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Neutrosophic Probability, Set, And Logic (first version)

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NEUTROSOPHIC PROBABILITY, SET, AND LOGIC

(first version)

Abstract.

This project is a part of a National Science Foundation interdisciplinary project proposal. Starting from a new viewpoint in philosophy, the neutrosophy, one extends the classical "probability theory", "fuzzy set" and "fuzzy logic" to <neutrosophic probability>, <neutrosophic set> and <neutrosophic logic> respectively.

They are useful în artificial intelligence, neural networks, evolutionary programming, neutrosophic dynamic systems, and quantum mechanics.

1) NEUTROSOPHY, A NEW BRANCH OF MATHEMATICAL PHILOSOPHY

A) **Etymology**:

Neutro-sophy [French *neutre* < Latin *neuter*, neutral, and *Greek sophia*, skill/wisdom] means knowledge of neutral thought.

B) **Definition**:

Neutrosophy is a new branch of philosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

C) **Characteristics**:

This mode of thinking:

- proposes new philosophical theses, principles, laws, methods, formulas, movements;
- interprets the uninterpretable;
- regards, from many different angles, old concepts, systems: showing that an idea, which is true in a given referential system, may be false in another one, and vice versa;
- measures the stability of unstable systems, and instability of stable systems.

D) **Methods of Neutrosophic Study**:

mathematization (neutrosophic logic, neutrosophic probability and statistics, duality), generalization, complementarity, contradiction, paradox, tautology, analogy, reinterpretation, combination, interference, aphoristic, linguistic, multidisciplinarity.

E) **Formalization**:

Let's note by <A> an idea or theory or concept, by <Non-A> what is not <A>, and by <Anti-A> the opposite of <A>. Also, <Neut-A> means what is neither <A>, nor <Anti-A>, i.e. neutrality in between the two extremes. And <A'> a version of <A>.

<Non-A> is different from <Anti-A>.

For example:

If <A> = white, then <Anti-A> = black (antonym),

but <Non-A> = green, red, blue, yellow, black, etc. (any color, except white), while <Neut-A> = green, red, blue, yellow, etc. (any color, except white and black), and <A'> = dark white, etc. (any shade of white).

<Neut-A> $\equiv$ <Neut-(Anti-A)>, neutralities of <A> are identical with neutralities of <Anti-A>.

<Non-A> $\supset$ <Anti-A>, and <Non-A> $\supset$ <Neut-A> as well,
also

<A> $\cap$ <Anti-A> = $\varnothing$

<A> $\cap$ <Non-A> = $\varnothing$

<A>, <Neut-A>, and <Anti-A> are disjoint two by two.
<Non-A> is the completitude of <A> with respect to the universal set.

F) **Main Principle**:

Between an idea <A> and its opposite <Anti-A>, there is a continuum-power spectrum of neutralities <Neut-A>.

G) **Fundamental Thesis**:

Any idea <A> is t% true, i% indeterminate, and f% false, where t+i+f = 100.

H) **Main Laws**:

Let <α> be an attribute, and $(a, i, b) \in [0, 100]^3$, with $a+i+b = 100$. Then:

- There is a proposition <P> and a referential system <R>,
such that <P> is a% <α>, i% indeterminate or <Neut-α>, and b% <Anti-α>.

- For any proposition <P>, there is a referential system <R>, such that <P> is a% <α>, i% indeterminate or <Neut-α>, and b% <Anti-α>.
- <α> is at some degree <Anti-α>, while <Anti-α> is at some degree <α>.

2) NEOTROSOPHIC PROBABILITY AND NEUTROSOPHIC STATISTICS

Let's first generalize the classical notions of "probability" and "statistics" for practical reasons.

A) Definitions:

Neutrosophic Probability studies the chance that a particular event E will occur, where that chance is represented by three coordinates (variables): t% true, i% indeterminate, and f% false, with $t+i+f = 100$ and $f, i, t \in [0, 100]$.

