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Neutrosophic Linear Programming Problem

Abdel-Nasser Hussian[1], Mai Mohamed[2], Mohamed Abdel-Baset[2,] and Florentin Smarandache[3]*

[1]Department of Computer Science, Faculty of Computers and Informatics, Zagazig University, Egypt.
[2]Department of Operations Research, Faculty of Computers and Informatics, Zagazig University, Sharqiyah, Egypt.
[3]Math & Science Department, University of New Mexico, Gallup, NM 87301, USA.

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Abstract: Smarandache presented neutrosophic theory as a tool for handling undetermined information, and together with Wang et al. introduced single valued neutrosophic sets that is a special neutrosophic set and can be used expediently to deal with real-world problems, especially in decision support. In this paper, we propose linear programming problems based on neutrosophic environment. Neutrosophic sets characterized by three independent parameters, namely truth-membership degree (T), indeterminacy-membership degree (I) and falsity-membership degree (F), which is more capable to handle imprecise parameters. We also transform the neutrosophic linear programming problem into a crisp programming model by using neutrosophic set parameters. To measure the efficiency of our proposed model we solved several numerical examples.

Keywords: linear programming problem; neutrosophic; neutrosophic sets.

1 Introduction

Linear programming is a method for achieving the best outcome (such as maximum profit or minimum cost) in a mathematical model represented by linear relationships. Decision making is a process of solving the problem and achieving goals under asset of constraints, and it is very difficult in some cases due to incomplete and imprecise information. And in Linear programming problems the decision maker may not be able to specify the objective function and/or constraints functions precisely. In 1995, Smarandache [5-7] introduce neutrosophy which is the study of neutralities as an extension of dialectics. Neutrosophic is the derivative of neutrosophy and it includes neutrosophic set, neutrosophic probability, neutrosophic statistics and neutrosophic logic. Neutrosophic theory means neutrosophy applied in many fields of sciences, in order to solve problems related to indeterminacy. Although intuitionistic fuzzy sets can only handle incomplete information not indeterminate, the neutrosophic set can handle both incomplete and indeterminate information.[2,5-7] Neutrosophic sets characterized by three independent degrees namely truth-membership degree (T), indeterminacy-membership degree(I), and falsity-membership degree (F), where T,I,F are standard or non-standard subsets of *]-0, 1+[*. The decision makers in neutrosophic set want to increase the degree of truth-membership and decrease the degree of indeterminacy and falsity membership.

The structure of the paper is as follows: the next section is a preliminary discussion; the third section describes the formulation of linear programing problem using the proposed model; the fourth section presents some illustrative examples to put on view how the approach can be applied; The last section summarizes the conclusions and gives an outlook for future research.

2 Some Preliminaries

2.1 Neutrosophic Set [2]

Let X be a space of points (objects) and $x\in X$. A neutrosophic set A in X is defined by a truth-membership function (x), an indeterminacy-membership function (x) and a falsity-membership function (x). $T(x)$, $I_A(x)$ and $F_A(x)$ are real standard or real nonstandard subsets of $]0-,1+[$.That is $T_A(x)$:$X\rightarrow]0-,1+[$, $I_A(x)$:$X\rightarrow]0-,1+[$ and $F_A(x)$:$X\rightarrow]0-,1+[$. There is no restriction on the sum of (x), (x) and (x), so

$$0-\leq \sup(T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3+.$$

In the following, we adopt the notations $\mu(x)$, $\sigma_A(x)$ and $\nu_A(x)$ instead of $T_A(x)$, $I_A(x)$ and $F_A(x)$, respectively. Also we write SVN numbers instead of single valued neutrosophic numbers.

*Corresponding author E-mail: analyst_mohamed@yahoo.com

2.2 Single Valued Neutrosophic Sets (SVNS)[2,7]

Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form

$$A=\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x\in X\}$$

where $\mu_A(x):X\to[0,1]$, $\sigma_A(x):X\to[0,1]$ and $\nu_A(x):X\to[0,1]$ with $0\le\mu_A(x)+\sigma_A(x)+\nu_A(x)\le 3$ for all $x\in X$. The intervals $\mu(x)$, $\sigma_A(x)$ and $\nu_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively.

For convenience, a SVN number is denoted by $A=(a,,)$, where $a,b,c\in[0,1]$ and $a+b+c\le 3$.

