends on the sense, on the metaphysical level, on the language and meta-language; the auto-reflexive propositions (with
reflection on themselves) depend on the mode of representation (objective/subjective, formal/informal, real/mental).
In a formal way, let's consider the world W as being generated by the formal system FS. One says that statement A belongs to the world W if A is a well-formed formula (wff) in W , i.e. a string of symbols from the alphabet of W that conforms to the grammar of the formal language endowing W. The grammar is conceived as a set of functions (formation rules) whose inputs are symbols strings and outputs "yes" or "no". A formal system comprises a formal language (alphabet and grammar) and a deductive apparatus (axioms and/or rules of inference). In a formal system the rules of inference are syntactically and typographically formal in nature, without reference to the meaning of the strings they manipulate.
Similarly for the neutrosophic falsehood-value, $\mathrm{NL}_{f}(\mathrm{~A})=1^{+}$if the statement A is false in all possible worlds, we call it "absolute falsehood", whereas $\operatorname{NL}_{f}(A)=1$ if the statement $A$ is false in at least one world, we call it "relative falsehood". Also, the neutrosophic indeterminacyvalue $\mathrm{NL}_{\mathrm{i}}(\mathrm{A})=1^{+}$if the statement A is indeterminate in all possible worlds, we call it "absolute indeterminacy", whereas $\mathrm{NL}_{\mathrm{i}}(\mathrm{A})=1$ if the statement A is indeterminate in at least one world, we call it "relative indeterminacy".
On the other hand, $\mathrm{NL}_{\mathrm{t}}(\mathrm{A})=-0$ if A is false in all possible world, whereas $\mathrm{NL}_{t}(\mathrm{~A})=0$ if A is false in at least one world; $\mathrm{NL}_{\mathrm{f}}(\mathrm{A})=-0$ if A is true in all possible world, whereas $\mathrm{NL}_{\mathrm{f}}(\mathrm{A})=0$ if $A$ is true in at least one world; and $\mathrm{NL}_{\mathrm{i}}(\mathrm{A})=-0$ if A is indeterminate in no possible world, whereas $\mathrm{NL}_{\mathrm{i}}(\mathrm{A})=0$ if A is not indeterminate in at least one world.
The ${ }^{-} 0$ and $1^{+}$monads leave room for degrees of super-truth (truth whose values are greater than 1 ), super-falsehood, and super-indeterminacy.
Here there are some corner cases:
There are tautologies, some of the form "B is $\mathrm{B}^{\prime}$ ", for which $\mathrm{NL}(\mathrm{B})=\left(1^{+},{ }^{-} 0,{ }^{-} 0\right)$, and contradictions, some of the form "C is not C ", for which $\mathrm{NL}(\mathrm{B})=\left(-0,{ }^{-} 0,1^{\dagger}\right)$.
While for a paradox, $\mathrm{P}, \mathrm{NL}(\mathrm{P})=(1,1,1)$. Let's take the Epimenides Paradox, also called the Liar Paradox, "This very statement is true". If it is true then it is false, and if it is false then it is true. But the previous reasoning, due to the contradictory results, indicates a high indeterminacy too. The paradox is the only proposition true and false in the same time in the same world, and indeterminate as well!
Let's take the Grelling's Paradox, also called the heterological paradox [Suber, 1999], "If an adjective truly describes itself, call it 'autological', otherwise call it 'heterological'. Is 'heterological' heterological? " Similarly, if it is, then it is not; and if it is not, then it is.
For a not well-formed formula, nwff, i.e. a string of symbols which do not conform to the syntax of the given logic, NL(nwff) $=\mathrm{n} / \mathrm{a}$ (undefined). A proposition which may not be considered a proposition was called by the logician Paulus Venetus flatus voci. NL(flatus voci) $=\mathrm{n} / \mathrm{a}$.

## 4. Operations with Standard and Non-Standard Real Subsets.

Let $S_{1}$ and $S_{2}$ be two (one-dimensional) standard or non-standard real subsets, then one defines:

### 4.1. Addition of sets:

$\mathrm{S}_{1} \oplus \mathrm{~S}_{2}=\left\{\mathrm{x} \mid \mathrm{x}=\mathrm{s}_{1}+\mathrm{s}_{2}\right.$, where $\mathrm{s}_{1} \in \mathrm{~S}_{1}$ and $\left.\mathrm{s}_{2} \in \mathrm{~S}_{2}\right\}$,
with $\inf \mathrm{S}_{1} \oplus \mathrm{~S}_{2}=\inf \mathrm{S}_{1}+\inf \mathrm{S}_{2}, \sup \mathrm{~S}_{1} \oplus \mathrm{~S}_{2}=\sup \mathrm{S}_{1}+\sup \mathrm{S}_{2}$;
and, as some particular cases, we have
$\{\mathrm{a}\} \oplus \mathrm{S}_{2}=\left\{\mathrm{x} \mid \mathrm{x}=\mathrm{a}+\mathrm{s}_{2}\right.$, where $\left.\mathrm{s}_{2} \in \mathrm{~S}_{2}\right\}$
with $\inf \{a\} \oplus S_{2}=a+\inf S_{2}, \sup \{a\} \oplus S_{2}=a+\sup S_{2} ;$
also $\left\{1^{+}\right\} \oplus \mathrm{S}_{2}=\left\{\mathrm{x} \mid \mathrm{x}=1^{+}+\mathrm{s}_{2}\right.$, where $\left.\mathrm{s}_{2} \in \mathrm{~S}_{2}\right\}$
with $\inf \left\{1^{+}\right\} \oplus \mathrm{S}_{2}=1^{+}+\inf \mathrm{S}_{2}$, $\sup \left\{1^{+}\right\} \oplus \mathrm{S}_{2}=1^{+}+\sup \mathrm{S}_{2}$.

### 4.2. Subtraction of sets:

$\mathrm{S}_{1} \ominus \mathrm{~S}_{2}=\left\{\mathrm{x} \mid \mathrm{x}=\mathrm{s}_{1}-\mathrm{S}_{2}\right.$, where $\mathrm{s}_{1} \in \mathrm{~S}_{1}$ and $\left.\mathrm{s}_{2} \in \mathrm{~S}_{2}\right\}$.
For real positive subsets (most of the cases will fall in this range) one gets
$\inf S_{1} \oplus S_{2}=\inf S_{1}-\sup S_{2}, \sup S_{1} \oplus S_{2}=\sup S_{1}-\inf S_{2}$;
and, as some particular cases, we have
$\{a\} \ominus S_{2}=\left\{x \mid x=a-s_{2}\right.$, where $\left.\mathrm{s}_{2} \in \mathrm{~S}_{2}\right\}$,
with $\inf \{a\} \ominus S_{2}=a-\sup S_{2}, \sup \{a\} \ominus S_{2}=a-\inf S_{2}$;
also $\left\{1^{+}\right\} \ominus \mathrm{S}_{2}=\left\{\mathrm{x} \mid \mathrm{x}=1^{+}-\mathrm{s}_{2}\right.$, where $\left.\mathrm{s}_{2} \in \mathrm{~S}_{2}\right\}$,
with $\inf \left\{1^{+}\right\} \ominus S_{2}=1^{+}-\sup S_{2}, \sup \left\{1^{+}\right\} \ominus S_{2}=1^{+}-\inf S_{2}$.

### 4.3. Multiplication of sets:

$\mathrm{S}_{1} \otimes \mathrm{~S}_{2}=\left\{\mathrm{x} \mid \mathrm{x}=\mathrm{s}_{1} \cdot \mathrm{~s}_{2}\right.$, where $\mathrm{s}_{1} \in \mathrm{~S}_{1}$ and $\left.\mathrm{s}_{2} \in \mathrm{~S}_{2}\right\}$.
For real positive subsets (most of the cases will fall in this range) one gets $\inf S_{1} \otimes S_{2}=\inf S_{1} \cdot \inf S_{2}, \sup S_{1} \otimes S_{2}=\sup S_{1} \cdot \sup S_{2} ;$
and, as some particular cases, we have
$\{a\} \otimes S_{2}=\left\{x \mid x=a \cdot s_{2}\right.$, where $\left.s_{2} \in S_{2}\right\}$,
with $\inf \{a\} \otimes S_{2}=a \cdot \inf S_{2}, \sup \{a\} \otimes S_{2}=a \cdot \sup S_{2} ;$
also $\left\{1^{+}\right\} \otimes \mathrm{S}_{2}=\left\{\mathrm{x} \mid \mathrm{x}=1^{+} \cdot \mathrm{s}_{2}\right.$, where $\left.\mathrm{s}_{2} \in \mathrm{~S}_{2}\right\}$,
with $\inf \left\{1^{+}\right\} \otimes S_{2}=1^{+} \cdot \inf S_{2}$, sup $\left\{1^{+}\right\} \otimes S_{2}=1^{+} \cdot \sup S_{2}$.

### 4.4. Division of a set by a number:

Let $k \in \mathbb{R}^{*}$, then $\mathrm{S}_{1} \varnothing \mathrm{k}=\left\{\mathrm{x} \mid \mathrm{x}=\mathrm{s}_{1} / \mathrm{k}\right.$, where $\left.\mathrm{s}_{1} \in \mathrm{~S}_{1}\right\}$,
Let $\left(T_{1}, I_{1}, F_{1}\right)$ and $\left(T_{2}, I_{2}, F_{2}\right)$ be standard or non-standard triplets of real subsets of $P(]^{-} 0,1^{+}[)^{3}$, where P(]$^{-} 0,1^{+}[)$is the set of all subsets of non-standard unit interval $]^{-} 0,1^{+}[$, then we define:
$\left(\mathrm{T}_{1}, \mathrm{I}_{1}, \mathrm{~F}_{1}\right)+\left(\mathrm{T}_{2}, \mathrm{I}_{2}, \mathrm{~F}_{2}\right)=\left(\mathrm{T}_{1} \oplus \mathrm{~T}_{2}, \mathrm{I}_{1} \oplus \mathrm{I}_{2}, \mathrm{~F}_{1} \oplus \mathrm{~F}_{2}\right)$,
$\left(\mathrm{T}_{1}, \mathrm{I}_{1}, \mathrm{~F}_{1}\right)-\left(\mathrm{T}_{2}, \mathrm{I}_{2}, \mathrm{~F}_{2}\right)=\left(\mathrm{T}_{1} \ominus \mathrm{~T}_{2}, \mathrm{I}_{1} \ominus \mathrm{I}_{2}, \mathrm{~F}_{1} \ominus \mathrm{~F}_{2}\right)$,
$\left(\mathrm{T}_{1}, \mathrm{I}_{1}, \mathrm{~F}_{1}\right) \cdot\left(\mathrm{T}_{2}, \mathrm{I}_{2}, \mathrm{~F}_{2}\right)=\left(\mathrm{T}_{1} \otimes \mathrm{~T}_{2}, \mathrm{I}_{1} \otimes \mathrm{I}_{2}, \mathrm{~F}_{1} \otimes \mathrm{~F}_{2}\right)$.

## 5. Neutrosophic Probability:

Is a generalization of the classical probability in which the chance that an event A occurs is $\mathrm{t} \%$ true - where $t$ varies in the subset $\mathrm{T}, \mathrm{i} \%$ indeterminate - where i varies in the subset I , and $\mathrm{f} \%$ false - where f varies in the subset F .
One notes $\operatorname{NP}(\mathrm{A})=(T, I, F)$.
It is also a generalization of the imprecise probability, which is an interval-valued distribution function.

## 6. Neutrosophic Statistics:

Is the analysis of the events described by the neutrosophic probability.

This is also a generalization of the classical statistics and imprecise statistics.

## 7. Neutrosophic Probability Space.

The universal set, endowed with a neutrosophic probability defined for each of its subset, forms a neutrosophic probability space.

Let $A$ and $B$ be two neutrosophic events, and $N P(A)=\left(T_{1}, I_{1}, F_{1}\right), N P(B)=\left(T_{2}, I_{2}, F_{2}\right)$ their neutrosophic probabilities. Then we define:
$\mathrm{NP}(\mathrm{A} \cap \mathrm{B})=\mathrm{NP}(\mathrm{A}) \cdot \mathrm{NP}(\mathrm{B})$.
$\mathrm{NP}(\neg \mathrm{A})=\left\{1^{+}\right\}-\mathrm{NP}(\mathrm{A})$.
$N P(A \cup B)=N P(A)+N P(B)-N P(A) \cdot N P(B)$.

1. $\mathrm{NP}($ impossible event $)=\left(\mathrm{T}_{\mathrm{imp}}, \mathrm{I}_{\mathrm{imp}}, \mathrm{F}_{\mathrm{imp}}\right)$,
where $\sup \mathrm{T}_{\mathrm{imp}} \leq 0, \inf \mathrm{~F}_{\mathrm{imp}} \geq 1$; no restriction on $\mathrm{I}_{\mathrm{imp}}$.
$\mathrm{NP}($ sure event $)=\left(\mathrm{T}_{\text {sur }}, \mathrm{I}_{\text {sur }}, \mathrm{F}_{\text {sur }}\right)$,
where $\inf \mathrm{T}_{\text {sur }} \geq 1$, $\sup \mathrm{F}_{\text {sur }} \leq 0$; no restriction on $\mathrm{I}_{\text {sur }}$.
$\mathrm{NP}($ totally indeterminate event $)=\left(\mathrm{T}_{\text {ind }}, \mathrm{I}_{\text {ind }}, \mathrm{F}_{\text {ind }}\right)$;
where inf $\mathrm{I}_{\text {ind }} \geq 1$; no restrictions on $\mathrm{T}_{\text {ind }}$ or $\mathrm{F}_{\text {ind }}$.
2. $N P(A) \in\{(T, I, F)$, where $T, I, F$ are real subsets which may overlap $\}$.
3. $N P(A \cup B)=N P(A)+N P(B)-N P(A \cap B)$.
4. $\mathrm{NP}(\mathrm{A})=\{1\}-\mathrm{NP}(\neg \mathrm{A})$.

## 8. Applications:

\#1. From a pool of refugees, waiting in a political refugee camp in Turkey to get the American visa, $\mathrm{a} \%$ have the chance to be accepted - where a varies in the set $\mathrm{A}, \mathrm{r} \%$ to be rejected - where $r$ varies in the set $R$, and $\mathrm{p} \%$ to be in pending (not yet decided) - where p varies in P .
Say, for example, that the chance of someone Popescu in the pool to emigrate to USA is (between) $40-60 \%$ (considering different criteria of emigration one gets different percentages, we have to take care of all of them), the chance of being rejected is $20-25 \%$ or $30-35 \%$, and the chance of being in pending is $10 \%$ or $20 \%$ or $30 \%$. Then the neutrosophic probability that Popescu emigrates to the Unites States is
$\mathrm{NP}($ Popescu $)=((40-60),(20-25) \mathrm{U}(30-35),\{10,20,30\})$, closer to the life's thinking.
This is a better approach than the classical probability, where 40 [ P (Popescu) [ 60, because from the pending chance - which will be converted to acceptance or rejection - Popescu might get extra percentage in his will to emigration, and also the superior limit of the subsets sum
$60+35+30>100$
and in other cases one may have the inferior sum $<0$, while in the classical fuzzy set theory the superior sum should be 100 and the inferior sum $\mu 0$. In a similar way, we could say about the element Popescu that Popescu( (40-60), (20-25)U(30-35), $\{10,20,30\})$ belongs to the set of accepted refugees. \#2. The probability that candidate C will win an election is say $25-30 \%$ true (percent of people voting for him), $35 \%$ false (percent of people voting against him), and $40 \%$ or $41 \%$ indeterminate (percent of people not coming to the ballot box, or giving a blank vote - not selecting anyone, or giving a negative vote - cutting all candidates on the list).

Dialectic and dualism don't work in this case anymore.
\#3. Another example, the probability that tomorrow it will rain is say $50-54 \%$ true according to meteorologists who have investigated the past years' weather, 30 or $34-35 \%$ false according to today's very sunny and droughty summer, and 10 or $20 \%$ undecided (indeterminate).
\#4. The probability that Yankees will win tomorrow versus Cowboys is $60 \%$ true (according to their confrontation's history giving Yankees' satisfaction), 30-32\% false (supposing Cowboys are actually up to the mark, while Yankees are declining), and 10 or 11 or $12 \%$ indeterminate (left to the hazard: sickness of players, referee's mistakes, atmospheric conditions during the game). These parameters act on players' psychology.

## 9. Remarks:

Neutrosophic probability is useful to those events which involve some degree of indeterminacy (unknown) and more criteria of evaluation - as quantum physics. This kind of probability is necessary because it provides a better representation than classical probability to uncertain events.

## 10. Generalizations of Other Probabilities.

In the case when the truth- and falsity-components are complementary, i.e. no indeterminacy and their sum is 1 , one falls to the classical probability. As, for example, tossing dice or coins, or drawing cards from a well-shuffled deck, or drawing balls from an urn.

An interesting particular case is for $\mathrm{n}=1$, with $0 \leq \mathrm{t}, \mathrm{i}, \mathrm{f} \leq 1$, which is closer to the classical probability.
For $\mathrm{n}=1$ and $\mathrm{i}=0$, with $0 \leq \mathrm{t}, \mathrm{f} \leq 1$, one obtains the classical probability.
From the intuitionistic logic, paraconsistent logic, dialetheism, faillibilism, paradoxism, pseudoparadoxism, and tautologism we transfer the "adjectives" to probabilities, i.e. we define the intuitionistic probability (when the probability space is incomplete), paraconsistent probability, faillibilist probability, dialetheist probability, paradoxist probability, pseudoparadoxist probability, and tautologic probability respectively.
Hence, the neutrosophic probability generalizes:

- the intuitionistic probability, which supports incomplete (not completely known/determined) probability spaces (for $0<\mathrm{n}<1$ and $\mathrm{i}=0,0 \leq \mathrm{t}, \mathrm{f} \leq 1$ ) or incomplete events whose probability we need to calculate;
- the classical probability (for $\mathrm{n}=1$ and $\mathrm{i}=0$, and $0 \leq \mathrm{t}, \mathrm{f} \leq 1$ );
- the paraconsistent probability (for $\mathrm{n}>1$ and $\mathrm{i}=0$, with both $\mathrm{t}, \mathrm{f}<1$ );
- the dialetheist probability, which says that intersection of some disjoint probability spaces is not empty (for $\mathrm{t}=\mathrm{f}=1$ and $\mathrm{i}=0$; some paradoxist probabilities can be denoted this way);
- the faillibilist probability (for $\mathrm{i}>0$ );
- the pseudoparadoxism (for $n_{-}$sup $>1$ or $n_{-}$inf $<0$ );
- the tautologism (for t_sup>1).