Neutrosophic Statistics is the analysis of such events.

B) Neutrosophic Probabi l ity Space:

The universal set, endowed with a neutrosophic probability defined for each of its subset, forms a neutrosophic probability space.

C) Applications:

1) The probability that candidate C will win an election is say 25% true (percent of people voting for him), 35% false (percent of people voting against him), and 40% indeterminate (percent of people not coming to the ballot box, or giving a blank vote - not selecting anyone, or giving a negative vote - cutting all candidates on the list).

Dialectic and dualism don't work in this case anymore.

2) Another example, the probability that tomorrow it will rain is say 50% true according to meteorologists who have investigated the past years' weather, 30% false according to today's very sunny and droughty summer, and 20% undecided (indeterminate).

3) NEUTROSOPHIC SET

Let's second generalize, in the same way, the fuzzy set.

A) Definition:

Neutrosophic Set is a set such that an element belongs to the set with a neutrosophic probability, i.e. t% is true that the element is in the set, f% false, and i% indeterminate.

B) **Neutrosophsc Set Operations**:

Let M and N be two neutrosophic sets.

One can say, by language abuse, that any element neutrosophically belongs to any set, due to the percentage of truth/indeterminacy/falsity which varies between 0 and 100.

For example: $x(50,20,30) \in M$ (which means, with a probability of 50% x is in M, with a probability of 30% x is not in M, and the rest is undecidable) $y(0,0,100) \in M$ (which normally means y is not for sure in M), or $z(0,100,0) \in M$ (which means one doesn't know absolutely anything about z's affiliation with M).

Let $0 \leq t_1, t_2, t' \leq 1$ represent the truth-probabilities, $0 \leq i_1, i_2, i' \leq 1$ the indeterminacy-probabilities, and $0 \leq f_1, f_2, f' \leq 1$ the falsity-probabilities of an element x to be in the set M and in the set N respectively, and of an element y to be in the set N, where $t_1 + i_1 + f_1 = 1$, $t_2 + i_2 + f_2 = 1$, and $t' + i' + f' = 1$.

One notes, with respect to the given sets,

$x = x(t_1, i_1, f_1) \in M$ and $x = x(t_2, i_2, f_2) \in N$,

by mentioning x's neutrosophic probability appurtenance.

And, similarly, $y = y(t', i', f') \in N$.

Also, for any $0 \leq x \leq 1$ one notes $1\text{-}x = x^{-}$.

Let $W(a,b,c) = (1\text{-}a) / (b+c)$ and $W(R) = W(R(t), R(i), R(f))$ for any tridimensional vector $R = (R(t), R(i), R(f))$.

Complement of M:

Let $N(x) = 1\text{-}x = x^{-}$. Therefore:

if $x(t_1, i_1, f_1) \in M$,

then $x(N(t_1), N(i_1)W(N), N(f_1)W(N)) \in C(M)$.

Intersection:

Let $C(x,y) = xy$, and $C(z_1,z_2) = C(z)$ for any bidimensional vector

$z = (z_1, z_2)$. Therefore:
if $x(t_1, i_1, f_1) \in M$, $x(t_2, i_2, f_2) \in N$,
then $x(C(t), C(i)W(C), C(f)W(C)) \in M \cap N$.

Union:

Let $D1(x,y) = x+y-xy = x+\bar{x}y = y+x\bar{y}$, and $D1(z_1,z_2) = D1(z)$ for any bidimensional vector $z = (z_1, z_2)$. Therefore:
if $x(t_1, i_1, f_1) \in M$, $x(t_2, i_2, f_2) \in N$,
then $x(Dl(t), Dl(i)W(Dl), D1(f)W(D1)) \in M \cup N$.

Cartesian Product:

if $x(t_1, i_1, f_1) \in M$, $y(t', i', f') \in N$,
then $(x(t_1, i_1, f_1), y(t', 1', f')) \in M \times N$.