2.3 Complement [3]

The complement of a single valued neutrosophic set A is denoted by $_C(A)$ and is defined by

$$T_c(A)(x) = F(A)(x)\,,$$
$$I_c(A)(x) = 1 - I(A)(x)\,,$$
$$F_c(A)(x) = T(A)(x),$$
$$\textit{for all } x \textit{ in } X$$

2.4 Union [3]

The union of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as C = AUB, whose truth-membership, indeterminacy membership and falsity-membership functions are given by

$$T(C)(x) = max\,(\,T(A)(x)\,, T(B)(x)\,)\,,$$
$$I(C)(x) = max\,(\,I(A)(x)\,, I(B)(x)\,)\,,$$
$$F(C)(x) = \;min(\,(A)(x)\,, F(B)(x)\,),$$
$$\textit{for all } x \textit{ in } X.$$

2.5 Intersection [3]

The intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as C = A∩B, whose truth-membership, indeterminacy membership and falsity-membership functions are given by

$$T(C)(x) = min\,(\,T(A)(x)\,, T(B)(x)\,)\,,$$

$$I(C)(x) = min\,(\,I(A)(x)\,, I(B)(x)\,)\,,$$

$$F(C)(x) = \;max(\,(A)(x)\,, F(B)(x)\,)\;\; \textit{for all } x \textit{ in } X.$$

3 Neutrosophic Linear Programming Problem

Linear programming problem with neutrosophic coefficients (NLPP) is defined as the following:

Maximize Z= $\sum_{j=1}^{n} c_j x_j$

Subject to

$$\sum_{j=1}^{n} \tilde{a}_{ij}^{n} x_j \le b_i \qquad 1\le i\le m \qquad (1)$$

$$x_j \ge 0, \qquad\qquad 1\le j\le n$$

where a_{ij}^{n} *is a neutrosophic number.*

The single valued neutrosophic number (a_{ij}^{n}) is given by A=(a,b,c) where a,b,c ∈[0,1] and a+b+c ≤ 3
The truth- membership function of *neutrosophic number* a_{ij}^{n} is defined as:

$$T\,a_{ij}^{n}(\mathrm{x})=\begin{cases}\frac{x-a_1}{a_2-a_1} & a_1\le x\le a_2\\ \frac{a_2-x}{a_3-a_2} & a_2\le x\le a_3\\ 0 & otherwise\end{cases} \qquad (2)$$

The indeterminacy- membership function of *neutrosophic number* a_{ij}^{n} is defined as:

$$I\,a_{ij}^{n}(\mathrm{x})=\begin{cases}\frac{x-b_1}{b_2-b_1} & b_1\le x\le b_2\\ \frac{b_2-x}{b_3-b_2} & b_2\le x\le b_3\\ 0 & otherwise\end{cases} \qquad (3)$$

And its falsity- membership function of *neutrosophic number* $\tilde{a_{ij}}^{n}$ is defined as:

$$F\,a_{ij}^{n}(\mathrm{x})=\begin{cases}\frac{x-C_1}{C_2-C_1} & C_1\le x\le C_2\\ \frac{c_2-x}{c_3-c_2} & C_2\le x\le C_3\\ 1 & otherwise\end{cases} \qquad (4)$$

Then we find the upper and lower bounds of the objective function for truth-membership, indeterminacy and falsity membership as follows:

$z_U^T = \max\{z(x_i^*)\}$ and $z_l^T = \min\{z(x_i^*)\}$ where $1\le i\le k$

$$z_{L=}^F z_L^T \text{ and } z_{u=}^F z_u^T - R(z_u^T - z_L^T)$$

$z_{U=}^I z_U^I$ *and* $z_{l=}^I z_{l=}^I - S(z_u^T - z_L^T)$

Where R ,S are predetermined real number in (0,1)

The truth membership, indeterminacy membership, falsity membership of objective function as follows:

$$T_O^{(Z)} = \begin{cases}1 & if\;\; z\ge z_u^T\\ \frac{z-z_L^T}{z_u^T-z_L^T} & if\;\; z_L^T\le z\le z_u^T\\ 0 & if\; z\;< z_L^T\end{cases} \qquad (5)$$

$$I_O^{(Z)} = \begin{cases}1 & if\; z\ge z_u^T\\ \frac{z-z_L^I}{z_u^I-z_L^I} & if\;\; z_L^T\le z\le z_u^T\\ 0 & if\; z\;< z_L^T\end{cases} \qquad (6)$$

$$F_O^{(Z)} = \begin{cases}1 & if\; z\ge z_u^T\\ \frac{z_u^F-z}{z_u^F-z_L^F} & if\;\; z_L^T\le z\le z_u^T\\ 0 & if\; z\;< z_L^T\end{cases} \qquad (7)$$