Compared with all other types of classical probabilities, the neutrosophic probability introduces a percentage of "indeterminacy" - due to unexpected parameters hidden in some probability spaces, and let each component $t$, $i$, f be even boiling over 1 to $1^{+}$ (overflooded) or freezing under 0 (underdried) to ${ }^{-} 0$.

For example: an element in some tautological probability space may have $t>1$, called "overprobable" (i.e. $t=1^{+}$). Similarly, an element in some paradoxist probability space may be "overindeterminate" (for $\mathrm{i}>1$ ), or "overunprobable" (for $\mathrm{f}>1$, in some unconditionally false appurtenances); or "underprobable" (for $t<0$, i.e. $t=0$, in some unconditionally false appurtenances), "underindeterminate" (for $\mathrm{i}<0$, in some unconditionally true or false appurtenances), "underunprobable" (for $\mathrm{f}<0$, in some unconditionally true appurtenances).
This is because we should make a distinction between unconditionally true ( $t>1$, and $f<0$ or $\mathrm{i}<0)$ and conditionally true appurtenances ( $\mathrm{t} \leq 1$, and $\mathrm{f} \leq 1$ or $\mathrm{i} \leq 1$ ).

## 11. Other Examples.

Let's consider a neutrosophic set a collection of possible locations (positions) of particle x . And let A and B be two neutrosophic sets.
One can say, by language abuse, that any particle $x$ neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between ${ }^{-} 0$ and $1^{+}$. For example: $\mathrm{x}(0.5,0.2,0.3)$ belongs to A (which means, with a probability of $50 \%$ particle x is in a position of A , with a probability of $30 \% \mathrm{x}$ is not in A , and the rest is undecidable); or $\mathrm{y}(0,0$, 1) belongs to $A$ (which normally means $y$ is not for sure in $A$ ); or $z(0,1,0)$ belongs to $A$ (which means one does know absolutely nothing about z's affiliation with A).
More general, $\mathrm{x}((0.2-0.3),(0.40-0.45) \cup[0.50-0.51],\{0.2,0.24,0.28\})$ belongs to the set A , which means:

- with a probability in between $20-30 \%$ particle x is in a position of A (one cannot find an exact approximate because of various sources used);
- with a probability of $20 \%$ or $24 \%$ or $28 \% \mathrm{x}$ is not in A;
- the indeterminacy related to the appurtenance of x to A is in between $40-45 \%$ or between 50 51\% (limits included).
The subsets representing the appurtenance, indeterminacy, and falsity may overlap, and n_sup $=30 \%+51 \%+28 \%>100 \%$ in this case.


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# Quantum Causality Threshold and Paradoxes 

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#### Abstract

: In this paper we consider two entangled particles and study all the possibilities: when both are immobile, or one of them is immobile, or both are moving in different directions, or one of them is moving in a different direction. Then we study the causality between them and the paradoxes, which are generated. We define the Causality Threshold of a particle A with respect to another particle B.


Keywords: entangled particles, causality, causality threshold, quantum paradoxes


#### Abstract

Abstrait:

Dans cet article nous considérons deux particles et nous étudions toutes les possibilités: quand les deux particles sont immobiles, ou bien l'une d'elles est immobile, ou bien les deux se déplacent dans différentes directions, ou bien l'une d'elles se déplace dans une direction différente. Ensuite, nous étudions la causalité entre les particles et les paradoxes qu'elles génèrent. Nous définissons le seuil de causalité d'une particle A par rapport á une autre particle B.


## 1. Perfect simultaneousness.

Let's consider two entangled particles A and B. \{Schrödinger introduced the notion "entangled" in order to describe the non-separable states [Belavkin (2002)]\}.

At the beginning, both are immobile, in the same space $S(A, B)$ and time $t$ (simultaneously), and none of them is in the causality cone of the other.
According to Einstein's Theory of Relativity, when a particle is moving with respect to the other, its time and space axes appear inclined from the perspective of the other particle, modifying what for this other particle is "before" or "after", but their causality cones remain the same. And, if both particles are moving with respect to each other, the appearance of the inclined time and space axes is reciprocal from the perspective of each other.

Let's define the Quantum Causality Threshold of the particle A with respect to the particle B , noted by $\tau_{\mathrm{A}, \mathrm{B}}$, to be the space-time when neither A nor B is a cause for the other on the $B$ space-time axis (i.e. when the position-time vector vertex $\mathrm{t}_{\mathrm{A}} \equiv \mathrm{B}$ ).
To change the causality of a particle A with respect to another particle B one has to pass through non-causality, i.e. one has to pass through their threshold.

Generally, $\tau_{\mathrm{A}, \mathrm{B}} \neq \tau_{\mathrm{B}, \mathrm{A}}$, because one can have $\mathrm{t}_{\mathrm{A}} \equiv \mathrm{B}$ but $\mathrm{t}_{\mathrm{B}} \neq \mathrm{A}$, or reciprocally [see, for example, Figure 1.1.1].
a) When $\tau_{\mathrm{A}, \mathrm{B}}=\tau_{\mathrm{B}, \mathrm{A}}$ there is no causality between A and B (and therefore there is no quantum causality paradox).
b) If one particle attains its threshold with respect to the other, and the other one does not, then there is a causality and a non-causality simultaneously (and thus a quantum causality paradox) [see Figures 1.1.1, 1.1.2, 1.2.1].
c) If no particle attains its threshold with respect to the other, one has two sub-cases: either opposite causalities (and thus, again, a quantum causality paradox) [see Figures 1.1.3, 1.1.4], or compatible causalities (and, consequently, there is no quantum causality paradox) [see Figures 1.2.2 (for t together with $\mathrm{t}^{\prime}$ time axes), Figure 1.2.3 (for t together with $\mathrm{t}^{\prime}$ " time axes)].

### 1.1. Moving particle(s) keeping the same direction.

### 1.1.1. Particle B is moving away from particle A



Figure 1.1.1

- $\mathrm{S}(\mathrm{A}, \mathrm{B})$ is the space (represented here by a plane) of both entangled particles A and B.
- The left red vertical ( t$)$ continuous line represents the time axis of the particle A.
- Similarly, the green ( t ) continuous line represents the time axis of the particle B.
- On the left side one has the double cone of causality of the particle A: the cone beneath $\mathrm{S}(\mathrm{A}, \mathrm{B})$ contains the events that are the cause for A (i.e. events that influenced $A$ ), and the cone above $S(A, B)$ contains the events that A is a cause for (i.e. events influenced by A).
- Similarly, the right double cone represents the cone of causality of the particle B.
- Beneath $S(A, B)$ it is the past time ("before $A$ "), lying on the $S(A, B)$ is the present time ("simultaneously with A"), and above $\mathrm{S}(\mathrm{A}, \mathrm{B})$ it is the future time ("after A").
- Similarly, because the particles A and B are in the same space, $S(A, B)$ separates the past, present, and future times for the particle B.

Relative to the same referential system, the particle A remains immobile, while the particle B starts moving in the opposite direction relative to A. [Figure 1.1.1]
Therefore, from the perspective of $B$, the entangled particles $A$ and $B$ are simultaneous, and none of them is the cause of the other ( $\mathrm{t}_{\mathrm{A}} \equiv \mathrm{B}$ on B 's time axis); while from the perspective of A , the particle A is a cause for the particle $B$ (i.e. $A<t_{B}$ on A's time axis).
Hence, it appears this quantum causality paradox: non-causality or causality simultaneously?
1.1.2. Particle B is moving closer to particle A


Figure 1.1.2
Relative to the same referential system, the particle A remains immobile, while the particle B starts moving in a direction towards A . [Figure 1.1.2]
Therefore, from the perspective of the particle $B$, the entangled particles $A$ and B are simultaneous, and none of them is the cause of the other $\left(\mathrm{t}_{\mathrm{A}} \equiv \mathrm{B}\right.$ on B 's time axis); while from the perspective of the particle $A$, the particle $B$ is a cause for the particle A (i.e. $\mathrm{tB}<\mathrm{A}$ on A's time axis).
Hence, again, it appears a similar quantum causality paradox: non-causality or causality simultaneously?

### 1.1.3. Both entangled particles are moving closer to each other



Figure 1.1.3
With respect to the same referential system, both particles A and B start moving towards each other. [Figure 1.1.3]
Therefore, from the perspective of the particle A, the particle B is a cause of the particle $A$ (i.e. $t_{B}<A$ on A's time axis), and reciprocally: from the perspective of the particle $B$, the particle $A$ is a cause of the particle $B$ (i.e. $t_{A}<$ B on B's time axis). Thus one obtains the following:
Quantum Causality Paradox: How is it possible that simultaneously A is a cause of B , and B is a cause of A ?

### 1.1.4. Both entangled particles are moving away from each other



Figure 1.1.4
With respect to the same referential system, both particles A and B start moving in opposite directions from each other. [Figure 1.1.4]
Therefore, from the perspective of $A$, the particle $A$ is a cause of the particle $B$ (i.e. $A<t_{B}$ on A's time axis), and reciprocally: from the perspective of $B$, the particle $B$ is a cause of the particle $A$ (i.e. $B<t_{A}$ on $B$ 's time axis). Thus, one obtains the following same statement:
Quantum Causality Paradox: How is it possible that simultaneously A is a cause of B , and B is a cause of A ?
This theoretical case is similar to the 2002 Suarez Experiment [1], the only difference being that in Suarez's experiment there is not a perfect simultaneousness between the particles A and B.

### 1.2. Moving particle(s) changing the direction.

1.2.1. With respect to the same referential system, the particle A is immobile; while the particle B is moving at the beginning in a direction towards A , and later B changes the direction moving away from A .
a) Then, from the perspective of $A$ : The particle $B$ is a cause for $A$ (i.e. $\mathrm{t}_{\mathrm{B}}<\mathrm{A}$ on A's time axis). Then B changes its movement in a direction away from A , consequently B attains its quantum threshold $\tau_{\mathrm{B}, \mathrm{A}}$, i.e. $\mathrm{t}^{\prime}{ }_{\mathrm{B}}$ $\equiv \mathrm{A}$ on A's time axis (now there is no causality between A and B ). B keeps moving further from A and crosses its quantum threshold, then A becomes a causality for B because $\mathrm{t}^{\prime}{ }_{\mathrm{B}}>\mathrm{A}$ on A's time axis.
b) While, from the perspective of $B$, there is no causality between $A$ and $B$, since $B \equiv t_{A}$ on all $B$ 's three time axes $t^{\prime}, t^{\prime \prime}$, $t^{\prime \prime}$. [Figure 1.2.1.]. Hence, this quantum causality paradox appears: simultaneously B is cause for A , and non-causality, and A is cause for B ?


Figure 1.2.1
1.2.2. Relative to the same referential system, the particle A is moving away from B ; while the particle B is moving at the beginning in a direction towards A , and later B changes the direction moving away from A .
a) Then from the perspective of A: B is a cause for A (i.e. $\mathrm{t}^{\mathrm{B}}<\mathrm{A}$ on A's time axis). Then $B$ changes its movement in a direction away from $A$, consequently B attains its quantum threshold $\tau_{\mathrm{B}, \mathrm{A}}$, i.e. $\mathrm{t}^{\prime}{ }_{\mathrm{B}} \equiv \mathrm{A}$ on A 's
time axis (now there is no causality among A and B ). B keeps moving further from A and crosses its quantum threshold, then A becomes a causality for $B$ because $t^{\prime \prime}{ }_{B}>A$ on A's time axis.
b) While from the perspective of $B$, the particle $B$ is always a cause for $A$, since $\mathrm{B}<\mathrm{t}_{\mathrm{A}}$ on all B's time axes $\mathrm{t}^{\prime}, \mathrm{t}^{\prime \prime}$, and $\mathrm{t}^{\prime \prime}$. [Figure 1.2.2]. Hence, this quantum causality paradox appears: simultaneously B is cause for A , and non-causality, and A is cause for B ?


Figure 1.2.2
1.2.3. With respect to the same referential system, the particle A is moving closer to B ; while the particle B is moving at the beginning in a direction towards A , and later B changes the direction moving away from A .
a) Then from the perspective of A : B is a cause for A (i.e. $\mathrm{t}_{\mathrm{B}}<\mathrm{A}$ on A 's time axis). Then $B$ changes its movement in a direction away from $A$, consequently B attains its quantum threshold $\tau_{\mathrm{B}, \mathrm{A}}$, i.e. $\mathrm{t}^{\prime}{ }_{\mathrm{B}} \equiv \mathrm{A}$ on A 's time axis (now there is no causality among A and B ). B keeps moving further from A and crosses its quantum threshold, then A becomes a cause for B, because $t^{\prime \prime}{ }_{B}>A$ on A's time axis.
b) While from the perspective of $B$, the particle $A$ is always a cause for $B$, since $t_{A}<B$ on all $B$ 's time axes $t^{\prime}, t^{\prime \prime}$, and $t^{\prime \cdots}$. [Figure 1.2.2]. Hence, this quantum causality paradox appears: simultaneously B is cause for A , and non-causality, and A is cause for B ?


Figure 1.2.3
2. Let's consider the non-simultaneousness, when the particles A and B are in the separate spaces, $\mathrm{S}(\mathrm{A})$ and $\mathrm{S}(\mathrm{B})$ respectively, and different time axes, $t$ and $\mathrm{t}^{\text {e }}$ respectively.

### 2.1. Moving particle(s) keeping the same direction.

2.1.1. With respect to the same referential system, both particles A and B are moving in the same direction but with different high speeds. [Figure 2.1.1] Therefore, from both perspectives, of A and of B, the particle B is cause for A.


Figure 2.1.1
2.1.2. With respect to the same referential system, both particles A and B are moving in the same direction and with the same high speeds. [Figure 2.1.2] Therefore, from both perspectives, of A and of B, neither one is the causality of the other.


Figure 2.1.2
2.1.3. With respect to the same referential system, both particles A and B are moving closer to each other and with different high speeds [Figure 2.1.3]. Therefore, from the perspective of $A$ the particle $B$ is a cause of $A$, and reciprocally, thus again one gets a quantum causality paradox.


Figure 2.1.3

### 2.2. Moving particle(s) changing the direction.

2.2.1. With respect to the same referential system, the particle A is moving towards B ; while the particle B is moving at the beginning in a direction towards A , and later B changes the direction moving away from A .
a) Then from the perspective of A: B is a cause for A (i.e. $\mathrm{t}_{\mathrm{B}}<\mathrm{A}$ on A's time axis). Then $B$ changes its movement in a direction away from $A$, consequently B attains its quantum threshold $\tau_{\mathrm{B}, \mathrm{A}}$, i.e. $\mathrm{t}^{\prime}{ }_{\mathrm{B}} \equiv \mathrm{A}$ on A 's time axis (now there is no causality among A and B ). B keeps moving further from A and crosses its quantum threshold, then A becomes a cause for $B$ because $t^{\prime}{ }_{B}>A$ on A's time axis.
b) While from the perspective of $B$, the particle $A$ is always a cause for $B$, since $\mathrm{t}_{\mathrm{A}}<\mathrm{B}$ on all B 's time axes $\mathrm{t}^{\prime}, \mathrm{t}^{\prime}$, and $\mathrm{t}^{\prime}$ ". [Figure 2.2.1-]. Hence, this quantum causality paradox appears: simultaneously B is cause for A , and non-causality, and A is cause for B ?


Figure 2.2.1
2.2.2. Relative to the same referential system, both particles are moving towards each other, and then both change the movement in the opposite directions.
Similarly, from both perspectives, of A and of B, there are normal causalities (corresponding to $t 1$ and $t^{\prime}$ time axes), non-causalities (corresponding to $t 2$ and t" time axes), and opposite causalities (corresponding to t 3 and $\mathrm{t}^{\text {" }}$ " time axes) [Figure 2.2.2].
Hence, one again, one arrives at quantum causality paradoxes.


Figure 2.2.2

## Acknowledgement:

I would like to thank Dr. Jean Dezert from ONERA, France, who made me aware of this paradox of the time and provided me the Poirier's article.

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# On Some New Ideas in Hadron Physics 

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#### Abstract

We shortly review a series of novel ideas on the physics of hadrons and nuclear matter. Despite being vastly different in scope and content, these models share a common attribute, in that they offer unconventional viewpoints on infrared QCD and nuclear phenomena. In a sense, they are reminiscent of the plethora of formulations that have been developed over the years on classical gravitation: many seemingly disparate approaches can be effectively used to describe and explore the same physics.


## 1 Introduction

Given the extent and complexity of hadron and nuclear phenomena, any attempt for an exhaustive review of new ideas is outright unpractical. We survey here only a limited number of models and guide the reader to appropriate references for further information. The paper is divided in several sections according to the following plan:

1. The first section discusses the Brightsen model and the Nuclear String hypothesis;
2. Models inspired by Kerr-Newman twistor model and the AdS/CFT conjecture are introduced in the second section;
3. The last section discusses CGLE model of hadron masses and non-equilibrium phase transitions in infrared QCD.
The selection of topics is clearly incomplete and subjective. As such, it may not necessarily reflect the prevalent opinion of theorists working in this field. Our intent is to simply stimulate a constructive exchange of ideas in this active area of research.

## 2 Brightsen mdodel and the nuclear string hypothesis

In this hadron model, developed by M.Pitkanen [1] based on his TGD theory, it is supposed that ${ }^{4} \mathrm{He}$ nuclei and $A<4$ nuclei and possibly also nucleons appear as basic building blocks of nuclear strings. This seems like some kind of improvement of the Close Packed Spheron model of L. Pauling in 1960s, which asserts that nuclei is composite form of small numbers of interacting boson-fermion nucleon clusters, i.e. ${ }^{3} \mathrm{He}$ (PNP), triton (NPN) and deuteron (NP). Another extension of Pauling model is known as Brightsen's cluster nuclei model, which has been presented and discussed by F. Smarandache and D. Rabounski [2].

Interestingly, it can be shown that the Close Packed model of nuclei may explain naturally why all the upper quarks have fractional electric charge at the order of $Q=+\frac{2}{\sqrt{3}}$. So far this is one of the most mysterious enigma in the hadron physics. But as described by Thompson [4], in a closed-packed crystal
sheet model, the displacement coefficients would be given by a matrix where the 1-1 component is:

$$
\begin{equation*}
c_{11}=\frac{2 \rho}{\sqrt{3}}-1 \tag{1}
\end{equation*}
$$

where the deformation can be described by the resolved distance between columns, written as $\rho d$. Here d represents diameter of the nuclei entity. Now it seems interesting to point out here that if we supposed that $\rho=1+\frac{\sqrt{3}}{2}$, then $c$ from equation (3) yields exactly the same value with the upper quark's electric charge mentioned above. In other words, this seems to suggest plausible deep link between $\mathrm{QCD} /$ quark charges and the close-packed nuclei picture [3].