Difference:

Let $D(x,y) = x-xy = x\bar{y}$, and $D(z_1,z_2) = D(z)$ for any bidimensional vector $z = (z_1, z_2)$. Therefore:
if $x(t_1, i_1, f_1) \in M$, $x(t_2, i_2, f_2) \in N$, then $x(D(t), D(i)W(D), D(f)W(D)) \in M \setminus N$, because $M \setminus N = M \cap C(N)$.

C) <u>**Applications**</u>:

From a pool of refugees, waiting in a political refugee camp to get the America visa of emigration, a% are accepted, r% rejected, and p% in pending (not yet decided), a+r+p=100. The chance of someone in the pool to emigrate to USA is not a% as in classical probability, but a% true and p% pending (therefore normally bigger than a%) - because later, the p% pending refugees will be distributed into the first two categories, either accepted or rejected.

Another example, a cloud is a neutrosophic set, because its borders are ambiguous, and each element (water drop) belongs with a neutrosophic probability to the set (i.e. there are separated water drops, around a compact mass of water drops, that we don't know how to consider them: in or out of the cloud).

We are not sure where the cloud ends nor where it begins, neither if some elements are or are not in the set. That's why the percent of indeterminacy is required: for a more organic, smooth, and especially accurate estimation.

4) NEUTROSOPHIC LOGIC, A GENERALIZATION OF FUZZY LOGIC

A) **Introduction**:

One passes from the classical {0, 1} Bivalent Logic of George Boole, to the Three-Valued Logic of Reichenbach (leader of the logical empiricism), then to the $\{0, a_1, ..., a_n, 1\}$ Plurivalent one of £ukasiewicz (and Post's m-valued calculus), and finally to the [0, 1] Infinite Logic as in mathematical analysis and probability: a Transcendental Logic (with values of the power of continuum), or Fuzzy Logic.

Falsehood is infinite, and truthhood quite alike; in between, at different degrees, indeterminacy as well.

Everything is G% good, I% indeterminate, and B% bad,
where G + I + B = 100.

Besides Diderot's dialectics on good and bad ("Rameau's Nephew", 1772), any act has its percentage of "good", "indeterminate", and of "bad" as well incorporated.

Rodolph Carnap said:

"Metaphysical propositions are neither true nor false, because they assert nothing, they contain neither knowledge nor error (...)". Hence, there are infinitely many statuses in between "Good" and "Bad", and generally speaking in between "A" and "Anti-A", like on the real number segment:

[0,	1]
False	True
Bad	Good
Non-sense	Sense
Anti-A	A

0 is the absolute falsity, 1 the absolute truth. In between each oppositing pair, normally in a vicinity of 0.5, are being set up the neutralities.

There exist as many states in between "True" and "False" as in between "Good" and "Bad". Irrational and transcendental standpoints belong to this interval.

Even if an act apparently looks to be only good, or only bad, the other haded side should be sought. The ratios

$$\frac{\text{Anti-A}}{\text{A}}, \qquad \frac{\text{Non-A}}{\text{A}}$$

vary indefinitely. They are transfinite.

If a statement is 30%T (true) and 6.0%I (indeterminate), then it is 10%F (false). This is somehow alethic, meaning pertaining to truthhood and falsehood in the same time.

In opposition to Fuzzy Logic, if a statement is 30%T doesn't involve it is 70%F. We have to study its indeterminacy as well.

B) **Definition of Neutrosophic Logic**:

This is a generalization (for the case of null indeterminacy) of the fuzzy logic.

Neutrosophic logic is useful in the real-world systems for designing control logic, and may work in quantum mechanics.

If a proposition P is t% true, doesn't necessarily mean it is 100-t% false as in fuzzy logic. There should also be a percent of indeterminacy on the values of P.