The neutrosophic set of the i^{th} constraint c_i is defined as:

$$T_{c_i}^{(x)} = \begin{cases} 1 & if \quad b_i \geq \sum_{j=1}^{n}(a_{ij}+d_{ij})x_j \\ \frac{b_i-\sum_{j=1}^{n} a_{ij\,x_j}}{\sum_{j=1}^{n} d_{ij\,x_j}} & if \quad \sum_{j=1}^{n} a_{ij\,x_j} \leq b_i < \sum_{j=1}^{n}(a_{ij}+d_{ij})x_j \\ 0 & if \quad b_i < \sum_{j=1}^{n} a_{ij\,x_j} \end{cases} \quad (8)$$

$$F_{c_i}^{(x)} = \begin{cases} 1 & if \quad b_i < \sum_{j=1}^{n} a_{ij\,x_j} \\ 1-T_{c_i}^{(x)} & if \quad \sum_{j=1}^{n} a_{ij\,x_j} \leq b_i < \sum_{j=1}^{n}(a_{ij}+d_{ij})x_j \\ 0 & if \; b_i \geq \sum_{j=1}^{n}(a_{ij}+d_{ij})x_j \end{cases} \quad (9)$$

$$I_{c_i}^{(x)} = \begin{cases} 0 \quad if \quad b_i < \sum_{j=1}^{n} a_{ij\,x_j} \\ \frac{b_i-\sum_{j=1}^{n} d_{ij\,x_j}}{\sum_{j=1}^{n} a_{ij\,x_j}} \; If \quad \sum_{j=1}^{n} a_{ij\,x_j} \leq b_i < \sum_{j=1}^{n}(a_{ij}+d_{ij})x_j \\ 0 \; if \; b_i \geq \sum_{j=1}^{n}(a_{ij}+d_{ij})x_j \end{cases} \quad (10)$$

4 Neutrosophic Optimization Model

In our neutrosophic model we want to maximize the degree of acceptance and minimize the degree of rejection and indeterminacy of the neutrosophic objective function and constraints. Neutrosophic optimization model can be defined as:

$$maxT_{(x)}$$
$$minF_{(x)}$$
$$minI_{(x)}$$

Subject to

$$\begin{gathered} T_{(X)} \geq F_{(x)} \\ T_{(X)} \geq I_{(x)} \\ 0 \leq T_{(X)} + I_{(x)} + F_{(x)} \leq 3 \\ T_{(X)}, \quad I_{(X)}, \quad F_{(X)} \geq 0 \\ x \geq 0 \end{gathered} \quad (11)$$

Where $T_{(x)}.F_{(x)}, I_{(x)}$ denotes the degree of acceptance, rejection and indeterminacy of x respectively.

The above problem is equivalent to the following:

$max\ \alpha,\ min\ \beta\ , min\ \theta$

Subject to

$$\begin{gathered} \alpha \leq \mathrm{T(x)} \\ \beta \leq \mathrm{F(x)} \\ \theta \leq \mathrm{I(x)} \\ \alpha \geq \beta \\ \alpha \geq \theta \\ 0 \leq \alpha + \beta + \theta \leq 3 \\ \mathrm{X} \geq 0 \end{gathered} \quad (12)$$

Where α denotes the minimal acceptable degree, β denote the maximal degree of rejection and θ denote maximal degree of indeterminacy.

The neutrosophic optimization model can be changed into the following optimization model:

$max(\alpha - \beta - \theta)$

Subject to

$$\begin{gathered} \alpha \leq \mathrm{T(x)} \\ \beta \geq \mathrm{F(x)} \\ \theta \geq \mathrm{I(x)} \\ \alpha \geq \beta \\ \alpha \geq \theta \\ 0 \leq \alpha + \beta + \theta \leq 3 \\ \alpha, \beta, \theta \geq 0 \\ x \geq 0 \end{gathered} \quad (13)$$

The previous model can be written as:

min (1- $\alpha)\beta\theta$

Subject to

$$\begin{gathered} \alpha \leq \mathrm{T(x)} \\ \beta \geq \mathrm{F(x)} \\ \theta \geq \mathrm{I(x)} \\ \alpha \geq \beta \\ \alpha \geq \theta \\ 0 \leq \alpha + \beta + \theta \leq 3 \\ x \geq 0 \end{gathered} \quad (14)$$

5 The Algorithm for Solving Neutrosophic Linear Programming Problem (NLPP)

Step 1: Solve the objective function subject to the constraints.