Interestingly, the origin of such fractional quark charge can also be described by a geometric icosahedron model [4]. In this model, the concept of quark generation and electroweak charge values are connected with (and interpreted as) the discrete symmetries of icosahedron geometry at its 12 vertices. Theoretical basis of this analog came from the fact that the gauge model of electroweak interactions is based on $\mathrm{SU}(2) \times \mathrm{U}(1)$ symmetry group of internal space. Meanwhile, it is known that $\mathrm{SU}(2)$ group corresponds to the $\mathrm{O}(3)$ group of 3D space rotations, hence it appears quite natural to connect particle properties with the discrete symmetries of the icosahedron polygon.

It is worth to mention here that there are some recent articles discussing plausible theoretical links between icosahedron model and close-packed model of nuclei entities, for instance by the virtue of Baxter theory [5]. Furthermore, there are other articles mentioning theoretical link between the close-packed model and Ginzburg-Landau theory. There is also link between Yang-Baxter theory and GinzburgLandau theory [6]. In this regards, it is well known that cluster hydrogen or cluster helium exhibit superfluidity $[7,8]$, therefore it suggests deep link between cluster model of Pauling or Brightsen and condensed matter physics (GinzburgLandau theory).

The Brightsen model supports a hypothesis that antimatter nucleon clusters are present as a parton (sensu Feynman) superposition within the spatial confinement of the proton
$\left({ }^{1} \mathrm{H}_{1}\right)$, the neutron, and the deuteron $\left({ }^{1} \mathrm{H}_{2}\right)$. If model predictions can be confirmed both mathematically and experimentally, a new physics is suggested. A proposed experiment is connected to orthopositronium annihilation anomalies, which, being related to one of known unmatter entity, orthopositronium (built on electron and positron), opens a way to expand the Standard Model.

Furthermore, the fact that the proposed Nuclear String hypothesis is derived from a theory which consists of manysheeted spacetime framework called TGD seems to suggest a plausible link between this model and Kerr-Schild twistor model as described below.

## 3 Multiparticle Kerr-Schild twistor model and AdS/ CFT Light-Front Holography model

Kerr's multiparticle solution can be obtained on the basis of the Kerr theorem, which yields a many-sheeted multi-twistorial spacetime over $M^{4}$ with some unusual properties. Gravitational and electromagnetic interaction of the particles occurs with a singular twistor line, which is common for twistorial structures of interacting particles [6].

In this regards the Kerr-Newman solution can be represented in the Kerr-Schild form [9]:

$$
\begin{equation*}
g_{\mu \nu}=\eta_{\mu \nu}+2 h k_{\mu} k_{v} \tag{2}
\end{equation*}
$$

where $\eta_{\mu \nu}$ is the metric of auxiliary Minkowski spacetime.
Then the Kerr theorem allows one to describe the Kerr geometry in twistor terms. And using the Kerr-Schild formalism, one can obtain exact asymptotically flat multiparticle solutions of the Einstein-Maxwell field equations. But how this model can yield a prediction of hadron masses remain to be seen. Nonetheless the axial stringy system corresponds to the Kerr-Schild null tetrad can be associated with superconducting strings. Interestingly one can find an interpretation of Dirac equation from this picture, and it is known that Dirac equation with an effective QCD potential can describe hadron masses.

What seems interesting from this Kerr-Schild twistor model, is that one can expect to give some visual interpretation of the electromagnetic string right from the solution of Einstein-Maxwell field equations. This would give an interesting clue toward making the string theory a somewhat testable result. Another approach to connect the superstring theory to hadron description will be discussed below, called Light-Front Holography model.

Brodsky et al. [10, 11] were able to prove that there are theoretical links, such that the Superstring theory reduces to AdS/CFT theory, and Ads/CFT theory reduces to the socalled Light Front Holography, which in turn this model can serve as first approximation to the Quantum Chromodynamics theory.

Starting from the equation of motion in QCD, they identify an invariant light front coordinate which allows separation of the dynamics of quark and gluon binding from the
kinematics of constituent spin and internal orbital angular momentum. Of most interesting here is that this method gives results in the from of 1-parameter light-front Schrödinger equation for QCD which determines the eigenspectrum and the light-front wavefunctions of hadrons for general spin and orbital angular momentum.

The light-front wave equation can be written as [8]:

$$
\begin{equation*}
\left(-\frac{d^{2}}{d \zeta^{2}}-\frac{1-4 L^{2}}{4 \zeta^{2}}+U(\zeta)\right) \phi(\zeta)=M^{2} \phi(\zeta) \tag{3}
\end{equation*}
$$

which is an effective single-variable light-front Schrödinger equation which is relativistic, covariant, and analytically tractable; here $M$ represents the mass spectra.

Nonetheless, whether this Light-Front Holography picture will yield some quantitative and testable predictions of hadron masses, remains to be seen.

## 4 Concluding note

We shortly review a series of novel ideas on the physics of hadrons and nuclear matter. Despite being vastly different in scope and content, these models share a common attribute, in that they offer unconventional viewpoints on hadron, nuclear phenomena, and infrared QCD. In a sense, they are reminiscent of the plethora of formulations that have been developed over the years on classical gravitation: many seemingly disparate approaches can be effectively used to describe and explore the same physics.

These very interesting new approaches, therefore, seem to suggest that there is a hitherto hidden theoretical links between different approaches.

In our opinion, these theoretical links worth to discuss further to prove whether they provide a consistent picture, in particular toward explanation of the hadron mass generation mechanism and spontaneous symmetry breaking process.

The present article is a first part of our series of review of hadron physics. Another part is under preparation.

## Acknowledgements

One of the authors (VC) wishes to express his gratitude to Profs. A. Yefremov and M. Fil'chenkov for kind hospitality in the Institute of Gravitation and Cosmology, PFUR.

Submitted on February 09, 2009 / Re-submitted on November 09, 2009
Accepted on November 27, 2009

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# On Some Novel Ideas in Hadron Physics. Part II 

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As a continuation of the preceding section, we shortly review a series of novel ideas on the physics of hadrons. In the present paper, emphasis is given on some different approaches to the hadron physics, which may be called as 'programs' in the sense of Lakatos. For clarity, we only discuss geometrization program, symmetries/ unification program, and phenomenology of inter-quark potential program.

## 1 Introduction

We begin the present paper by reiterating that given the extent and complexity of hadron and nuclear phenomena, any attempt for an exhaustive review of new ideas is outright unpractical. Therefore in this second part, we limit our short review on a number of scientific programs (in the sense of Lakatos). Others of course may choose different schemes or categorization. The main idea for this scheme of approaches was attributed to an article by Lipkin on hadron physics. accordingly, we describe the approaches as follows:

1. The geometrization approach, which was based on analogy between general relativity as strong field and the hadron physics;
2. Models inspired by (generalization of) symmetry principles;
3. Various composite hadron models;
4. The last section discusses phenomenological approach along with some kind of inter-quark QCD potential.

To reiterate again, the selection of topics is clearly incomplete, and as such it may not necessarily reflect the prevalent opinion of theorists working in this field (for more standard review the reader may wish to see [1]). Here the citation is far from being complete, because we only cite those references which appear to be accessible and also interesting to most readers.

Our intention here is to simply stimulate a healthy exchange of ideas in this active area of research, in particular in the context of discussions concerning possibilities to explore elementary particles beyond the Standard Model (as mentioned in a number of papers in recent years).

## 2 Geometrization approach

In the preceding section we have discussed a number of hadron or particle models which are essentially based on geometrical
theories, for instance Kerr-Schild model or Topological Geometrical Dynamics [1].

However, we can view these models as part of more general approach which can be called 'geometrization' program. The rationale of this approach can be summarized as follows (to quote Bruchholz):
"The deeper reason is that the standard model is based on Special Relativity while gravitation is the principal item of General Relativity."[3]

Therefore, if we follow this logic, then it should be clear that the Standard Model which is essentially based on Quantum Electrodynamics and Dirac equation, is mostly special relativistic in nature, and it only explains the weak field phenomena (because of its linearity). And if one wishes to extend these theories to explain the physical phenomena corresponding to the strong field effects (like hadrons), then one should consider the nonlinear effects, and therefore one begins to introduce nonlinear Dirac-Hartree-Fock equation or nonlinear Klein-Gordon equation (we mentioned this approach in the preceding section).

Therefore, for instance, if one wishes to include a consistent general relativistic approach as a model of strong fields, then one should consider the general covariant generalization of Dirac equation [4]:

$$
\begin{equation*}
\left(i \gamma^{k}(x) \bar{\nabla}_{k}-m\right) \psi(x)=0 \tag{1}
\end{equation*}
$$

Where the gamma matrices are related to the 4 -vector relative to General Coordinate Transformations (GCT). Then one can consider the interaction of the Dirac field with a scalar external field U which models a self-consistent quark system field (by virtue of changing $m \rightarrow m+\mathbf{U}$ ) [4].

Another worth-mentioning approach in this context has been cited by Bruchholz [3], i.e. the Geilhaupt's theory which is based on some kind of Higgs field from GTR and Quantum Thermodynamics theory.

In this regards, although a book has been written discussing some aspects of the strong field (see Grib et al. [4]), actually this line of thought was recognized not so long ago,
as cited in Jackson and Okun [5]:
" The close mathematical relation between non-Abelian gauge fields and general relativity as connections in fiber bundles was not generally realized until much later".

Then began the plethora of gauge theories, both including or without gravitational field. The essential part of these GTR-like theories is to start with the group of General Coordinate Transformations (GCT). It is known then that the finite dimensional representations of GCT are characterized by the corresponding ones of the $\operatorname{SL}(4, R)$ which belongs to GL(4,R) [6]. In this regards, Ne'eman played the pioneering role in clarifying some aspects related to double covering of SL( $n, R$ ) by GL( $n, R$ ), see for instance [7]. It can also be mentioned here that spinor $\operatorname{SL}(2, C)$ representation of GTR has been discussed in standard textbooks on General Relativity, see for instance Wald (1983). The SL(2,C) gauge invariance of Weyl is the most well-known, although others may prefer SL(6,C), for instance Abdus Salam et al. [8].

Next we consider how in recent decades the progress of hadron physics were mostly driven by symmetries consideration.

## 3 Symmetries approach

Perhaps it is not quite an exaggeration to remark here that most subsequent developments in both elementary particle physics and also hadron physics were advanced by YangMills' effort to generalize the gauge invariance [9]. And then Ne'eman and Gell-Mann also described hadrons into octets of $\operatorname{SU}(3)$ flavor group.

And therefore, it becomes apparent that there are numerous theories have been developed which intend to generalize further the Yang-Mills theories. We only cite a few of them as follows.

We can note here, for instance, that Yang-Mills field somehow can appear more or less quite naturally if one uses quaternion or hypercomplex numbers as basis. Therefore, it has been proved elsewhere that Yang-Mills field can be shown to appear naturally in Quaternion Space too [9].

Further generalization of Yang-Mills field has been discussed by many authors, therefore we do not wish to reiterate all of them here. Among other things, there are efforts to describe elementary particles (and hadrons) using the most generalized groups, such as E8 or E11, see for instance [17].

Nonetheless, it can be mentioned in this regards, that there are other symmetries which have been considered (beside the SL( $6, C$ ) mentioned above), for instance $\mathrm{U}(12)$ which has been considered by Ishida and Ishida, as generalizations of $\mathrm{SU}(6)$ of Sakata, Gursey et al. [11].

One can note here that Gursey's approach was essentially to extend Wigner's idea to elementary particle physics using $\mathrm{SU}(2)$ symmetry. Therefore one can consider that Wigner has played the pioneering role in the use of groups and symme-
tries in elementary particles physics, although the mathematical aspects have been presented by Weyl and others.

## 4 Composite model of hadrons

Beside the group and symmetrical approach in Standard Model, composite model of quarks and leptons appear as an equivalent approach, as this method can be traced back to FermiYang in 1949, Sakata in 1956, and of course the Gell-MannNe'eman [11]. Nonetheless, it is well known that at that time quark model was not favorite, compared to the geometricalunification program, in particular for the reason that the quarks have not been observed.

With regards to quarks, Sakata has considered in 1956 three basic hadrons (proton, neutron, and alpha- particle) and three basic leptons (electron, muon, neu- trino). This Nagoya School was quite inuential and the Sakata model was essentially transformed into the quark model of Gell-Mann, though with more abstract interpretation. It is perhaps more interesting to remark here, that Pauling's closed-packed spheron model is also composed of three sub-particles.

The composite models include but not limited to su- perconductor models inspired by BCS theory and NJL (Nambu-Jona-Lasinio theory). In this context, we can note that there are hadron models as composite bosons, and other models as composite fermions. For instance, hadron models based on BCS theory are essentially com- posite fermions. In developing his own models of com- posite hadron, Nambu put forward a scheme for the the- ory of the strong interactions which was based on and has resemblance with the BCS theory of superconductivity, where free electrons in superconductivity becomes hy- pothetical fermions with small mass; and energy gap of superconductor becomes observed mass of the nucleon. And in this regards, gauge invariance of superconductivity becomes chiral invariance of the strong interaction. Nambu's theory is essentially non-relativistic.

It is very interesting to remark here that although QCD is the correct theory for the strong interactions it cannot be used to compute at all energy and momentum scales. For many purposes, the original idea of Nambu- Jona-Lasinio woks better.

Therefore, one may say that the most distinctive as- pect between geometrization program to describe hadron models and the composite models (especially Nambu's BCS theory), is that the first approach emphasizes its theoretical correspondence to the General Relativity, metric tensors etc., while the latter emphasizes analogies between hadron physics and the strong field of super- conductors. [4]

In the preceding section we have mentioned another composite hadron models, for instance the nuclear string and Brightsen cluster model. The relativistic wave equa- tion for the composite models is of course rather com- plicated (compared to the 1-entity model of particles)[11].

## 5 Phenomenology with Inter-Quark potential

While nowadays most physicists prefer not to rely on the phenomenology to build theories, it is itself that has has its own virtues, in particular in studying hadron physics. It is known that theories of electromagnetic fields and gravitation are mostly driven by some kind of geometrical principles. But to describe hadrons, one does not have much choices except to take a look at experiments data before begin to start theorizing, this is perhaps what Gell-Mann meant while emphasizing that physicists should sail between Scylla and Charybdis. There- fore one can observe that hadron physics are from the beginning affected by the plentitude of analogies with human senses, just to mention a few: strangeness, flavor and colour. In other words one may say that hadron physics are more or less phenomenology-driven, and symmetries consideration comes next in order to explain the observed particles zoo.

The plethora of the aforementioned theories actually boiled down to either relativistic wave equation (Klein- Gordon) or non-relativistic wave equation, along with some kind of interquark potential. The standard picture of course will use the QCD linear potential, which can be derived from Maxwell equations.

But beside this QCD linear potential, there are other types of potentials which have been considered in the literature, to mention a few of them:
a. Trigononometric Rosen-Morse potential [13] of the form:

$$
\begin{equation*}
\nu_{t}(|z|)=-2 b . \cot |z|+a(a+1)^{2} \csc |z|, \tag{2}
\end{equation*}
$$

where $z=\frac{r}{d}$.
b. PT-Symmetric periodic potential [14];
c. An Interquark qq-potential from Yang-Mills theory has been considered in [15];
d. An alternative PT-Symmetric periodic potential has been derived from radial biquaternion Klein-Gordon equation [16]. Interestingly, we can note here that a recent report by Takahashi et al. indicates that periodic potential could explain better the cluster deuterium reaction in $\mathrm{Pd} / \mathrm{PdO} / \mathrm{ZrO} 2$ nanocomposite-samples in a joint research by Kobe University in 2008. This experiment in turn can be compared to a previous excellent result by Arata-Zhang in 2008 [18]. What is more interesting here is that their experiment also indicates a drastic mesoscopic effect of $\mathrm{D}(\mathrm{H})$ absorption by the Pd-nanocomposite-samples.
Of course, there is other type of interquark potentials which have not been mentioned here.

## 6 Concluding note

We extend a bit the preceding section by considering a number of approaches in the context of hadron theories. In a
sense, they are reminiscent of the plethora of formulations that have been developed over the years on classical gravitation: many seemingly disparate ap- proaches can be effectively used to describe and explore the same physics.

It can be expected that those different approaches of hadron physics will be advanced further, in particular in the context of possibility of going beyond Standard Model.

## Acknowledgements

One of the authors (VC) wishes to express his gratitude to Profs. A. Yefremov and M. Fil'chenkov for kind hos- pitality in the Institute of Gravitation and Cosmology, PFUR. Special thanks to Prof. A. Takahashi for dis- cussion of his experimental results of Pd lattice with his team.

Submitted on December 10, 2009
Accepted on January 01, 2010

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# A Derivation of Maxwell Equations in Quaternion Space 

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#### Abstract

Quaternion space and its respective Quaternion Relativity (it also may be called as Rotational Relativity) has been defined in a number of papers including [1], and it can be shown that this new theory is capable to describe relativistic motion in elegant and straightforward way. Nonetheless there are subsequent theoretical developments which remains an open question, for instance to derive Maxwell equations in Q-space. Therefore the purpose of the present paper is to derive a consistent description of Maxwell equations in Q- space. First we consider a simplified method similar to the Feynman's derivation of Maxwell equations from Lorentz force. And then we present another derivation method using Dirac decomposition, introduced by Gersten (1999). Further observation is of course recommended in order to refute or verify some implication of this proposition.


## 1 Introduction

Quaternion space and its respective Quaternion Relativity (it also may be called as Rotational Relativity has been defined in a number of papers including [1], and it can be shown that this new theory is capable to describe relativistic motion in elegant and straightforward way. For instance, it can be shown that the Pioneer spacecraft's Doppler shift anomaly can be explained as a relativistic effect of Quaternion Space [11]. The Yang-Mills field also can be shown to be consistent with Quaternion Space [1]. Nonetheless there are subsequent theoretical developments which remains an open issue, for instance to derive Maxwell equations in Q-space [1].

Therefore the purpose of the present article is to derive a consistent description of Maxwell equations in Q-space. First we consider a simplified method similar to the Feynman's derivation of Maxwell equations from Lorentz force. And then we present another method using Dirac decomposition, introduced by Gersten (1999). In the first section we will shortly review the basics of Quaternion space as introduced in [1].

Further observation is of course recommended in order to verify or refute the propositions outlined herein.

## 2 Basic aspects of Q-relativity physics

In this section, we will review some basic definitions of quaternion number and then discuss their implications to quaternion relativity (Q-relativity) physics [1].