A better approach of the logical value of P is f% false, i% indeterminate, and t% true, where $t+i+f=100$ and $t, i, f \in [0, 100]$, called neutrosophic logical value of P, and noted by $n(P) = (t,i,f)$.

Neutrosophic Logic means the study of neutrosophic logical values of the propositions.

There exist, for each individual event, PRO parameters, CONTRA parameters, and NEUTER parameters which influence the above values. Indeterminacy results from any hazard which may occur, from unknown parameters, or from new arising conditions.

This resulted from practice.

C) **Aplications**:

1) The candidate C, who runs for election in a metropolis M of p people with right to vote, will win.

This proposition is, say, 25% true (percent of people voting for him), 35% false (percent of people voting against him), and 40% indeterminate (percent of people not coming to the ballot box, or giving a blank vote - not selecting anyone, or giving a negative vote - cutting all candidates on the list).

2) Tomorrow it will rain.

This proposition is, say, 50% true according to meteorologists who have investigated the past years' weather, 30% false according to today's very sunny and droughty summer, and 20% undecided.

3) This is a heap.

As an application to the sorites paradoxes, we may now say this proposition is t% true, f% false, and i% indeterminate (the neutrality comes for we don't know exactly where is the difference between a heap and a non-heap; and, if we approximate the border, our 'accuracy' is subjective).

We are not able to distinguish the difference between yellow and red as well if a continuum spectrum of colors is painted on a wall imperceptibly changing from one into another.

D) Definition of Neutrosophic Logical Connectors:

One uses the definitions of neutrosophic probability and neutrosophic set.

Let , $0 \le t_1, t_2 \le 1$ represent the thrut-probabilities,

$0 \le i_1, i_2 \le 1$ the indeterminacy-probabilities, and

$0 \le f_1, f_2 \le 1$ the falsity-probabilities of two events P_1 and P_2 respectively, where $t_1 + i_1 + f_1 = 1$ and $t_2 + i_2 + f_2 = 1$. One notes the neutrosophic logical values of P_1 and P_2 by

$n(P_1) = (t_1, i_1, f_1)$ and $n(P_2) = (t_2, i_2, f_2)$ ·

Also, for any $0 \le x \le 1$ one notes $1\text{-}x = x$.

Let $W(a,b,c) = (1\text{-}a) / (b+c)$ and $W(R) = W(R(t), R(i), R(f))$ for any tridimensional vector $R = (R(t), R(i), R(f))$.

Negation:

Let $N(x) = 1\text{-}x = x$. Then:

$n(\neg P_1) = (N(t_1), N(i_1)W(N), N(f_1)W(N))$.

Conjunction: Let $C(x,y) = xy$, and $C(z_1,z_2) = C(z)$ for any bidimensional vector $z = (z_1, z_2)$. Then:

$n(P_1 \wedge P_2) = (C(t), C(i)W(C), C(f)W(C))$.

(And, in a similar way, generalized for n propositions.)

Weak or inclusive disjunction:

Let $D1(x,y) = x+y\text{-}xy = x+\bar{x}y = y+x\bar{y}$, and $D1(z_1,z_2) = D1(z)$ for any bidimensional vector $z = (z_1,z_2)$. Then:

$n(P_1 \vee P_2) = (D1(t), D1(i)W(D1), D1(f)W(D1))$.
(And, in a similar way, generalized for n propositions.)

Strong or exclusive disjunction:
Let $D2(x,y) = x(1-y)+y(1-x)-xy(1-x)(1-y) = x\bar{y}+\bar{x}y-xy\bar{x}\bar{y}$, and $D2(z_1,z_2) = D2(z)$ for any bidimensional vector $z = (z_1, z_2)$. Then:
$n(P1 \underline{\vee} P2) = (D2(t), D2(i)W(D2), D2(f)W(D2))$.
(And, in a similar way, generalized for n propositions.)