Step 2: Create the decision set which include the highest degree of truth-membership and the least degree of falsity and indeterminacy memberships.

Step 3: Declare goals and tolerance.

Step 4: Construct membership functions.

Step 5: Set α, β, θ in the interval*]-0, 1+[* for each neutrosophic number .

Step 6: Find the upper and lower bound of objective function as we illustrated previously in section 3.

Step 7: Construct neutrosophic optimization model as in equation (13).

6 Numerical Examples

To measure the efficiency of our proposed model we solved four numerical examples.

6.1 Illustrative Example#1

$$\max \tilde{5}x_1 + \tilde{3}x_2$$
$$s.t.$$
$$\tilde{4}x_1 + \tilde{3}x_2 \le \tilde{12} \qquad (15)$$
$$\tilde{1}x_1 + \tilde{3}x_2 \le \tilde{6}$$
$$x_1, x_2 \ge 0$$

where

$c_1 = \tilde{5} = \{(4,5,6),(0.5,0.8,0.3)\}$;

$c_2 = \tilde{3} = \{(2.5,3,3.2),(0.6,0.4,0)\}$;

$a_{11} = \tilde{4} = \{(3.5,4,4.1),(0.75,0.5,0.25)\}$;

$a_{12} = \tilde{3} = \{(2.5,3,3.2),(0.2,0.8,0.4)\}$;

$a_{21} = \tilde{1} = \{(0,1,2),(0.15,0.5,0)\}$;

$a_{22} = \tilde{3} = \{(2.8,3,3.2),(0.75,0.5,0.25)\}$;

$b_1 = \tilde{12} = \{(11,12,13),(0.2,0.6,0.5)\}$;

$b_2 = \tilde{6} = \{(5.5,6,7.5),(0.8,0.6,0.4)\}$.

The equivalent crisp formulation is:

$$\max\ 1.3125x_1 + 0.0158x_2$$
$$s.t.$$
$$2.5375x_1 + 0.54375x_2 \le 2.475$$
$$0.3093x_1 + 1.125x_2 \le 2.1375$$
$$x_1, x_1 \ge 0$$

The optimal solution is $x_1 = 0.9754$; $x_2 = 0$; with optimal objective value 1.2802

6.2 Illustrative Example#2

$$\max \tilde{25}x_1 + \tilde{48}x_2$$
$$s.t.$$
$$15x_1 + 30x_2 \le 45000$$
$$24x_1 + 6x_2 \le 24000$$
$$21x_1 + 14x_2 \le 28000$$
$$x_1, x_2 \ge 0 \qquad (16)$$

where

$c_1 = \tilde{25} = \{(19,25,33),(0.8,0.1,0.4)\}$;

$c_2 = \tilde{48} = \{(44,48,54),(0.75,0.25,0)\}$.

The corresponding crisp linear programs given as follows:

$$\max\ 11.069x_1 + 22.8125x_2$$
$$s.t.$$
$$15x_1 + 30x_2 \le 45000$$
$$24x_1 + 6x_2 \le 24000$$
$$x_1, x_1 \ge 0$$

The optimal solution is $x_1 = 0$; $x_2 = 1500$; with optimal objective value 34218.75

6.3 Illustrative Example#3

$$\max 25x_1 + 48x_2$$
$$s.t.$$
$$\tilde{15}x_1 + \tilde{30}x_2 \le \tilde{45000}$$
$$\tilde{24}x_1 + \tilde{6}x_2 \le \tilde{24000}$$
$$\tilde{21}x_1 + \tilde{14}x_2 \le \tilde{28000}$$
$$x_1, x_2 \ge 0$$

where (17)

$a_{11} = \tilde{15} = \{(14,15,17),(0.75,0.5,0.25)\}$;

$a_{12} = \tilde{30} = \{(25,30,34),(0.25,0.7,0.4)\}$;

$a_{21} = \tilde{24} = \{(21,24,26),(0.4,0.6,0)\}$;

$a_{22} = \tilde{6} = \{(4,6,8),(0.75,0.5,0.25)\}$;

$a_{31} = \tilde{21} = \{(17,21,22),(1,0.25,0)\}$;

$a_{32} = \tilde{14} = \{(12,14,19),(0.6,0.4,0)\}$;

$b_1 = \tilde{45000} = \{(44980,45000,45030),(0.3,0.4,0.8)$;

$b2 = \tilde{24000} = \{(23980,24000,24050),(0.4,0.25,0.5)\}$;

$b_3 = \tilde{28000} = \{(27990,28000,28030),(0.9,0.2,0)\}$.