Quaternion number belongs to the group of "very good" algebras: of real, complex, quaternion, and octonion [1], and normally defined as follows [1]:

$$
\begin{equation*}
Q \equiv a+b i+c j+d k \tag{1}
\end{equation*}
$$

Where $a, b, c, d$ are real numbers, and $i, j, k$ are imaginary quaternion units. These Q -units can be represented either via $2 \times 2$ matrices or $4 \times 4$ matrices. There is quaternionic multiplication rule which acquires compact form [1]:

$$
\begin{equation*}
1 q_{k}=q_{k} 1=q_{k}, q_{j} q_{k}=-\delta_{j k}+\epsilon_{j k n} q_{n} \tag{2}
\end{equation*}
$$

Where $\delta_{k n}$ and $\epsilon_{j k n}$ represents 3-dimensional symbols of Kronecker and Levi-Civita, respectively.

In the context of Quaternion Space [1], it is also possible to write the dynamics equations of classical mechanics for an inertial observer in constant Q-basis. $\mathrm{SO}(3, \mathrm{R})$ - invariance of two vectors allow to represent these dynamics equations in Q-vector form [1]:

$$
\begin{equation*}
m \frac{d^{2}}{d t^{2}}\left(x_{k} q_{k}\right)=F_{k} q_{k} \tag{3}
\end{equation*}
$$

Because of antisymmetry of the connection (generalised angular velocity) the dynamics equations can be written in vector components, by conventional vector notation [1]:

$$
\begin{equation*}
m(\vec{a}+2 \vec{\Omega} \times \vec{v}+\vec{\Omega} \times \vec{r}+\vec{\Omega} \times(\vec{\Omega} \times \vec{r}))=\vec{F} \tag{4}
\end{equation*}
$$

Therefore, from equation (4) one recognizes known types of classical acceleration, i.e. linear, coriolis, angular, centripetal.

From this viewpoint one may consider a generalization of Minkowski metric interval into biquaternion form [1]:

$$
\begin{equation*}
d z=\left(d x_{k}+i d t_{k}\right) q_{k} \tag{5}
\end{equation*}
$$

With some novel properties, i.e.:

- temporal interval is defined by imaginary vector;
- space-time of the model appears to have six dimensions (6D);
- vector of the displacement of the particle and vector of corresponding time change must always be normal to each other, or:

$$
\begin{equation*}
d x_{k} d t_{k}=0 \tag{6}
\end{equation*}
$$

One advantage of this Quaternion Space representation is that it enables to describe rotational motion with great clarity.

After this short review of Q-space, next we will discuss a simplified method to derive Maxwell equations from Lorentz force, in a similar way with Feynman's derivation method using commutative relation [2][10].

## 3 An intuitive approach from Feynman's derivative

A simplified derivation of Maxwell equations will be discussed here using similar approach known as Feynman's derivation [2][3][10].

We can introduce now the Lorentz force into equation (4), to become:

Which is a linear function of the velocities. We now may consider that the right hand side of equation (10) consists of a scalar potential [2]:

$$
\begin{equation*}
\phi(x, t)=m a \cdot x-\frac{m}{2}(\vec{\Omega} \times x)^{2} \tag{11}
\end{equation*}
$$

And a vector potential:

$$
\begin{equation*}
A(x, t) \equiv m \dot{x} \cdot \vec{\Omega} \times x \tag{12}
\end{equation*}
$$

So that

$$
\begin{equation*}
V(x, \dot{x}, t)=\phi(x, t)-\dot{x} \cdot A(x, t) \tag{13}
\end{equation*}
$$

Then the equation of motion (9) may now be written in Lorentz form as follows [2]:

$$
\begin{equation*}
m \ddot{x}=E(x, t)+x \times H(x, t) \tag{14}
\end{equation*}
$$

With

$$
\begin{equation*}
E=-\frac{\partial A}{\partial t}-\nabla \phi=-m \Omega \times x-m a+m \Omega \times(x \times \Omega) \tag{15}
\end{equation*}
$$

$m\left(\frac{d \vec{v}}{d t}+2 \vec{\Omega} \times \vec{v}+\vec{\Omega} \times \vec{r}+\vec{\Omega} \times(\vec{\Omega} \times \vec{r})\right)=q_{\otimes}\left(\vec{E}+\frac{1}{c} \vec{v} \times \vec{B}\right)^{\text {And }}$
Or

$$
\begin{equation*}
\left(\frac{d \vec{v}}{d t}\right)=\frac{q_{\otimes}}{m}\left(\vec{E}+\frac{1}{c} \vec{v} \times \vec{B}\right)-2 \vec{\Omega} \times \vec{v}-\vec{\Omega} \times \vec{r}-\vec{\Omega} \times(\vec{\Omega} \times \vec{r}) \tag{8}
\end{equation*}
$$

We note here that q variable here denotes electric charge, not quaternion number.

Interestingly, equation (4) can be compared directly to equation (8) in [2]:
$m \ddot{x}=F-m\left(\frac{d \vec{v}}{d t}\right)+m \vec{r} \times \vec{\Omega}+m 2 \dot{x} \times \vec{\Omega}+m \vec{\Omega} \times(\vec{r} \times \vec{\Omega})$,
In other words, we find an exact correspondence between quaternion version of Newton second law (3) and equation (9), i.e. the equation of motion for particle of mass $m$ in a frame of reference whose origin has linear acceleration a and an angular velocity $\vec{\Omega}$ with respect to the reference frame [2].

Since we want to find out an "electromagnetic analogy" for the inertial forces, then we can set $\mathrm{F}=0$. The equation of motion (9) then can be derived from Lagrangian $\mathrm{L}=\mathrm{T}-\mathrm{V}$, where T is the kinetic energy and V is a velocity-dependent generalized potential [2]:

$$
\begin{equation*}
V(x, \dot{x}, t)=m a \cdot x-m \dot{x} \cdot \vec{\Omega} \times x-\frac{m}{2}(\vec{\Omega} \times x)^{2} \tag{10}
\end{equation*}
$$

$$
\begin{equation*}
H=\nabla \times A=2 m \Omega \tag{7}
\end{equation*}
$$

At this point we may note [2, p. 303] that Maxwell equations are satisfied by virtue of equations (15) and (16). The correspondence between Coriolis force and magnetic force, is known from Larmor method. What is interesting to remark here, is that the same result can be expected directly from the basic equation of Quaternion Space (3) [1]. The aforementioned simplified approach indicates that it is indeed possible to find out Maxwell equations in Quaternion space, in particular based on our intuition of the direct link between Newton second law in Q-space and Lorentz force (We can remark that this parallel between classical mechanics and electromagnetic field appears to be more profound compared to simple similarity between Coulomb and Newton force).

As an added note, we can mention here, that the aforementioned Feynman's derivation of Maxwell equations is based on commutator relation which has classical analogue in the form of Poisson bracket. Then there can be a plausible way to extend directly this 'classical' dynamics to quaternion extension of Poisson bracket [14], by assuming the dynamics as element of the type: $r \in H \wedge H$ of the type: $r=a i \wedge j+$ $b i \wedge k+c j \wedge k$, from which we can define Poisson bracket on H. But in the present paper we don't explore yet such a possibility.

In the next section we will discuss more detailed derivation of Maxwell equations in Q-space, by virtue of Gersten's method of Dirac decomposition [4].

## 4 A new derivation of Maxwell equations in Quaternion Space by virtue of Dirac decomposition.

In this section we present a derivation of Maxwell equations in Quaternion space based on Gersten's method to derive Maxwell equations from one photon equation by virtue of Dirac decomposition [4]. It can be noted here that there are other methods to derive such a 'quantum Maxwell equations' (i.e. to find link between photon equation and Maxwell equations), for instance by Barut quite a long time ago (see ICTP preprint no. IC/91/255).

We know that Dirac deduces his equation from the relativistic condition linking the Energy E, the mass $m$ and the momentum p [5]:

$$
\begin{equation*}
\left(E^{2}-c^{2} \vec{p}^{2}-m^{2} c^{4}\right) I^{(4)} \Psi=0 \tag{17}
\end{equation*}
$$

Where $I^{(4)}$ is the $4 \times 4$ unit matrix and $\Psi$ is a 4 -component column (bispinor) wavefunction. Dirac then decomposes equation (17) by assuming them as a quadratic equation:

$$
\begin{equation*}
\left(A^{2}-B^{2}\right) \Psi=0 \tag{18}
\end{equation*}
$$

Where

$$
\begin{gather*}
A=E,  \tag{19}\\
B=c \vec{p}+m c^{2} \tag{20}
\end{gather*}
$$

The decomposition of equation (18) is well known, i.e. $(A+B)(A-B)=0$, which is the basic of Dirac's decomposition method into $2 \times 2$ unit matrix and Pauli matrix [4][12].

By virtue of the same method with Dirac, Gersten found in 1999 [4] a decomposition of one photon equation from relativistic energy condition (for massless photon [5]):

$$
\begin{equation*}
\left(\frac{E^{2}}{c^{2}}-\vec{p}^{2}\right) I^{(3)} \Psi=0 \tag{21}
\end{equation*}
$$

Where $I^{(3)}$ is the $3 \times 3$ unit matrix and $\Psi$ is a 3-component column wavefunction. Gersten then found [4] equation (21) decomposes into the form:
$\left[\frac{E}{c} I^{(3)}-\vec{p} \cdot \vec{S}\right]\left[\frac{E}{c} I^{(3)}+\vec{p} \cdot \vec{S}\right] \vec{\Psi}-\left(\begin{array}{c}p_{x} \\ p_{y} \\ p_{z}\end{array}\right)(\vec{p} \cdot \vec{\Psi})=0$
where $\vec{S}$ is a spin one vector matrix with components [4]:

$$
\begin{align*}
& S_{x}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & -i \\
0 & -i & 0
\end{array}\right)  \tag{23}\\
& S_{y}=\left(\begin{array}{ccc}
0 & 0 & i \\
0 & 0 & 0 \\
-i & 0 & 0
\end{array}\right) \tag{24}
\end{align*}
$$

$$
S_{z}=\left(\begin{array}{ccc}
0 & -i & 0  \tag{25}\\
-i & 0 & 0 \\
0 & 0 & 0
\end{array}\right)
$$

And with the properties:

$$
\begin{equation*}
\left[S_{x}, S_{y}\right]=i S_{z},\left[S_{x}, S_{z}\right]=i S_{y},\left[S_{y}, S_{z}\right]=i S_{x}, \vec{S}^{2}=2 I^{(3)} \tag{26}
\end{equation*}
$$

Gersten asserts that equation (22) will be satisfied if the two equations [4][5]:

$$
\begin{gather*}
{\left[\frac{E}{c} I^{(3)}+\vec{p} \cdot \vec{S}\right] \vec{\Psi}=0}  \tag{27}\\
\vec{p} \cdot \vec{\Psi}=0 \tag{28}
\end{gather*}
$$

are simultaneously satisfied. The Maxwell equations [9] will be obtained by substitution of E and p with the ordinary quantum operators (see for instance Bethe, Field Theory):

$$
\begin{equation*}
E \rightarrow i \hbar \frac{\partial}{\partial t} \tag{29}
\end{equation*}
$$

and

$$
\begin{equation*}
p \rightarrow-i h \nabla \tag{30}
\end{equation*}
$$

And the wavefunction substitution:

$$
\begin{equation*}
\vec{\Psi}=\vec{E}-i \vec{B} \tag{31}
\end{equation*}
$$

Where E and B are electric and magnetic elds, respectively. With the identity:

$$
\begin{equation*}
(\vec{p} \cdot \vec{S}) \vec{\Psi}=\hbar \nabla \times \vec{\Psi} \tag{32}
\end{equation*}
$$

Then from equation (27) and (28) one will obtain:

$$
\begin{gather*}
i \frac{\hbar}{c} \frac{\partial(\vec{E}-i \vec{B})}{\partial t}=-\hbar \nabla \times(\vec{E}-i \vec{B})  \tag{33}\\
\nabla \cdot(\vec{E}-i \vec{B})=0 \tag{34}
\end{gather*}
$$

Which are the Maxwell equations if the electric and magnetic fields are real [4][5].

We can remark here that the combination of E and B as introduced in (31) is quite well known in literature [6][7]. For instance, if we use positive signature in (31), then it is known as Bateman representation of Maxwell equations $\left(\operatorname{div} \vec{\epsilon}=0 ; \operatorname{rot} \vec{\epsilon}=\frac{\partial \epsilon}{\partial t} ; \epsilon=\vec{E}+i \vec{B}\right)$.
But the equation (31) with negative signature represents the complex nature of Electromagnetic fields [6], which indicates that these fields can also be represented in quaternion form.

Now if we represent in other form $\vec{\epsilon}=\vec{E}-i \vec{B}$ as more conventional notation, then equation (33) and (34) will get a quite simple form:

$$
\begin{gather*}
i \frac{\hbar}{c} \frac{\partial \vec{\epsilon}}{\partial t}=-\hbar \nabla \times \vec{\epsilon}  \tag{35}\\
\nabla \cdot \vec{\epsilon}=0 \tag{36}
\end{gather*}
$$

Now to consider quaternionic expression of the above results from Gersten [4], one can begin with the same linearization procedure just as in equation (5):

$$
\begin{equation*}
d z=\left(d x_{k}+i d t_{k}\right) q_{k} \tag{37}
\end{equation*}
$$

Which can be viewed as the quaternionic square root of the metric interval dz:

$$
\begin{equation*}
d z^{2}=d x^{2}-d t^{2} \tag{38}
\end{equation*}
$$

Now consider the relativistic energy condition (for massless photon [5]) similar to equation (21):

$$
\begin{equation*}
E^{2}=p^{2} c^{2} \Rightarrow\left(\frac{E^{2}}{c^{2}}-\vec{p}^{2}\right)=k^{2} \tag{39}
\end{equation*}
$$

It is obvious that equation (39) has the same form with (38), therefore we may find its quaternionic square root too, then we find:

$$
\begin{equation*}
k=\left(E_{q k}+i \vec{p}_{q k}\right) q_{k} \tag{40}
\end{equation*}
$$

Where q represents the quaternion unit matrix. Therefore the linearized quaternion root decomposition of equation (21) can be written as follows [4]:

$$
\begin{align*}
& {\left[\frac{E_{q k} q_{k}}{c} I^{(3)}+i \vec{p}_{q k} q_{k} \cdot \vec{S}\right]\left[\frac{E_{q k} q_{k}}{c} I^{(3)}+i \vec{p}_{q k} q_{k} \cdot \vec{S}\right] \vec{\Psi}} \\
& -\left(\begin{array}{l}
p_{x} \\
p_{y} \\
p_{z}
\end{array}\right)\left(i \vec{p}_{q k} q_{k} \cdot \vec{\Psi}\right)=0 \tag{41}
\end{align*}
$$

Accordingly, equation (41) will be satisfied if the two equations:

$$
\begin{gather*}
{\left[\frac{E_{q k} q_{k}}{c} I^{(3)}+i \vec{p}_{q k} q_{k} \cdot \vec{S}\right] \overrightarrow{\Psi_{k}}=0}  \tag{42}\\
i \vec{p}_{q k} q_{k} \cdot \vec{\Psi}_{k}=0 \tag{43}
\end{gather*}
$$

are simultaneously satisfied. Now we introduce similar wavefunction substitution, but this time in quaternion form:

$$
\begin{equation*}
\vec{\Psi}_{q k}=\vec{E}_{q k}-i \vec{B}_{q k}=\vec{\epsilon}_{q k} \tag{44}
\end{equation*}
$$

And with the identity:

$$
\begin{equation*}
\left(\vec{p}_{q k} q_{k} \cdot \vec{S}\right) \vec{\Psi}_{k}=\hbar \nabla_{k} \times \vec{\Psi}_{k} \tag{45}
\end{equation*}
$$

Then from equation (42) and (43) one will obtain the Maxwell equations in Quaternion-space as follows:

$$
\begin{gather*}
i \frac{\hbar}{c} \frac{\partial \vec{\epsilon}_{q} k}{\partial t}=-\hbar \nabla_{k} \times \vec{\epsilon}_{q k}  \tag{46}\\
\nabla_{k} \cdot \vec{\epsilon}_{q k}=0 \tag{47}
\end{gather*}
$$

Now the remaining question is to define quaternion differential operator in the right hand side of (46) and (47).

In this regards one can choose some definitions of quaternion differential operator, for instance the 'Moisil-Theodoresco operator'[8] :

$$
\begin{equation*}
D[\varphi]=\operatorname{grad} \varphi=\sum_{k=1}^{3} i_{k} \partial_{k} \varphi=i_{1} \partial_{1} \varphi+i_{2} \partial_{2} \varphi+i_{3} \partial_{3} \varphi \tag{48}
\end{equation*}
$$

Where we can define here that $i_{1}=i ; i_{2}=j ; i_{3}=k$ to represent $2 \times 2$ quaternion unit matrix, for instance. Therefore the differential of equation (44) now can be expressed in similar notation of (48) :

$$
\begin{align*}
& D[\vec{\Psi}]=D[\vec{\epsilon}]=i_{1} \partial_{1} E_{1}+i_{2} \partial_{2} E_{2}+i_{3} \partial_{3} E_{3}  \tag{49}\\
& -i\left(i_{1} \partial_{1} B_{1}+i_{2} \partial_{2} B_{2}+i_{3} \partial_{3} B_{3}\right),
\end{align*}
$$

This expression indicates that both electric and magnetic fields can be represented in unified manner in a biquaternion form.

Then we define quaternion differential operator in the right-hand-side of equation (46) by an extension of the conventional definition of curl:

$$
\nabla \times A_{q k}=\left|\begin{array}{ccc}
i & j & k  \tag{50}\\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
A_{x} & A_{y} & A_{z}
\end{array}\right|
$$

To become its quaternion counterpart, where $\mathrm{i}, \mathrm{j}, \mathrm{k}$ represents quaternion matrix as described above. This quaternionic extension of curl operator is based on the known relation of multiplication of two arbitrary complex quaternions $q$ and $b$ as follows:

$$
\begin{equation*}
q \cdot b=q_{0} b_{0}-\langle\vec{q}, \vec{b}\rangle+[\vec{q} \times \vec{b}]+q_{0} \vec{b}+b_{0} \vec{q}, \tag{51}
\end{equation*}
$$

where

$$
\begin{equation*}
\langle\vec{q}, \vec{b}\rangle:=\sum_{k=1}^{3} q_{k} b_{k} \in C \tag{52}
\end{equation*}
$$

And

$$
[\vec{q} \times \vec{b}]:=\left|\begin{array}{ccc}
i & j & k  \tag{53}\\
q_{1} & q_{2} & q_{3} \\
b_{1} & b_{2} & b_{3}
\end{array}\right|
$$

We can note here that there could be more rigorous approach to define such a quaternionic curl operator.[7]

In the present paper we only discuss derivation of Maxwell equations in Quaternion Space using the decomposition method described by Gersten [4][5]. Further extension to Proca equations in Quaternion Space seems possible too using the same method [5], but it will not be discussed here.