Material conditional (implication):
Let $I(x,y) = 1-x+xy = \bar{x}+xy = 1-x\bar{y}$, and $I(z_1,z_2) = I(z)$ for any bidimensional vector $z = (z_1,z_2)$. Then:
$n(P_1 \rightarrow P_2) = (I(t), I(i)W(I), I(f)W(I))$.

Material biconditional (echivalence):
Let $E(x,y) = (1-x+xy)(1-y+xy) = (\bar{x}+xy)(\bar{y}+xy) = (1-x\bar{y})(1-\bar{x}y)$, and $E(z_1,z_2) = E(z)$ for any bidimensional vector $z = (z_1,z_2)$.
$n(P \leftrightarrow Q) = (E(t), E(i)W(E), E(f)W(E))$.

Sheffer's connector:
Let $S(x,y) = 1-xy$, and $S(z_1,z_2) = S(z)$ for any bidemensional vector $z = (z_1,z_2)$.
$n(P \mid Q) = n(\neg P \vee \neg Q) = (S(t), S(i)W(S), S(f)W(S))$.

Peirce's connector:
Let $P(x,y) = (1-x)(1-y) = \bar{x}\bar{y}$, and $P(z_1,z_2) = P(z)$ for any bidimensional vector $z = (z_1,z_2)$.
$n(P \downarrow Q) = n(\neg P \wedge \neg Q) = (P(t), P(i)W(P), P(f)W(P))$.

E) **Properties of Neutrosophic Logical Connectors**:

Let's note by $t(P)$ the truth-component of the neutrosophic value $n(P)$, and $t(P) = p$, $t(Q) = q$.

a) Conjunction:
$t(P \wedge Q) \min\leq \{p,q\}$.

$$\bigwedge_{k=1}^{\infty} t(P) = 0 \text{ if } t(P) \neq 1$$

b) Weak disjunction:
$t(P \vee Q) \geq \max\{p,q\}$.

$$\bigvee_{k=1}^{\infty} t(P) = 1 \text{ if } t(P) \neq 0$$

c) Implication:
$t(P \to P) = 1$ if $t(P) = 0$ or 1, and $\rangle$ p otherwise.

$$\lim_{t(P)\to 0} t(P \to Q) = 1$$

$$\lim_{t(Q)\to 1} t(P \to Q) = 1$$

$$\lim_{t(P)\to 1} t(P \to Q) = q$$

$$\lim_{t(Q)\to 0} t(P \to Q) = 1\text{-}p$$

d) Equivalence:
$t(P \leftrightarrow Q) = t(Q \leftrightarrow P) = t(\neg P \leftrightarrow \neg Q)$

$$\lim_{\substack{t(P)\to 0 \\ t(Q)\to 0}} t(P \leftrightarrow Q) = 1$$

$$\lim_{\substack{t(P)\to 1 \\ t(Q)\to 1}} t(P \leftrightarrow Q) = 1$$

$$\lim_{\substack{t(P)\to 0\\ t(Q)\to 1}} t(P \leftrightarrow Q) = 0$$

$$\lim_{\substack{t(P)\to 1\\ t(Q)\to 0}} t(P \leftrightarrow Q) = 0$$

$$\lim_{t(P)\to 0} t(P \leftrightarrow Q) = 1-q$$

$$\lim_{t(P)\to 1} t(P \leftrightarrow Q) = q$$

Let $q \neq 0, 1$ be constant, and one notes

$P_{max}(q) = (q^2-3q+1)/(2q^2-2q)$. Then:

$\max\limits_{0 \le t(P) \le 1} t(P \leftrightarrow Q)$ occurs when:

$P = P_{max}(q)$ if $p_{max}(q) \in [0, 1]$,
or $p = 0$ if $p_{max}(q) < 0$,
or $p = 1$ if $p_{max}(q) > 1$,
because the equivalence connector is described by a parabola of equation

$e_q(p) = (q^2-q)p^2 + (-q^2+3q-1)p + (1-q)$,
which is concave down.