The associated crisp linear programs model will be:

$$\max\ 25x_1 + 48x_2$$
$$s.t.$$
$$5.75x_1 + 6.397x_2 \le 9282$$
$$10.312x_1 + 6.187x_2 \le 14178.37$$

$x_1, x_1 \geq 0$

The optimal solution is $x_1 = 0$; $x_2 = 1450.993$; with optimal objective value 69647.65

6.4 Illustrative Example#4

$$
\begin{aligned}
&\max 7x_1 + 5x_2 \\
&s.t. \\
&\tilde{1}x_1 + \tilde{2}x_2 \leq 6 \\
&\tilde{4}x_1 + \tilde{3}x_2 \leq 12 \\
&x_1, x_2 \geq 0
\end{aligned} \tag{18}
$$

where

$a_{11} = \tilde{1} = \{(0.5,1,2),(0.2, 0.6,0.3)\}$;

$a_{12} = \tilde{2} = \{(2.5,3,3.2),(0.6,0.4,0.1)\}$;

$a_{21} = \tilde{4} = \{(3.5,4,4.1),(0.5,0.25,0.25)\}$;

$a_{22} = \tilde{3} = \{(2.5,3,3.2),(0.75,0.25,0)\}$;

The associated crisp linear programs model will be:

$$
\begin{gathered}
max\ 7x_1 + 5x_2 \\
\text{s.t.} \\
0.284x_1 + 1.142x_2 \leq 6 \\
1.45x_1 + 1.36x_2 \leq 12 \\
x_1, x_1 \geq 0
\end{gathered}
$$

The optimal solution is $x_1 = 4.3665$; $x_2 = 4.168$; with optimal objective value 63.91

The result of our NLP model in this example is better than the results obtained by intuitionistic fuzzy set [4].

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7 Conclusion and Future Work

Neutrosophic sets and fuzzy sets are two hot research topics. In this paper, we propose linear programming model based on neutrosophic environment, simultaneously considering the degrees of acceptance, indeterminacy and rejection of objectives, by proposed model for solving neutrosophic linear programming problems (NlPP). In the proposed model, we maximize the degrees of acceptance and minimize indeterminacy and rejection of objectives. NlPP was transformed into a crisp programming model using truth membership, indeterminacy membership, and falsity membership functions. We also give a numerical examples to show the efficiency of the proposed method. As far as future directions are concerned, these will include studying the duality theory of linear programming problems based on neutrosophic environment.

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Abdel Nasser H. Zaied, is prof. of Information Systems, Dean, Faculty of Computers and Informatics, Zagazig University, Egypt. He previously worked as an associate professor of Industrial Engineering, Zagazig University Egypt, an assistant professor of Technology Management, Arabian Gulf University, Bahrain; and as visiting professor at Oakland University, USA. He supervised 12 PhD. thesis and 45 MSc. thesis, and examined 8 PhD. thesis and 47 MSc thesis. He published 30 research papers in International and Regional Journals and 22 research papers in International and National conferences. His areas of research are: Systems Analysis and Design; Information Security; Knowledge Management; Quality Management Systems, Information Security and project Management, Electronic

applications.

Mai Mohamed , received her BS degree and master degree from Zagazig University ,faculty of computers and informatics, Egypt . she is currently research interest is computation intelligence, neural networks, and Neutrosophic logic .

Mohamed Abdel-Baset Received his B.Sc., M.Sc and the Ph.D in information technology from Zagazig University. He is a lecturer in the operations Research Department, Faculty of Computers and Informatics, Zagazig University.

His current research interests are Optimization, Operations Research, Data Mining, Computational Intelligence, Applied Statistics and Decision support systems. He is also a reviewer in different international journals and conferences. He has published more than 100 articles in international journals and conference proceedings.

Florentin Smarandache polymath, professor of mathematics

University of New Mexico, 705 Gurley Ave., Gallup, New Mexico 87301, USA.

http://fs.gallup.unm.edu/