In the next section we will discuss some physical implications of this new derivation of Maxwell equations in Quaternion Space.

## 5 A few implications: de Broglie's wavelength and spin

In the foregoing section we derived a consistent description of Maxwell equations in Q-Space by virtue of Dirac- Gersten's decomposition. Now we discuss some plausible implications of the new proposition.

First, in accordance with Gersten, we submit the viewpoint that the Maxwell equations yield wavefunctions which can be used as guideline for interpretation of quantum mechanics [4][5]. The one-to-one correspondence between classical and quantum wave interpretation actually can be expected not only in the context of Feynman's derivation of Maxwell equations from Lorentz force, but also from known exact correspondence between commutation relation and Poisson bracket [2][3]. Furthermore, the proposed quaternion yields to a novel viewpoint of both the wavelength, as discussed below, and also mechanical model of spin [13].

The equation (39) implies that momentum and energy could be expressed in quaternion form. Now by introducing the definition of de Broglie's wavelength
$\left(\lambda_{D B}=\frac{\hbar}{p} \rightarrow p_{D B}=\frac{\hbar}{\lambda}\right)$,
then one obtains an expression in terms of wavelength:
$k=\left(E_{k}+i \vec{p}_{k}\right) q_{k}=\left(E_{k} q_{k}+i \vec{p}_{k} q_{k}\right)=\left(E_{k} q_{k}+i \frac{\hbar}{\lambda_{k}^{D B} q_{k}}\right)$
In other words, now we can express de Broglie's wavelength in a consistent Q-basis:

$$
\begin{equation*}
\lambda_{D B-Q}=\frac{\hbar}{\sum_{k=1}^{3}\left(p_{k}\right) q_{k}}=\frac{\hbar}{v_{\text {group }} \sum_{k=1}^{3}\left(m_{k}\right) q_{k}} \tag{55}
\end{equation*}
$$

Therefore the above equation can be viewed as an Extended De Broglie wavelength in Q-space. This equation means that the mass also can be expressed in Q-basis. In the meantime, a quite similar method to define quaternion mass has also been considered elsewhere (Gupta [13]), but it has not yet been expressed in Dirac equation form as presented here.

Further implications of this new proposition of quaternion de Broglie requires further study, and therefore it is excluded from the present paper.

## 6 Concluding remarks

In the present paper we derive a consistent description of Maxwell equations in Q-space. First we consider a simplified method similar to the Feynman's derivation of Maxwell equations from Lorentz force. And then we present another method to derive Maxwell equations by virtue of Dirac decomposition, introduced by Gersten (1999).

In accordance with Gersten, we submit the viewpoint that the Maxwell equations yield wavefunctions which can be used as guideline for interpretation of quantum mechanics. The one-to-one correspondence between classical and quantum wave interpretation asserted here actually can be expected not only in the context of Feynman's derivation of Maxwell equations from Lorentz force, but also from known exact correspondence between commutation relation and Poisson bracket [2][4].

A somewhat unique implication obtained from the above results of Maxwell equations in Quaternion Space, is that it suggests that the DeBroglie wavelength will have quaternionic form. Its further implications, however, are beyond the scope of the present paper.

In the present paper we only discuss derivation of Maxwell equations in Quaternion Space using the decomposition method described by Gersten [4][5]. Further extension to Proca equations in Quaternion Space seems possible too using the same method [5], but it will not be discussed here.

This proposition, however, deserves further theoretical considerations. Further observation is of course recommended in order to refute or verify some implications of this result.

## Acknowledgements

One of the authors (VC) wishes to express his gratitude to Profs. A. Yefremov and M. Fil'chenkov for kind hospitality in the Institute of Gravitation and Cosmology, PFUR. Special thanks also to Prof. V.V. Kassandrov for excellent guide to Maxwell equations, and to Prof. Y.P. Rybakov for discussions on the interpretation of de Broglie's wavelength

Submitted on December 11, 2009 Accepted on January 02, 2010

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# An Exact Mapping from Navier-Stokes Equation to Schrödinger Equation via Riccati Equation 

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#### Abstract

In the present article we argue that it is possible to write down Schrödinger representation of Navier-Stokes equation via Riccati equation. The proposed approach, while differs appreciably from other method such as what is proposed by R. M. Kiehn, has an advantage, i.e. it enables us extend further to quaternionic and biquaternionic version of Navier-Stokes equation, for instance via Kravchenko's and Gibbon's route. Further observation is of course recommended in order to refute or verify this proposition.


## 1 Introduction

In recent years there were some attempts in literature to find out Schrödinger-like representation of Navier-Stokes equation using various approaches, for instance by R. M. Kiehn [1,2]. Deriving exact mapping between Schrödinger equation and Navier-Stokes equation has clear advantage, because Schrodinger equation has known solutions, while exact solution of Navier-Stokes equation completely remains an open problem in mathematical-physics. Considering wide applications of Navier-Stokes equation, including for climatic modelling and prediction (albeit in simplified form called "geostrophic flow" [9]), one can expect that simpler expression of Navier-Stokes equation will be found useful.

In this article we presented an alternative route to derive Schrödinger representation of Navier-Stokes equation via Riccati equation. The proposed approach, while differs appreciably from other method such as what is proposed by R. M. Kiehn [1], has an advantage, i.e. it enables us to extend further to quaternionic and biquaternionic version of NavierStokes equation, in particular via Kravchenko's [3] and Gibbon's route $[4,5]$. An alternative method to describe quaternionic representation in fluid dynamics has been presented by Sprössig [6]. Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

## 2 From Navier-Stokes equation to Schrödinger equation via Riccati

Recently, Argentini [8] argues that it is possible to write down ODE form of 2D steady Navier-Stokes equations, and it will lead to second order equation of Riccati type.

Let $\rho$ the density, $\mu$ the dynamic viscosity, and f the body force per unit volume of fluid. Then the Navier-Stokes equation for the steady flow is [8]:

$$
\begin{equation*}
\rho(v \cdot \nabla v)=-\nabla p+\rho \cdot f+\mu \cdot \Delta v . \tag{1}
\end{equation*}
$$

After some necessary steps, he arrives to an ODE version of 2D Navier-Stokes equations along a streamline [8, p.5] as
follows:

$$
\begin{equation*}
u_{1} \cdot \dot{u}_{1}=f_{1}-\frac{\dot{q}}{\rho}+v \cdot \dot{u}_{1} \tag{2}
\end{equation*}
$$

where $v=\frac{\mu}{\rho}$ is the kinematic viscosity. He [8, p.5] also finds a general exact solution of equation (2) in Riccati form, which can be rewritten as follows:

$$
\begin{equation*}
\dot{u}_{1}-\alpha \cdot u_{1}^{2}+\beta=0 \tag{3}
\end{equation*}
$$

where:

$$
\begin{equation*}
\alpha=\frac{1}{2 v}, \quad \beta=-\frac{1}{v}\left(\frac{\dot{q}}{\rho}-f_{1}\right) s-\frac{c}{v} \tag{4}
\end{equation*}
$$

Interestingly, Kravchenko [3, p. 2] has argued that there is neat link between Schrödinger equation and Riccati equation via simple substitution. Consider a 1 -dimensional static Schrödinger equation:

$$
\begin{equation*}
\ddot{u}+v \cdot u=0 \tag{5}
\end{equation*}
$$

and the associated Riccati equation:

$$
\begin{equation*}
\dot{y}+y^{2}=-v . \tag{6}
\end{equation*}
$$

Then it is clear that equation (5) is related to (6) by the inverted substitution [3]:

$$
\begin{equation*}
y=\frac{\dot{u}}{u} . \tag{7}
\end{equation*}
$$

Therefore, one can expect to use the same method (7) to write down the Schrödinger representation of Navier-Stokes equation. First, we rewrite equation (3) in similar form of equation (6):

$$
\begin{equation*}
\dot{y}_{1}-\alpha \cdot y_{1}^{2}+\beta=0 . \tag{8}
\end{equation*}
$$

By using substitution (7), then we get the Schrödinger equation for this Riccati equation (8):

$$
\begin{equation*}
\ddot{u}-\alpha \beta \cdot u=0 \tag{9}
\end{equation*}
$$

where variable $\alpha$ and $\beta$ are the same with (4). This Schrödinger representation of Navier-Stokes equation is remarkably simple and it also has advantage that now it is possible to generalize it further to quaternionic (ODE) Navier-Stokes
equation via quaternionic Schrödinger equation, for instance using the method described by Gibbon et al. [4, 5].

## 3 An extension to biquaternionic Navier-Stokes equation via biquaternion differential operator

In our preceding paper [10, 12], we use this definition for biquaternion differential operator:
$\diamond=\nabla^{q}+i \nabla^{q}=\left(-i \frac{\partial}{\partial t}+e_{1} \frac{\partial}{\partial x}+e_{2} \frac{\partial}{\partial y}+e_{3} \frac{\partial}{\partial z}\right)+$
$+i\left(-i \frac{\partial}{\partial T}+e_{1} \frac{\partial}{\partial X}+e_{2} \frac{\partial}{\partial Y}+e_{3} \frac{\partial}{\partial Z}\right)$,
where $e_{1}, e_{2}, e_{3}$ are quaternion imaginary units obeying (with ordinary quaternion symbols: $e_{1}=i, e_{2}=j, e_{3}=k$ ): $i^{2}=j^{2}=k^{2}=-1, i j=-j i=k, j k=-k j=i, k i=-i k=j$ and quaternion Nabla operator is defined as [13]:

$$
\begin{equation*}
\nabla^{q}=-i \frac{\partial}{\partial t}+e_{1} \frac{\partial}{\partial x}+e_{2} \frac{\partial}{\partial y}+e_{3} \frac{\partial}{\partial z} \tag{11}
\end{equation*}
$$

(Note that (10) and (11) include partial time-differentiation.)
Now it is possible to use the same method described above [10, 12] to generalize the Schrödinger representation of Navier-Stokes (9) to the biquaternionic Schrödinger equation, as follows.

In order to generalize equation (9) to quaternion version of Navier-Stokes equations (QNSE), we use first quaternion Nabla operator (11), and by noticing that $\Delta \equiv \nabla \nabla$, we get:

$$
\begin{equation*}
\left(\nabla^{q} \bar{\nabla}^{q}+\frac{\partial^{2}}{\partial t^{2}}\right) u-\alpha \beta \cdot u=0 \tag{12}
\end{equation*}
$$

We note that the multiplying factor $\alpha \beta$ in (12) plays similar role just like $V(x)-E$ factor in the standard Schrödinger equation [12]:

$$
\begin{equation*}
-\frac{\hbar^{2}}{2 m}\left(\nabla^{q} \bar{\nabla}^{q}+\frac{\partial^{2}}{\partial t^{2}}\right) u+(V(x)-E) u=0 \tag{13}
\end{equation*}
$$

Note: we shall introduce the second term in order to "neutralize" the partial time-differentiation of $\nabla^{q} \bar{\nabla}^{q}$ operator.

To get biquaternion form of equation (12) we can use our definition in equation (10) rather than (11), so we get [12]:

$$
\begin{equation*}
\left(\diamond \bar{\diamond}+\frac{\partial^{2}}{\partial t^{2}}-i \frac{\partial^{2}}{\partial T^{2}}\right) u-\alpha \beta \cdot u=0 \tag{14}
\end{equation*}
$$

This is an alternative version of biquaternionic Schrödinger representation of Navier-Stokes equations. Numerical solution of the new Navier-Stokes-Schrödinger equation (14) can be performed in the same way with [12] using Maxima software package [7], therefore it will not be discussed here.

We also note here that the route to quaternionize Schrödinger equation here is rather different from what is described by Gibbon et al. [4, 5], where the Schrödinger-equivalent to Euler fluid equation is described as [5, p. 4]:

$$
\begin{equation*}
\frac{D^{2} w}{D t^{2}}-(\nabla Q) w=0 \tag{15}
\end{equation*}
$$

and its quaternion representation is [5, p. 9]:

$$
\begin{equation*}
\frac{D^{2} w}{D t^{2}}-q_{b} \otimes w=0 \tag{16}
\end{equation*}
$$

with Riccati relation is given by:

$$
\begin{equation*}
\frac{D_{a}^{q}}{D t+q_{a} \otimes q_{a}}=q_{b} \tag{17}
\end{equation*}
$$

Nonetheless, further observation is of course recommended in order to refute or verify this proposition (14).

## Acknowledgement

Special thanks to Prof. W. Sprössig for remarks on his paper [6]. VC would like to dedicate the article to Prof. R. M. Kiehn.

Submitted on November 12, 2007
Accepted on November 30, 2007

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# A Study of the Schrödinger-Langevin Equation with PT-Symmetric Periodic Potential and its Application to Deuteron Cluster, and Relation to the SelfOrganized Criticality Phenomena 

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## Introduction

One of the most reported problem related to the CMNS (condensed matter nuclear science, or LENR), is the low probability of Coulomb barrier tunneling. It is supposed by standard physics that tunneling is only possible at high enough energy (by solving Gamow function).

However, a recent study by A. Takahashi (2008, 2009) and experiment by Arata etc. (2008) seem to suggest that it is not impossible to achieve a working experiment to create the CMNS process.

In accordance with Takahashi's EQPET/TSC model [1][2][3], the proposed study will find out some analytical and numerical solutions to the problem of barrier tunneling for cluster deuterium, in particular using Langevin method to solve the time-independent Schrödinger equation. It is hoped that the result can answer some of these mysteries.

One of the results of recent experiments is the lack of signature of D-D reaction as in standard fusion process; this is part of the reason to suggest that D-D fusion doesn't take place [1]. However, Takahashi suggests new possible reaction in the context of cluster deuterium, called 4D fusion [1][2][3], this mechanism seems to enable reaction at low temperature (CMNS). His result (2009) can be summarized as follows:
"The ultimate condensation is possible only when the double Platonic symmetry of 4D/TSC is kept in its dynamic motion. The sufficient increase (super screening) of barrier factor is also only possible as far as the Platonic symmetric 4D/TSC system is kept. Therefore, there should be always 4 deuterons in barrier penetration and fusion process, so that 4 d simultaneous fusion should take place predominantly. The portion of 2D (usual) fusion rate is considered to be negligible."

In this respect it can be noted that there are recent reports suggesting that hydrogen cluster can get reaction at very low temperature, forming the condition of superfluidity [5]. This seems to happen too in the context of Takahashi TSC condensate dynamics. Other study worth mentioning here is one that discussed molecular chessboard dynamics of deuterium [6].

The difference between this proposed study and recent work of Takahashi based on Langevin equation for cluster deuterium is that we focus on solution of Schrödinger-Langevin equation [7][8] with PT-Symmetric periodic potential as we discussed in the preceding paper and its Gamow integral. The particular implications of this study to deuteron cluster will be discussed later.

Another differing part from the previous study is that in this study we will also seek clues on possibility to consider this low probability problem as an example of self-organized criticality phenomena. In other words, the time required before CMNS process can be observed is actually the time required to trigger the critical phenomena. To our present knowledge, this kind of approach has never been studied before, although self-organized criticality related to Schrödinger equation approximation to Burger's turbulence has been discussed in Boldyrev [12]. Nonetheless there is recent study suggesting link between diffusion process and the self-organized criticality phenomena.

The result of this study will be useful to better understanding of anomalous phenomena behind Condensed matter nuclear science.

## Schrödinger-Langevin equation

The Langevin equation is considered as equivalent and therefore has often been used to solve the time-independent Schrodinger, in particular to study molecular dynamics.

Here we only cite the known Langevin equation [7, p. 29]:

$$
\begin{align*}
& d X_{t}=p_{t} \cdot d t  \tag{1}\\
& d p=-\partial_{x} \lambda_{o}\left(X_{t}\right) d t+K p_{t} d t+d W_{t} \sqrt{2 T K} \tag{2}
\end{align*}
$$

Takahashi \& Yabuuchi also used quite similar form of the stochastic non-linear Langevin equation [8] in order to study the dynamics of TSC condensate motion.

## Schrödinger equation with PT-Symmetric periodic potential

Consider a PT-Symmetric potential of the form [9][10]:

$$
\begin{equation*}
V=k_{1} \cdot \sin (b \cdot r), \tag{3}
\end{equation*}
$$

where

$$
\begin{equation*}
b=\frac{|m|}{\sqrt{-i-1}} . \tag{4}
\end{equation*}
$$

Hence, the respective Schrödinger equation with this potential can be written as follows:

$$
\begin{equation*}
\Psi^{\prime \prime}(r)=-k^{2}(r) \cdot \Psi(r) \tag{5}
\end{equation*}
$$

where

$$
\begin{equation*}
k(r)=\frac{2 m}{\hbar^{2}}[E-V(r)]=\frac{2 m}{\hbar^{2}}\left[E-k_{1} \cdot \sin (b \cdot r)\right] \tag{6}
\end{equation*}
$$

For the purpose of finding Gamow function, in area near $x=a$ we can choose linear approximation for Coulomb potential, such that:

$$
\begin{equation*}
V(x)-E=-\alpha(x-a) \tag{7}
\end{equation*}
$$

Substitution to Schrodinger equation yields:

$$
\begin{equation*}
\Psi "+\frac{2 m \alpha}{\hbar^{2}}(x-a) \Psi=0 \tag{8}
\end{equation*}
$$

which can be solved by virtue of Airy function.

## Gamow integral

In principle, the Gamow function can be derived as follows [11]:

$$
\begin{equation*}
\frac{d^{2} y}{d x^{2}}+P(x) y=0 \tag{9}
\end{equation*}
$$

Separating the variables and integrating, yields:

$$
\begin{equation*}
\int \frac{d^{2} y}{y}=\int-P(x) \cdot d x \tag{10}
\end{equation*}
$$

or

$$
\begin{equation*}
\left.y \cdot d y=\exp \left(-\int P(x) \cdot d x\right)+\mathrm{C}\right) \tag{11}
\end{equation*}
$$

To find solution of Gamow function, therefore the integral below must be evaluated:

$$
\begin{equation*}
\gamma=\sqrt{\frac{2 m}{\hbar^{2}}[V(x)-E]} \tag{12}
\end{equation*}
$$

For the purpose of analysis we use the same data from Takahashi's EQPET model [3],[4], i.e. $\mathrm{b}=5.6 \mathrm{fm}$, and $\mathrm{r}_{0}=5 \mathrm{fm}$. Here we assume that $\mathrm{E}=\mathrm{V}_{\mathrm{b}}=0.257 \mathrm{MeV}$. Therefore the integral becomes:

$$
\begin{equation*}
\Gamma=0.218 \sqrt{m} \cdot \int_{r 0}^{b}\left(k_{1} \sin (b r)-0.257\right)^{1 / 2} . d r \tag{13}
\end{equation*}
$$

By setting boundary condition (either one or more of these conditions):
(a) at $\mathrm{r}=0$ then $\mathrm{V}_{\mathrm{o}}=-\mathrm{V}_{\mathrm{b}}-0.257 \mathrm{MeV}$
(b) at $\mathrm{r}=5.6 \mathrm{fm}$ then $\mathrm{V} 1=k_{1} \sin (b r)-0.257=0.257 \mathrm{Mev}$, therefore one can find estimate of m .
(c) Using this procedure solution of the equation (11) can be found.