5) NEOTROSOPHIC TOPOLOGY

A) <u>**Definition**</u>:

Let's construct a **Neutrosophic Topology** on NT = [0, 1], considering the associated family of subsets (0, p), for $0 \le p \le 1$, the whole set [0 , 1], and the empty set Ø = (0 , 0), called open sets, which is closed under set union and finite intersection. The union is defined as $(0, p) \cup (0, q) = (0, d)$, where $d = p+q-pq$, and the intersection as $(0, p) \cap (0, q) = (0, c)$, where $c = pq$. The complementary of (0, p) is (0, n), where $n = 1-p$, which is a closed set.

B) **Neutrosophic Topological Space**:

The interval NT, endowed with this topology, forms a neutrosophic topological space.

C) **Isomorphicity**:

Neutrosophic Logical Space, Neutrosophic Topological Space, and Neutrosophic Probability Space are all isomorphic.

A method of Neutrosophy is the:

6) **TRANSDISCIPLINARITY**:

A) **Introduction**:

Transdisciplinarity means to find common features to uncommon entities: <A> $\cap$ <Non-A> $\neq \emptyset$, even if they are disjunct.

B) **Multi-Structure and Multi-Space**:

Let S_1 and S_2 be two distinct structures, induced by the group of laws L which verify the axiom groups A_1 and A_2 respectively, such that A_1 is strictly included in A_2.

One says that the set M, endowed with the properties:

a) M has an S_1-structure,

b) there is a proper subset P (different from the empty set, from the unitary element, and from M) of the initial set M which has an S_2-structure,

c) M doesn't have an S_2-structure,

is called an **S_1-structure with respect to the S_2-structure.**

Let $S_1, S_2, \ldots, S_k$ be distinct space-structures.

We define the **Multi-Space** (or **k-structured-space**) as a set M such that for each structure S_i, $1 \leq i \leq k$, there is a proper (different from $\emptyset$ and from M) subset M_i of it which has that structure. The $M_1, M_2, \ldots, M_k$ proper subsets are different two by two.

Let's introduce new terms:

C) **Psychomathematics**:

A discipline which studies psychological processes in connection with mathematics.

D) **Mathematical Modeling of Psychological Process**:

Weber's law and Fechner's law on sensations and stimuli are improved.

E) **Psychoneutrosophy**:

Psychology of neutral thought, action, behavior, sensation, perception, etc. This is a hybrid field deriving from theology, philosophy, economics, psychology, etc.

For example, to find the psychological causes and effects of individuals supporting neutral ideologies (neither capitalists, nor communists), politics (not in the left, not in the right), etc.

F) **Socioneutrosophy**:

Sociology of neutralities.

For example the sociological phenomena and reasons which determine a country or group of people or class to remain neuter in a military, political, ideological, cultural, artistic, scientific, economical, etc. international or internal war (dispute).

G) **Econoneutrosophy**:

Economics of non-profit organizations, groups, such as: churches, philanthropic associations, charities, emigrating foundations, artistic or scientific societies, etc.

How they function, how they survive, who benefits and who loses, why are they necessary, how they improve, how they interact with for-profit companies.

These terms are in the process of development.

References :

[1] McNeil, F. Martin, Thro, Ellen, "Fuzzy Logic / A Practical Approach", Foreword by Ronald R. Yager, Academic Press, Florida, 1994.

[2] Smarandache, Florentin, "Collected Papers", Vol. II, University of Kishinev, Kishinev, 1997.

[3] "The Florentin Smarandache papers", Special Collection, Center for American History,,Archives of American Mathematics, University of Texas at Austin, TX.

[4] "The Florentin Smarandache papers", Special Collection, Hayden Library, Arizona State University, Tempe, AZ.

["Proceedings of the Second Symposium", Editor Şerban C. Andronescu, Romanian Academy of Scientists, American Branch, City University of New York, American Institute for Writing Research, New York, 81-92, 1999.]