The interpretation of this Gamow function is the tunneling rate of the fusion reaction of cluster of deuterium (for the given data) corresponding to Takahashi data [4], with the difference that here we consider a PT-symmetric periodic potential.

The numerical study will be performed with standard package like Maxima etc. Some plausible implications in Cosmology modeling will also be discussed.

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Ver. 1, 23 Feb. 2009.

## Numerical Solutions

# A Note on Computer Solution of Wireless Energy Transmit via Magnetic Resonance 

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#### Abstract

In the present article we argue that it is possible to find numerical solution of coupled magnetic resonance equation for describing wireless energy transmit, as discussed recently by Karalis (2006) and Kurs et al. (2007). The proposed approach may be found useful in order to understand the phenomena of magnetic resonance. Further observation is of course recommended in order to refute or verify this proposition.


## 1 Introduction

In recent years there were some new interests in methods to transmit energy without wire. While it has been known for quite a long-time that this method is possible theoretically (since Maxwell and Hertz), until recently only a few researchers consider this method seriously.

For instance, Karalis et al [1] and also Kurs et al. [2] have presented these experiments and reported that efficiency of this method remains low. A plausible way to solve this problem is by better understanding of the mechanism of magnetic resonance [3].

In the present article we argue that it is possible to find numerical solution of coupled magnetic resonance equation for describing wireless energy transmit, as discussed recently by Karalis (2006) and Kurs et al. (2007). The proposed approach may be found useful in order to understand the phenomena of magnetic resonance.

Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

## 2 Numerical solution of coupled-magnetic resonance equation

Recently, Kurs et al. [2] argue that it is possible to represent the physical system behind wireless energy transmit using coupled-mode theory, as follows:

$$
\begin{align*}
a_{m}(t)=\left(i \omega_{m}\right. & \left.-\Gamma_{m}\right) a_{m}(t)+ \\
& +\sum_{n \neq m} i \kappa_{n m} a_{n}(t)-F_{m}(t) . \tag{1}
\end{align*}
$$

The simplified version of equation (1) for the system of two resonant objects is given by Karalis et al. [1, p.2]:

$$
\begin{equation*}
\frac{d a_{1}}{d t}=-i\left(\omega_{1}-i \Gamma_{1}\right) a_{1}+i \kappa a_{2} \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{d a_{2}}{d t}=-i\left(\omega_{2}-i \Gamma_{2}\right) a_{2}+i \kappa a_{1} . \tag{3}
\end{equation*}
$$

These equations can be expressed as linear 1st order ODE as follows:

$$
\begin{equation*}
f^{\prime}(t)=-i \alpha f(t)+i \kappa g(t) \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
g^{\prime}(t)=-i \beta g(t)+i \kappa f(t), \tag{5}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha=\left(\omega_{1}-i \Gamma_{1}\right) \tag{6}
\end{equation*}
$$

and

$$
\begin{equation*}
\beta=\left(\omega_{2}-i \Gamma_{2}\right) \tag{7}
\end{equation*}
$$

Numerical solution of these coupled-ODE equations can be found using Maxima [4] as follows. First we find test when parameters (6) and (7) are set up to be 1 . The solution is:

$$
\begin{aligned}
& \text { (\%i5) 'diff(f(x),x)+\%i*f=\%i*b*g(x); } \\
& \text { (\%o5) 'diff(f(x),x,1)+\%i*f=\%i*b*g(x) } \\
& \text { (\%i6) 'diff(g(x),x)+\%i*g=\%i*b*f(x); } \\
& \text { (\%o6) }{ }^{\prime} \operatorname{diff}(\mathrm{g}(\mathrm{x}), \mathrm{x}, 1)+\% \mathrm{i} * \mathrm{~g}=\% \mathrm{i} * \mathrm{~b} * \mathrm{f}(\mathrm{x}) \\
& \text { (\%i7) desolve([\%o5,\%o6],[f(x),g(x)]); }
\end{aligned}
$$

The solutions for $f(x)$ and $g(x)$ are:

$$
\begin{align*}
f(x)= & \frac{[i g(0) b-i f(x)] \sin (b x)}{b}- \\
& \quad-\frac{[g(x)-f(0) b] \cos (b x)}{b}+\frac{g(x)}{b}  \tag{8}\\
g(x)= & \frac{[i f(0) b-i g(x)] \sin (b x)}{b}- \\
& \quad-\frac{[f(x)-g(0) b] \cos (b x)}{b}+\frac{f(x)}{b} . \tag{9}
\end{align*}
$$

Translated back to our equations (2) and (3), the solutions for $\alpha=\beta=1$ are given by:

$$
\begin{align*}
a_{1}(t)= & \frac{\left[i a_{2}(0) \kappa-i a_{1}\right] \sin (\kappa t)}{\kappa}- \\
& \quad-\frac{\left[a_{2}-a_{1}(0) \kappa\right] \cos (\kappa t)}{\kappa}+\frac{a_{2}}{\kappa} \tag{10}
\end{align*}
$$

$$
\begin{align*}
& f(x)=e^{-(i c-i a) t / 2}\left[\frac{[2 i f(0) c+2 i g(0) b-f(0)(i c-i a)] \sin \left(\frac{\sqrt{c^{2}-2 a c+4 b^{2}+a^{2}}}{2} t\right)}{\sqrt{c^{2}-2 a c+4 b^{2}+a^{2}}}+\right. \\
& \left.+\frac{f(0) \cos \left(\frac{\sqrt{c^{2}-2 a c+4 b^{2}+a^{2}}}{2} t\right)}{\sqrt{c^{2}-2 a c+4 b^{2}+a^{2}}}\right]  \tag{13}\\
& g(x)=e^{-(i c-i a) t / 2}\left[\frac{[2 i f(0) c+2 i g(0) a-g(0)(i c-i a)] \sin \left(\frac{\sqrt{c^{2}-2 a c+4 b^{2}+a^{2}}}{2} t\right)}{\sqrt{c^{2}-2 a c+4 b^{2}+a^{2}}}+\right. \\
& \left.+\frac{g(0) \cos \left(\frac{\sqrt{c^{2}-2 a c+4 b^{2}+a^{2}}}{2} t\right)}{\sqrt{c^{2}-2 a c+4 b^{2}+a^{2}}}\right]  \tag{14}\\
& a_{1}(t)=e^{-(i \beta-i \alpha) t / 2}\left(\frac{\left[2 i a_{1}(0) \beta+2 i a_{2}(0) \kappa-(i \beta-i \alpha) a_{1}\right] \sin \left(\frac{\xi}{2} t\right)}{\xi}-\frac{a_{1}(0) \cos \left(\frac{\xi}{2} t\right)}{\xi}\right)  \tag{15}\\
& a_{2}(t)=e^{-(i \beta-i \alpha) t / 2}\left(\frac{\left[2 i a_{2}(0) \beta+2 i a_{1}(0) \kappa-(i \beta-i \alpha) a_{2}\right] \sin \left(\frac{\xi}{2} t\right)}{\xi}-\frac{a_{2}(0) \cos \left(\frac{\xi}{2} t\right)}{\xi}\right) \tag{16}
\end{align*}
$$

and

$$
\begin{align*}
a_{2}(t)= & \frac{\left[i a_{1}(0) \kappa-i a_{2}\right] \sin (\kappa t)}{\kappa}- \\
& \quad-\frac{\left[a_{1}-a_{2}(0) \kappa\right] \cos (\kappa t)}{\kappa}+\frac{a_{1}}{\kappa} \tag{11}
\end{align*}
$$

Now we will find numerical solution of equations (4) and (5) when $\alpha \neq \beta \neq 1$. Using Maxima [4], we find:

$$
\begin{aligned}
& (\% \mathrm{i} 12))^{\prime} \operatorname{diff}(\mathrm{f}(\mathrm{t}), \mathrm{t})+\% \mathrm{i}^{*} \mathrm{a}^{*} \mathrm{f}(\mathrm{t})=\% \mathrm{i}^{*} \mathrm{~b}^{*} \mathrm{~g}(\mathrm{t}) \\
& (\% \mathrm{o} 12) \quad \operatorname{diff}(\mathrm{f}(\mathrm{t}), \mathrm{t}, 1)+\% \mathrm{i}^{*} \mathrm{a}^{*} \mathrm{f}(\mathrm{t})=\% \mathrm{i}^{*} \mathrm{~b}^{*} \mathrm{~g}(\mathrm{t}) \\
& (\% \mathrm{i} 13))^{\prime} \operatorname{diff}(\mathrm{g}(\mathrm{t}), \mathrm{t})+\% \mathrm{i}^{*} \mathrm{c}^{*} \mathrm{~g}(\mathrm{t})=\% \mathrm{i}^{*} \mathrm{~b}^{*} \mathrm{f}(\mathrm{t}) \\
& (\% \mathrm{o} 13) \\
& \left(\% \operatorname{diff}(\mathrm{~g}(\mathrm{t}), \mathrm{t}, 1)+\% \mathrm{i}^{*} \mathrm{c}^{*} \mathrm{~g}(\mathrm{t})=\% \mathrm{i}^{*} \mathrm{~b}^{*} \mathrm{f}(\mathrm{t})\right. \\
& (\% \mathrm{i} 14) \operatorname{desolve}([\% \mathrm{o} 12, \% \mathrm{o} 13],[\mathrm{f}(\mathrm{t}), \mathrm{g}(\mathrm{t})])
\end{aligned}
$$

and the solution is found to be quite complicated: these are formulae (13) and (14).

Translated back these results into our equations (2) and (3), the solutions are given by (15) and (16), where we can define a new "ratio":

$$
\begin{equation*}
\xi=\sqrt{\beta^{2}-2 \alpha \beta+4 \kappa^{2}+\alpha^{2}} \tag{12}
\end{equation*}
$$

It is perhaps quite interesting to remark here that there is no "distance" effect in these equations.

Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

## Acknowledgment

VC would like to dedicate this article to R.F.F.

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# Numerical Solution of Quantum Cosmological Model Simulating Boson and Fermion Creation 

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#### Abstract

A numerical solution of Wheeler-De Witt equation for a quantum cosmological model simulating boson and fermion creation in the early Universe evolution is presented. This solution is based on a Wheeler-De Witt equation obtained by Krechet, Fil'chenkov, and Shikin, in the framework of quantum geometrodynamics for a Bianchi-I metric.


## 1 Introduction

It is generally aserted that in the early stage of Universe evolution, the quantum phase predominated the era. Therefore there are numerous solutions have been found corresponding to the Wheeler-DeWitt equation which governs this phase [2]. In the present paper we present another numerical solution of Wheeler-De Witt equation for a quantum cosmological model simulating boson and fermion creation in the early Universe evolution for a Bianchi-type I metric [1].

The solution is based on Wheeler-De Witt equation for a Bianchi-I metric obtained by Krechet, Fil'chenkov, and Shikin [1], in the framework of quantum geometrodynamics. Albeit the essence of the solution is quite similar from the solution given in [1] using Bessel function, in the present paper we present numerical result using Maxima. For comparison with other solutions of 1-d hydrogen problem, see [3] and [4].

## 2 Solution of Wheeler-DeWitt equation for boson and fermion creation

In the evolution of the Universe after inflation, a scalar field describing de Sitter vacuum was supposed to decay and its energy is converted into the energy of fermions and heavy vector-particles (the so-called $X$ and $Y$ bosons) [2].

In the framework of quantum geometrodynamics, and for a Bianchi-I metric, the Wheeler-De Witt equation has been obtained by Krechet, Fil'chenkov, and Shikin, which reduces to become (Eq. 23 in [1]):

$$
\begin{equation*}
T^{\prime \prime}-\frac{2 i C}{3 \tau} T^{\prime}-(E-V) T=0 \tag{1}
\end{equation*}
$$

where $T^{\prime \prime}$ and $T^{\prime}$ represent second and first differentiation of $T$ with respect to $r$. The resulting equation appears quite similar to radial 1-dimensional Schrödinger equation for a hydrogen-like atom [3], with the potential energy is given by [1]:

$$
\begin{equation*}
U(r)=\frac{\beta}{\tau}+\frac{\varepsilon_{o}}{\tau^{4 / 3}} \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
E=\frac{8}{3} \kappa\left(\frac{\Lambda}{\kappa}-\frac{M^{2}}{2 \lambda}\right) \tag{3}
\end{equation*}
$$

has here a continuous spectrum.
The solution of equation (1) has been presented in [1] based on modified Bessel function. Its interpretation is that in this quantum cosmological model an initial singularity is absent.

As an alternative to the method presented in [1], the numerical solution can be found using Maxima software package, as follows. All solutions are given in terms of $E$ as constant described by (3).
(a) Condition where $V=0$
'diff(y,r,2)-E*y-(2*\%i*C/3/t)*y=0; ode2(\%o1,y,r);

The result is given by:

$$
\begin{equation*}
y=K 1 \sin (a)+K 2 \cos (a) \tag{5}
\end{equation*}
$$

where:

$$
\begin{equation*}
\mathrm{a}=(\mathrm{r} / \sqrt{3}) \sqrt{-3 \mathrm{E}-2 \mathrm{iC} / \mathrm{t}} . \tag{6}
\end{equation*}
$$

(b) Condition where $V \lessgtr 0$

$$
\begin{align*}
& \text { 'diff }(\mathrm{y}, \mathrm{r}, 2)-\mathrm{E} * \mathrm{y}-(2 * \% \mathrm{i} * \mathrm{C} / 3 / \mathrm{t})^{*} \mathrm{y}-\left(\mathrm{b} / \mathrm{t}+\mathrm{e} / \mathrm{t}^{4 / 3}\right) * \mathrm{y}=0 \\
& \text { ode2(\%o2,y,r); } \tag{7}
\end{align*}
$$

The result is given by:

$$
\begin{equation*}
\mathrm{y}=\mathrm{K} 1 \sin (\mathrm{~d})+\mathrm{K} 2 \cos (\mathrm{~d}), \tag{8}
\end{equation*}
$$

where:

$$
\begin{equation*}
\mathrm{d}=\left(\mathrm{r} /\left(\sqrt{3} \mathrm{t}^{2 / 3}\right) \sqrt{-3 \mathrm{Et}^{4 / 3}-2 \mathrm{iCt}^{1 / 3}-3 \mathrm{e}-3 \mathrm{bt}^{1 / 3}} .\right. \tag{9}
\end{equation*}
$$

As a result, the solution given above looks a bit different compared to the solution obtained in [1] based on the modified Bessel function.

## 3 A few implications

For the purpose of stimulating further discussions, a few implications of the above solution of Wheeler-DeWitt equation (in the form of 1-d Schrödinger equation) are pointed as follows:
(a) Considering that the Schrödinger equation can be used to solve the Casimir effect (see for instance Silva [5], Alvarez \& Mazzitelli [6]), therefore one may expect that there exists some effects of Casimir effect in cosmological scale, in a sense that perhaps quite similar to Unruh radiation which can be derived from the Casimir effective temperature. Interestingly, Anosov [7] has pointed out a plausible deep link between Casimir effect and the fine structure constant by virtue of the entropy of coin-tossing problem. However apparently he did not mention yet another plausible link between the Casimir effective temperature and other phenomena at cosmological scale;
(b) Other implication may be related to the Earth scale effects, considering the fact that Schrödinger equation corresponds to the infinite dimensional Hilbert space. In other words one may expect some effects with respect to Earth eigen oscillation spectrum, which is related to the Earth's inner core interior. This is part of gravitational geophysical effects, as discussed by Grishchuk et al. [8]. Furthermore, this effect may correspond to the so-called Love numbers. Other phenomena related to variation to gravitational field is caused by the Earth inner core oscillation, which yields oscillation period $T \sim 3-7$ hours. Interestingly, a recent report by Cahill [9] based on the Optical fibre gravitational wave detector gave result which suggests oscillation period of around 5hours. Cahill concluded that this observed variation can be attributed to Dynamical 3-space. Nonetheless, the Figure 6 c in [9] may be attributed to Earth inner core oscillation instead. Of course, further experiment can be done to verify which interpretation is more consistent.

## Acknowledgements

One of the authors (VC) wishes to express his gratitude to Profs. A. Yefremov and M. Fil'chenkov for kind hospitality in the Institute of Gravitation and Cosmology, PFUR.

Submitted on February 06, 2009
Accepted on February 23, 2009

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## Academic Freedom

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(Drepturile Omului în Domeniul Ştiinţific)

## Articolul 1: Introducere

Inceputul secolului al 21-lea reflectă mai mult decât oricând în istoria omenirii, rolul adânc şi significant al ştiinţei şi tehnologiei în activităţile umane.

Natura atotpătrunzătoare şi universală a ştiinței şi tehnologiei moderne a dat naştere unei percepţii comune că viitoarele descoperiri importante pot fi făcute, în principal sau în exclusivitate, numai de grupuri mari de cercetare finanțate de guvernări sau de firme mari, care au access la instrumente foate scumpe precum şi la un număr mare de personal de support.

Această percepție comună, este totuşi nerealistă şi contrazice modul adevărat în care sunt făcute descoperirile ştiințifice. Proiecte tehnologice mari şi scumpe, oricât de complexe, sunt numai rezultatul aplicării profundei intuiții științifice a unor grupuri mici de cercetători dedicaţi sau a unor oameni de ştiință solitari, care de multe ori lucrează izolaţi. Un om de ştiință care lucrează singur, este, acum precum şi în viitor, aşa cum a fost şi în trecut, capabil să facă o descoperire, care poate influența substanțial soarta omenirii şi poate schimba faţa întegii planete pe care o locuim pentru aşa de puţin timp.

Descoperirile cele mai importante sunt făcute de persoane care lucreaza ca subalterni în diverse agenții guvernamentale, instituții de învăţământ şi cercetare, sau intreprinderi comerciale. În consecinţă, cercetătorul este foare frecvent forțat sau umbrit de directorii instituţiilor şi firmelor, care, având planuri diferite, caută să controleze şi să aplice descoperirile ştiintifice şi cercetările pentru profit personal sau pentru organizație, sau prestigiu personal.

Recordul istoric al decoperirilor ştiinţifice abundă în cazuri de represiune şi ridiculizare făcute de cei la putere, dar în ultimii ani acestea au fost dezvăluite si corectate de către inexorabilul progres al necesităţii practice şi iluminare intelectuală. Tot aşa de rău arată şi istoria distrugerii şi degradării produse prin plagiarism şi denaturare intenţionată, făcute de necinstiţi, motivaţi de invidie şi lăcomie. Şi aşa este şi azi.

Intenția acestei Declaraţii este să sprijine şi să dezvolte doctrina fundamentală că cercetarea ştiinţifică trebuie să fie

[^7]liberă de influența ascunsă şi făţis represivă a directivelor birocratice, politice, religioase, pecuniare şi, de asemenea, creația științifică este un drept al omului, nu mai mic decât alte drepturi similare şi speranțe disperate care sunt promulgate in acorduri şi legi internaţionale.

Toţi oamenii de ştiință care sunt de acord vor trebui să respecte aceasta Declarație, ca o indicație a solidarităţii cu comunitatea ştiințifică internațională care este preocupată de acest subiect, şi să asigure drepturile cetăţenilor lumii la creație ştiințifică fără amestec, în acordanță cu talentul şi dispoziţia fiecăruia, pentru progresul ştiinței și conform abilităţii lor maxime ca cetățeni decenți într-o lume indecentă, în avantajul Omenirii. Stiinţa şi tehnologia au fost pentru prea multă vreme servanții asupririi.

## Articolul 2: Cine este un cercetător ştiințific

Un cercetător ştiinţific este orice persoană care se preocupă de știință. Orice persoană care colaborează cu un cercetător în dezvoltarea şi propunerea ideilor şi a informaţiilor întrun project sau aplicaţie, este de asemenea un cercetător. Deținerea unor calificări formale nu este o cerință prealabilă pentru ca o persoană să fie un cercetător științific.

## Articolul 3: Unde este produsă ştiinţa

Cercetarea științifică poate să aibă loc oriunde, de exemplu, la locul de muncă, în timpul studiilor, în timpul unui program academic sponsorizat, în grupuri, sau ca o persoană singură acasă făcând o cercetare independentă.

## Articolul 4: Libertatea de a alege o temă de cercetare

Mulți cercetători care lucrează pentru nivele mai avansate de cercetare sau în alte programe de cercetare la instituţii academice, cum sunt universităţile şi facultăţile de studii avansate, sunt descurajaţi, de personalul de conducere academic sau de oficiali din administraţie, de a lucra în domeniul lor preferat de cercetare, şi aceasta nu din lipsa mijloacelor de suport, ci din cauza ierarhiei academice sau a altor oficialităţi, care pur şi simplu nu aprobă o direcţie de cercetare să se dezvolte la potenţialul ei, ca să nu deranjeze dogma convențională, teoriile favorite, sau subvenționarea altor proiecte care ar putea fi discreditate de cercetarea propusă. Autoritatea majorităţii ortodoxe este destul de frecvent invocată ca să stopeze un proiect de cercetare, astfel încât autorităţile şi bugetul să nu fie deranjate. Această practică
comună este o obstrucție deliberată a gândirii libere, este neștiințifică la extrem, şi este criminală. Aceasta nu poate fi tolerată.

Un cercetător care lucrează pentru orice instituţie academică, organizație, sau agenție trebuie să fie complet liber în alegerea unei teme de cercetare şi să fie limitat doar de suportul material şi de expertiza intelectuală care poate fi oferită de instituția academică, organizația, sau agenția respectivă. Dacă un cercetător îşi desfăsoară activitatea lui de cercetare fiind membru al unui grup de cercetători, atunci directorii de cercetare şi liderii grupului îşi vor limita rolul lor doar la capacitatea de recomandare şi consultanță în ceea ce priveşte alegerea unei teme de cercetare relevante de către un cercetător din grup.

## Articolul 5: Libertatea de alegere a metodelor de cercetare

În multe cazuri personalul administrativ sau academic de conducere impune o anumită presiune asupra unor cercetători, care fac parte dintr-un program de cercetare care se desfăşoară într-un mediu academic, ca să-i forţeze să adopte alte metode de cercetare decât acelea alese de ei, motivul fiind nu altul decât o preferință personală, o prejudecată, o procedură instituțională, ordine editorială, ori autoritate colectivă. Această practică, care este destul de răspândită, este o eliminare deliberată a libertăţii de gândire, şi această nu poate fi permisă.

Un cercetator academic sau dintr-o instituţie care nu lucrează pentru profit are dreptul să dezvolte o temă de cercetare în orice mod rezonabil, utilizând orice mijloace rezonabile pe care el le consieră că vor fi cele mai eficiente. Doar cercetătorul însuşi ia decizia finală asupra modului cum cercetarea va fi efectuată.

Dacă un cercetator academic, sau dintr-o instituţie care nu lucrează pentru profit, lucrează ca un membru al unui grup de cercetători academici, sau dintr-o instituţie care nu lucrează pentru profit, conducătorii de proiect şi directorii de cercetare vor avea doar un rol de îndrumători şi consultanți şi nu trebuie în nici un fel să influențeze, să intervină, sau să limiteze metodele de cercetare sau tema de cercetare ale unui cercetător din grup.

## Articolul 6: Libertatea de participare şi colaborare în cercetare

În practicarea ştiinței moderne există un element semnificant de rivalitate instituţională, concomitent cu elemente de invidie personală şi de prezervare a reputaţiei cu orice preţ, indiferent de realităţile ştiințifice. Aceasta de multe ori a condus la faptul că cercetătorii au fost împiedicaţi să nominalizeze asistența colegilor competenți care fac parte din instituții rivale sau alţii care nu au nici o afiliaţie academică. Această practică este de asemenea o obstrucție deliberată a progresului ştiințific.

Dacă un cercetator ştiințific dintr-o instituție care nu lucrează pentru profit cere asistența unui alt cercetător şi dacă acel cercetător este de accord, cercetătorul are libertatea de a invita celălalt cercetător să-i ofere orice asistență, cu condiția ca asistența să fie în cadrul bugetului de cercetare stabilit. Dacă asistența este independentă de buget, cercetătorul are libertatea să angajeze cercetătorul colaborator la discreția lui, făra absolut nici o intervenție din partea nici unei alte persoane.

## Articolul 7: Libertatea de a nu fi de accord în discuţii ştiințifice

Datorită invidiei ascunse şi a intereselor personale, ştiinţa modernă nu apreciază discuții deschise şi nu acceptă în mod categoric pe acei cercetători care pun la îndoială teoriile ortodoxe. Deseori, cercetători cu abilităţi deosebite, care arată deficiențele într-o teorie actuală sau într-o interpretare a datelor, sunt denumiţi excentrici, astfel ca vederile lor să poată fi ignorate cu uşurință. Ei sunt făcuţi de râs în public şi în discuţii personale şi sunt opriţi în mod sistematic de a participa la convenții, seminarii, sau colocvii ştiințifice, astfel ca ideile lor să nu poată să găsească o audiență. Falsificări deliberate ale datelor şi reprezentarea greşită a teoriei sunt acum unelte frecvente ale celor fără scrupule, în eliminarea dovezilor, atât tehnice cât şi istorice. Comitete internaţionale de cercetători rău-intenţionaţi au fost formate şi aceste comitete organizează şi conduc convenţii internaţionale, unde numai cei care sunt de accord cu ei sunt admişi să prezinte lucrări, indiferent de calitatea acestora. Aceste comitete extract sume mari de bani din bugetul public ca să suporte proiectele lor preferate, folosind falsități şi minciuni. Orice obiecțiune la propunerile lor, pe baze ştiințifice, este trecută sub tăcere prin orice mijloace la dispoziţia lor, aşa ca banii să poată să continue să se verse la conturile proiectelor lor şi să le garanteze posturi bine plătite. Cercetătorii care s-au opus au fost daţi afară la cererea acestor comitete, alţii au fost împiedicaţi, de către o rețea de complici corupţi, de a obține posturi academice. În alte situaţii unii au fost daţi afară de la candidatura pentru titluri academice avansate, cum ar fi doctoratul, pentru că şi-au exprimat idei care nu sunt de accord cu teoria la modă, chiar dacă această teorie ortodoxă la modă este în vigoare de multă vreme. Ei ignoră complet faptul fundamental că nici o teorie ştiinţifică nu este definitivă şi inviolabilă, şi prin urmare este deschisă pentru discuții şi re-examinare. De asemenea ei ignoră faptul că un fenomen ar putea să aibă mai multe explicaţii plauzibile, și în mod răutăcios discreditează orice explicaţie care nu este de acord cu opinia ortodoxă, folosind fără nici o restricție argumente neştiințifice să explice opiniile lor părtinitoare.

Toți cercetătorii trebuie să fie liberi să discute cercetările lor şi cercetările altora, fără frica de a fi ridiculizați, fără nici o bază materială, în public sau în discuţii particulare,
sau să fie acuzaţi, criticaţi, nerespectaţi sau discreditaţi în alte feluri, cu afirmaţii nesubstanţiate. Nici un cercetător nu trebuie să fie pus într-o poziţie în care situaţia sau reputaţia lui vor fi riscate, datorită exprimării unei opinii ştiinţifice. Libertatea de exprimare ştiințifică trebuie să fie supremă. Folosirea autorităţii în respingerea unui argument ştiinţific este neştiinţifică şi nu trebuie să fie folosită ca să oprească, să anuleze, să intimideze, să ostracizeze, sau să reducă la tăcere ori să interzică în orice fel un cercetător. Înterzicerea deliberată a faptelor sau argumentelor ştiințifice, fie prin fapte sau prin omitere, şi falsificarea deliberată a datelor, ca să suporte un argument sau ca să discrediteze un punct de vedere opus, este o decepţie ştiinţifică, care poate fi numită crimă ştiințifică. Principiile de evidență trebuie să fie călăuza discuției ştiințifice, fie că acea evidența este fizică sau teoretică sau o combinaţie a lor.

## Articolul 8: Libertatea de a publica rezultate ştiinţifice

O cenzură deplorabilă a articolelor știinţifice a devenit acum practica standard a editorilor multor jurnale de specialitate şi arhive electronice, şi a grupurilor lor de aşa zişi referenți experţi. Referenții sunt, în majoritate, protejaţi prin anonimitate aşa încât un autor nu le poate verifica aşa zisa lor expertiză. Lucrările sunt acum de obicei respinse dacă autorul nu este de accord sau contrazice teorii preferate şi ortodoxia majoritară. Multe lucrări sunt acum respinse în mod automat bazat pe faptul că în bibliografie apare citat un cercetător care nu este în graţiile editorilor, referenţilor, sau al altor cenzori experţi, cu nici un fel de considerație față de conținutul lucrării. Există o listă neagră a cercetătorilor care sunt în opoziție şi această listă este comunicată între conducerile editurilor. Toate acestea duc la o crasă prejudecare şi o represiune greşită împotriva gândirii libere şi trebuie condamnate de comunitatea internațională a cercetătorilor.

Toți cercetătorii trebuie să aibă dreptul să prezinte rezultatele cercetărilor lor științifice, în totalitate sau parțial, la conferințe ştiințifice relevante, şi să le publice în jurnale ştiințifice tipărite, arhive electronice sau in altă media. Cercetătorilor nu trebuie să li se respingă lucrările sau rapoartele lor când sunt prezentate spre publicare în jurnale ştiințifice, arhive electronice, sau in altă media, numai pentru motivul că lucrările lor pun sub semn de întrebare opinia majoritară curentă, este în contradicție cu opiniile unei conduceri editoriale, zdruncină bazele altor proiecte de cercetare prezente sau de viitor ale altor cercetători, este în conflict cu orice dogmă politică sau doctrină religioasă, sau cu opinia personală a cuiva, şi nici un cercetător ştiințific nu trebuie să fie pe lista neagră sau cenzurat şi împiedicat de la publicare de nici o altă persoană. Nici un cercetător științific nu trebuie să blocheze, modifice, sau să se amestece în orice mod la publicarea lucrării unui cercetător deoarece îi sunt promise cadouri sau alte favoruri.

## Artiolul 9: Publicând articole ştiințifice în calitate de co-autor

In cercurile ştiinţifice este un secret bine cunoscut, că mulți co-autori ai lucrărilor de cercetare au foarte puţin sau nimic în comun cu rezultatele prezentate. Mulţi conducători de teze ale studenților, de exemplu, nu au nici o problemă să-şi pună numele pe lucrările candidaților pe care numai formal îi coordonează. În multe cazuri dintre acestea, persoana care de fapt scrie lucrarea are o inteligență superioară celei a coordinatorului. In alte situaţii, din nou, pentru motive de notorietate, reputaţie, bani, prestigiu, şi altele, neparticipanți sunt incluşi în lucrare în calitate de co-autori. Adevăraţii autori ai acestor lucrări pot să obiecteze numai cu riscul de a fi penalizaţi mai târziu într-un mod sau altul, sau chiar riscând să fie excluşi de la candidatura pentru grade superioare de cercetare sau din grupul de cercetare. Mulți au fost de fapt eliminați din aceste motive. Această teribilă practică nu poate fi tolerată. Numai acele persone responsabile pentru cercetare trebuie să fie creditaţi ca autori.

Cercetatorii nu trebuie să invite alte persoane să fie coautori şi nici un cercetător nu ar trebui să admită ca numele lui să fie inclus în calitate de co-autor la o lucrare științifică, dacă nu au avut o contribuție substanțială la cercetarea prezentată în lucrare. Nici un cercetător nu trebuie să se lase forțat de nici un reprezentant al unei instituţii academice, firmă, agenție guvernamentală, sau orice altă persoană să devină co-autor la o lucrare, dacă ei nu au avut o contribuție significantă pentru acea lucrare, şi nici un cercetător nu trebuie să accepte să fie co-autor în schimb pentru pentru cadouri sau alte gratuităţi. Nici o persoană nu trebuie să încurajeze sau să încerce să încurajeze un cercetător, în orice modalitate, să admită ca numele său să fie inclus în calitate de co-autor al unei lucrări științifice pentru care ei nu au adus o contribuție semnificativă.

## Articolul 10: Independența afiliației

Mulţi cercetători sunt angajaţi prin contracte de scurtă durată. Odată cu terminarea contractului se termină şi afiliaţia academică. Este frecventă practica conducerii editurilor ca persoanelor fără afiliație academică sau comercială să nu li se publice lucrările. Când cercetătorul nu este afiliat, el nu are resurse şi deci are oportunităţi reduse să participe şi să prezinte lucrări la conferințe. Aceasta este o practică vicioasă care trebuie stopată. Ştiința nu recunoaşte afiliaţie.

Nici un cercetător nu trebuie să fie împiedicat de la prezentarea de lucrări la conferințe, colocvii sau seminarii, de la publicarea în orice media, de la acces la biblioteci academice sau publicaţii ştiințifice, de la participarea la şedințe academice, sau de la prezentarea de prelegeri, din cauză că nu are o afiliere cu instituţii academice, institute de cercetare, laboratoare guvernamentale sau comericale, sau cu orice altă organizaţie.

## Articolul 11: Acces deschis la informația ştiinţifică

Multe cărți ştiințifice de specialitate şi multe jurnale ştiinţifice au un profit mic sau nici un profit, de aceea editorii refuză să le publice fără o contribuţie monetară de la institțiile academice, agenții guvernamentale, fundații filantropice, şi altele. În aceste circumstanțe editorii ar trebui să dea acces liber la versiunile electronice ale publicațiilor, şi să se străduiască să menţină costul pentru tipărirea materialului la minim.

Toți cercetătorii trebuie să se străduiască să se asigure ca lucrările lor să fie gratuite şi accesibile la comunitatea ştiințifică internaţională, sau, dacă nu este posibil, la un preţ modest. Toți cercetătorii trebuie să ia măsuri active ca să ofere cărțile lor tehnice la cel mai mic preț posibil, pentru ca informația ştiințifică să devină accesibilă marii comunităţi ştiințifice internaționale.

## Articolul 12: Responsabiltatea eticǎ a cercetătorilor

Istoria este martoră că descoperirile ştiințifice sunt folosite în ambele direcţii, bune şi rele, pentru binele unora şi pentru distrugerea altora. Deoarece progresul ştiinței şi tehnologiei nu poate fi oprit, trebuie să avem metode de control asupra applicațiilor rău făcătoare. Doar guvernele alese democratic, eliberate de religie, de rasism şi alte prejudicii, pot să protejeze civilizaţia. Doar guvernele, tribunalele şi comitetele alese democratic pot proteja dreptul la o creaţie ştiințifică liberă. Astăzi, diferite state nedemocratice şi regime totalitare performă o activă cercetare în fizica nucleară, chimie, virologie, inginerie genetică, etc. ca să producă arme nucleare, chimice şi biologice. Nici un cercetător nu trebebuie să colaboreze voluntar cu state nedemocratice sau regime totalitare. Orice cercetător forțat să lucreze în crearea de arme pentru astfel de state trebuie să găsească mijloace de a încetini progresul programelor de cercetare şi să reducă rezultatele ştiinţifice, astfel încât civilizaţia şi democraţia în cele din urmă să triumfe.

Toți cercetătorii au o responsabilitate morală pentru descoperirile şi rezultatele lor ştiințifice. Nici un cercetător să nu se angajeze de bună voie în proiectarea sau construcția a nici unui fel de armament pentru state cu regimuri nedemocratice sau totalitare sau să accepte ca talentele şi cunoştiințele lor să fie aplicate în crearea de arme care vor conduce la distrugerea Omenirii. Un cercetător ştiințific trebuie să trăiască aplicând dictonul că toate guvernele nedemocratice şi violarea drepturilor umane sunt crime.

14 martie, 2007

# Déclaration de la Liberté Académique 

## (Les Droits de l'Homme dans le Domaine Scientifique)

## Article 1: Préambule

Le début du 21ème siècle reflète, plus qu'aucun autre temps de l'histoire, la profondeur et l'importance de la science et la technologie dans les affaires humaines.

La nature puissante et influente de la science et la technologie modernes a fait naître une perception commune voulant que les prochaines grandes découvertes ne peuvent être faites principalement ou entièrement que par des groupes de recherche qui sont financés par des gouvernements ou des sociétés et ont accès à une instrumentation dispendieuse et à des hordes de personnel de soutien.

Cette perception est cependant mythique et donne une fausse idée de la façon dont des découvertes scientifiques sont faites. Les grands et coûteux projets technologiques, aussi complexes qu'ils soient, ne sont que le résultat de l'application de la perspicacité des petits groupes de recherche ou d'individus dévoués, travaillant souvent seuls ou séparément. Un scientifique travaillant seul est, maintenant et dans le futur, comme dans le passé, capable de faire une découverte qui pourrait influencer le destin de l'humanité.

Les découvertes les plus importantes sont généralement faites par des individus qui sont dans des positions subalternes au sein des organismes gouvernementaux, des établissements de recherche et d'enseignement, ou des entreprises commerciales. Par conséquent, le rechercheur est trop souvent restraint par les directeurs d'établissements ou de la société, qui ont des ambitions différentes, et veulent contrôler et appliquer les découvertes et la recherche pour leur bienêtre personnel, leur agrandissement, ou pour le bien-être de leur organisation.

L'histoire est remplie d'exemples de suppression et de ridicule par l'établissement. Pourtant, plus tard, ceux-ci ont été exposés et corrigés par la marche inexorable de la nécessité pratique et de l'éclaicissement intellectuel. Tristement, la science est encore marquée par la souillure du plagiat et l'altération délibérée des faits par les sans-scrupules qui sont motivés par l'envie et la cupidité; cette pratique existe encore aujourd'hui.

L'intention de cette Déclaration est de confirmer et pro-

[^8]mouvoir la doctrine fondamentale de la recherche scientifique; la recherche doit être exempte d'influences suppressive, latente et manifeste, de directives bureaucratiques, politiques, religieuses et pécuniaires. La création scientifique doit être un droit de l'homme, tout comme les droits et espérances tels que proposés dans les engagements internationaux et le droit international.

Tous les scientifiques doivent respecter cette Déclaration comme étant signe de la solidarité dans la communauté scientifique internationale. Ils défendront les droits à la création scientifique libre, selon leurs différentes qualifications, pour l'avancement de la science et, à leur plus grande capacité en tant que citoyens honnêtes dans un monde malhonnête, pour permettre un épanouissement humain. La science et la technologie ont été pendant trop longtemps victimes de l'oppression.

## Article 2: Qu'est-ce qu'un scientifique

Un scientifique est une personne qui travaille en science. Toute personne qui collabore avec un scientifique en développant et en proposant des idées et des informations dans la recherche, ou son application, est également un scientifique. Une formation scientifique formelle n'est pas un prérequis afin d'être un scientifique.

## Article 3: Le domaine de la science

La recherche scientifique existe n'importe où, par exemple, au lieu de travail, pendant un cours d'éducation formel, pendant un programme universitaire commandité, dans un groupe, ou en tant qu'individu à sa maison conduisant une recherche indépendante.

## Article 4: Liberté du choix du thème de recherche

Plusieurs scientifiques qui travaillent dans des échelons plus élevés de recherche tels que les établissements académiques, les universités et les institutions, sont empêchés de choisir leurs sujets de recherche par l'administration universitaire, les scientifiques plus haut-placés ou par des fonctionnaires administratifs. Ceci n'est pas par manque d'équipements, mais parce que la hiérarchie académique et/ou d'autres fonctionnaires n'approuvent pas du sujet d'une enquête qui pourrait déranger le dogme traditionnel, les théories favorisées, ou influencer négativement d'autres projets déjà proposés. L'autorité plutôt traditionnelle est souvent suscitée pour
faire échouer un projet de recherche afin de ne pas déranger l'autorité et les budgets. Cette pratique commune est une obstruction délibérée à la science, ainsi que la pensée scientifique et démontre un élément anti-scientifique à l'extrême; ces actions sont criminelles et ne peuvent pas être tolérées.

Un scientifique dans n'importe quel établissement académique, institution ou agence, doit être complètement libre quant au choix d'un thème de recherche. Il peut être limité seulement par l'appui matériel et les qualifications intellectuelles offertes par l'établissement éducatif, l'agence ou l'institution. Quand un scientifique effectue de la recherche collaborative, les directeurs de recherche et les chefs d'équipe seront limités aux rôles de consultation et de recommandation par rapport au choix d'un thème approprié pour un scientifique dans leur groupe.

## Article 5: Liberté de choisir ses méthodes et ses techniques de recherche

Souvent les scientifiques sont forcés par le personnel administratif ou académique à adopter des méthodes de recherches contraires à celles que le scientifique préfère. Cette pression exercée sur un scientifique contre son gré est à cause de la préférence personnelle, le préjugé, la politique institutionnelle, les préceptes éditoriaux, ou même l'autorité collective. Cette pratique répandue va à l'encontre la liberté de pensée et ne peut pas être permise ni toléreé.

Un scientifique travaillant à l'extérieur du secteur commercial doit avoir le droit de développer un thème de recherche de n'importe quelle manière et moyens raisonnables qu'il considère les plus efficaces. La décision finale sur la façon dont la recherche sera executée demeure celle du scientifique lui-même.

Quand un scientifique travaille en collaboration, il doit avoir l'indépendance de choisir son thème et ses méthodes de recherche, tandis que les chefs de projets et les directeurs auront seulement des droits de consultatition et de recommandation, sans influencer, atténuer ou contraindre les méthodes de recherches ou le thème de recherche d'un scientifique de leur groupe.

## Article 6: Liberté de participation et de collaboration en recherche

La rivalité entre les différentes institutions dans la science moderne, la jalousie personnelle et le désir de protéger sa réputation à tout prix empêchent l'entraide parmi des scientifiques qui sont aussi compétents les uns que les autres mais qui travaillent dans des établissements rivaux. Un scientifique doit avoir recours à ses collègues dans un autre centre de recherche.

Quand un premier scientifique qui n'a aucune affiliation commerciale a besoin de l'aide et qu'il invite un autre scientifique, ce deuxième est libre d'accepter d'aider le premier
si l'aide demeure à l'intérieur du budget déjà établi. Si l'aide n'est pas dépendante des considérations budgetaires, le premier scientifique a la liberté d'engager le deuxième à sa discrétion sans l'interposition des autres. Le scientifique pourra ainsi rémunérer le deuxième s'il le désire, et cette décision demeure à sa discrétion.

## Article 7: Liberté du désaccord dans la discussion scientifique

À cause de la jalousie et des intérêts personnels, la science moderne ne permet pas de discussion ouverte et bannit obstinément ces scientifiques qui remettent en cause les positions conventionnelles. Certains scientifiques de capacité exceptionnelle qui précisent des lacunes dans la théorie ou l'interprétation courante des données sont étiquetés comme cinglés, afin que leurs opinions puissent être facilement ignorées. Ils sont raillés en public et en privé et sont systématiquement empêchés de participer aux congrès scientifiques, aux conférences et aux colloques scientifiques, de sorte que leurs idées ne puissent pas trouver une audience. La falsification délibérée des données et la présentation falsifiée des théories sont maintenant les moyens utlilisés habituellement par les sans-scrupules dans l'étouffement des faits, soit techniques soit historiques. Des comités internationaux de mécréants scientifiques ont été formés et ces mêmes comités accueillent et dirigent des conventions internationales auxquelles seulement leurs acolytes sont autorisés à présenter des articles sans tenir compte de la qualité du travail. Ces comités amassent de grandes sommes d'argent de la bourse publique et placent en premier leurs projets commandités et fondés par la déception et le mensonge. N'importe quelle objection à leurs propositions, pour protéger l'intégrité scientifique, est réduite au silence par tous leur moyens, de sorte que l'argent puisse continuer à combler leurs comptes et leur garantir des emplois bien payés. Les scientifiques qui s'y opposent se font renvoyer à leur demande; d'autres ont été empêchés de trouver des positions académiques par ce réseau de complices corrompus. Dans d'autres situations certains ont vu leur candidature expulsée des programmes d‘études plus élevés, tels que le doctorat, après avoir ébranlé une théorie à la mode, même si une théorie plus conventionnelle existe depuis plus longtemps. Le fait fondamental qu'aucune théorie scientifique est ni définitive ni inviolable, et doit être ré-ouverte, dicutée et ré-examinée, ils l'ignorent complètement. Souvent ils ignorent le fait qu'un phénomène peut avoir plusieurs explications plausibles, et critiquent avec malveillance n'importe quelle explication qui ne s'accorde pas avec leur opinion. Leur seul recours est l'utilisation d'arguments non scientifiques pour justifier leurs avis biaisés.

Tous les scientifiques seront libres de discuter de leur recherche et la recherche des autres sans crainte d'être ridiculisés, sans fondement matériel, en public ou en privé, et sans êtres accusés, dénigrés, contestés ou autrement critiqués
par des allégations non fondées. Aucun scientifique ne sera mis dans une position dans laquelle sa vie ou sa réputation sera en danger, dû à l'expression de son opinion scientifique. La liberté d'expression scientifique sera primordiale. L'autorité ne sera pas employée dans la réfutation d'un argument scientifique pour bâillonner, réprimer, intimider, ostraciser, ou autrement pour contraindre un scientifique à l'obéissance ou lui faire obstacle. La suppression délibérée des faits ou des arguments scientifiques, par acte volontaire ou par omission, ainsi que la modification délibérée des données pour soutenir un argument ou pour critiquer l'opposition constitue une fraude scientifique qui s'élève jusqu'à un crime scientifique. Les principes de l'évidence guideront toutes discussions scientifiques, que cette évidence soit concrète, théorique ou une combinaison des deux.

## Article 8: Liberté de publier des résultats scientifiques

La censure déplorable des publications scientifiques est maintenant devenue la norme des bureaux de rédaction, des journaux et des archives électroniques, et leurs bandes de soit-dits arbitres qui prétent être experts. Les arbitres sont protégés par l'anonymat, de sorte qu'un auteur ne puisse pas vérifier l'expertise prétendue. Des publications sont maintenant rejetées si l'auteur contredit, ou est en désaccord avec, la théorie préférée et la convention la plus acceptée. Plusieurs publications sont rejetées automatiquement parce qu'il y a un des auteurs dans la liste qui n'a pas trouvé faveur avec les rédacteurs, les arbitres, ou d'autres censureurs experts, sans respect quelconque pour le contenu du document. Les scientifiques discordants sont mis sur une liste noire et cette liste est communiquée entre les bureaux de rédaction des participants. Cet effet culmine en un penchant biaisé et une suppression volontaire de la libre pensée, et doit être condamné par la communauté scientifique internationale.

Tous les scientifiques doivent avoir le droit de présenter leurs résultats de recherche, en entier ou en partie, aux congrès scientifiques appropriés, et d'éditer ceux-ci dans les journaux scientifiques, les archives électroniques, et tous les autres médias. Aucun scientifique ne se fera rejeter ses publications ou rapports quand ils seront soumis pour publication dans des journaux scientifiques, des archives électroniques, ou d'autres médias, simplement parce que leur travail met en question l'opinion populaire de la majorité, fait conflit avec les opinions d'un membre de rédaction, contredit les prémisses de bases d'autres recherche ou futurs projets de recherche prévus par d'autres scientifiques, sont en conflit avec quelque sorte de dogme politique, religieuse, ou l'opinion personnelle des autres. Aucun scientifique ne sera mis sur une liste noire, ou sera autrement censuré pour empêcher une publication par quiconque. Aucun scientifique ne bloquera, modifiera, ou interfèrera autrement avec la publication du travail d'un scientifique sachant qu'il aura des faveurs ou bénifices en le faisant.

## Article 9: Les publications à co-auteurs

C'est un secret mal gardé parmi les scientifiques que beaucoup de co-auteurs de publications ont réellement peu, ou même rien, en rapport avec la recherche présentée. Les dirigeants de recherche des étudiants diplômés, par exemple, préfèrent leurs noms inclus avec celui des étudiants sous leur surveillance. Dans de tels cas, c'est l'élève diplômé qui a une capacité intellectuelle supérieure à son dirigeant. Dans d'autres situations, pour des fins de notoriété et de réputation, d'argent, de prestige et d'autres raisons malhonnêtes, des personnes qui n'ont rien contribué sont incluses en tant que co-auteurs. Les vrais auteurs peuvent s'y opposer, mais seront pénalisés plus tard d'une manière quelconque, voir même l'expulsion de leur candidature pour un diplôme plus élevé, ou une mise à pied d'une équipe de recherche. C'est un vécu réel de plusieurs co-auteurs dans ces circonstances. Cette pratique effroyable ne doit pas être tolérée. Pour maintenir l'intégrité de la science, seulement les personnes chargées de la recherche devraient être reconnues en tant qu'auteurs.

Aucun scientifique n'invitera quiconque n'a pas collaboré avec lui à être inclus en tant que co-auteur, de même, aucun scientifique ne permettera que son nom soit inclus comme coauteur d'une publication scientifique sans y avoir contribué de manière significative. Aucun scientifique ne se laissera contraindre par les représentants d'un établissement académique, par une société, un organisme gouvernemental, ou qui que ce soit à inclure leur nom comme co-auteur d'une recherche s'il n'y a pas contribué de manière significative. Un scientifique n'acceptera pas d'être co-auteur en échange de faveurs ou de bénéfices malhonnêtes. Aucune personne ne forcera un scientifique d'aucune manière à mettre son nom en tant que co-auteur d'une publication si le scientifique n'y a pas contribué de manière significative.

## Article 10: L'indépendance de l'affiliation

Puisque des scientifiques travaillent souvent à contrats à court terme, quand le contrat est terminé, l'affiliation académique du scientifique est aussi terminée. C'est souvent la politique des bureaux de rédaction que ceux sans affiliation académique ou commerciale ne peuvent pas être publiés. Sans affiliation, beaucoup de ressources ne sont pas disponibles au scientifiques, aussi les occasions de présenter des entretiens et des publications aux congrès sont réduites. Cette pratique vicieuse doit être arrêtée. La science se déroule indépendamment de toutes affiliations.

Aucun scientifique ne sera empêché de présenter des publications aux congrès, aux colloques ou aux séminaires; un scientifique pourra publier dans tous les médias, aura accès aux bibliothèques académiques ou aux publications scientifiques, pourra assister à des réunions scientifiques, donner des conférences, et ceci même sans affiliation avec un établissement académique, un institut scientifique, un
laboratoire gouvernemental ou commercial ou tout autre organisation.

## Article 11: L'accès à l'information scientifique

La plupart des livres de science et les journaux scientifiques ne font pas de profits, donc les éditeurs sont peu disposés à les éditer sans une contribution financière des établissements académiques, des organismes gouvernementaux, des fondations philanthropiques et leur semblables. Dans ces cas, les éditeurs commerciaux doivent permettre le libre accès aux versions électroniques des publications et viser à garder le coût d'imprimerie à un minimum.

Les scientifiques s'efforceront d'assurer la disponibilité de leurs ouvrages à la communauté internationale gratuitement, ou à un coût minimum. Tous les scientifiques doivent faire en sorte que les livres de techniques soient disponibles à un coût minimum pour que l'information scientifique puisse être disponible à une plus grande communauté scientifique internationale.

## Article 12: La responsabilité morale des scientifiques

L'histoire a démontré que des découvertes scientifiques sont parfois utilisées à des fins extrèmes, soit bonnes, soit mauvaises, au profit de certains et à la ruine des autres. Puisque l'avancement de la science et de la technologie continue toujours, des moyens d'empêcher son application malveillante doivent être établis. Puisqu'un gouvernement élu de manière démocratique, sans biais religieux, racial ou autres biais peut sauvegarder la civilisation, ainsi seulement le gouvernement, les tribunaux et les comités élu de manière démocratique peuvent sauvegarder le droit de la création scientifique libre et intègre. Aujourd'hui, divers états anti-démocratiques et régimes totalitaires font de la recherche active en physique nucléaire, en chimie, en virologie, en génétique, etc. afin de produire des armes nucléaires, chimiques ou biologiques. Aucun scientifique ne devrait volontairement collaborer avec les états anti-démocratiques ou les régimes totalitaires. Un scientifique qui est contraint à travailler au développement des armes pour de tels états doit trouver des moyens pour ralentir le progrès de cette recherche et réduire son rendement, de sorte que la civilisation et la démocratie puissent finalement régner.

Tous les scientifiques ont la responsabilité morale de leurs créations et découvertes. Aucun scientifique ne prendra volontairement part dans les ébauches ou la construction d'armes pour des états anti-démocratiques et/ou des régimes totalitaires, et n'appliquera ni ses connaissances ni son talent au développement d'armes nuisibles à l'humanité. Un scientifique suivra le maxime que tous les gouvernements antidémocratiques et l'abus des droits de l'homme sont des crimes.

Le 10 avril, 2007


[^0]:    *Elleipein - "to fall short"; hyperballein - "to throw beyond" (Greek).

[^1]:    ${ }^{\dagger}$ Here is a space of $n$ dimensions.
    ${ }^{\ddagger}$ Landau and Lifshitz in The Classical Theory of Fields [4] use the signature $(-+++)$, where the three-dimensional part of the four-dimensional impulse vector is real. We, following Eddington [5], use the signature (+---), because in this case the three-dimensional observable impulse, being the projection of the four-dimensional impulse vector on an observer's spatial section, is real. Here $\alpha, \beta=0,1,2,3$, while $i, k=1,2,3$.

[^2]:    *Because we use the signature (+---).

[^3]:    ${ }^{\dagger}$ Note, $d s^{2}=0$ is true not only at $c^{2} d \tau^{2}=d \sigma^{2}=0$, but also when $c^{2} d \tau^{2}=d \sigma^{2} \neq 0$ (in the isotropic region, where light propagates). The properly observed time interval is determined as $d \tau=\sqrt{g_{00}} d t+$ $+\frac{g_{0 i}}{c \sqrt{g 00}} d x^{i}$, where the coordinate time interval is $d t \neq 0[4,5,6,7]$.

[^4]:    ${ }^{*}$ Here $u^{i}=d x^{i} / d t$ is so-called the coordinate velocity.

[^5]:    ${ }^{1}$ Published in Bulletin of Pure and Applied Sciences, Vol. 23D, No. 2, 139-140, 2004.

[^6]:    *At the time of writing, we are informed that Arbab's forthcoming paper will discuss a more comprehensive and theoretical approach of his hypothesis [15]. Our remarks here are limited to his papers discussed in this issue, and also in his earler paper [16].

[^7]:    *Original text published in English: Progress in Physics, 2006, v. 1, 57-60. Online - http://www.ptep-online.com/

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[^8]:    *Original text published in English: Progress in Physics, 2006, v. 1, 57-60. Online - http://www.ptep-online.com/

    Le texte originaire en anglais par Dmitri Rabounski, rédacteur en chef de la revue Progress in Physics. E-mail: rabounski@yahoo.com

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