New Operations on Intuitionistic Fuzzy Soft Sets Based on First Zadeh's Logical Operators

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Abstract – In this paper , we have defined First Zadeh's implication , First Zadeh's intuitionistic fuzzy conjunction and intuitionistic fuzzy disjunction of two intuitionistic fuzzy soft sets and some their basic properties are studied with proofs and examples.

Keywords *– Fuzzy sets, Intuitionistic fuzzy sets, Fuzzy soft sets, Intuitionistic fuzzy soft sets.*

1. Introduction

The concept of the intuitionistic fuzzy (IFS , for short) was introduced in 1983 by Atanassov [1] as an extension of Zadeh's fuzzy set. All operations, defined over fuzzy sets were transformed for the case the IFS case .This concept is capable of capturing the information that includes some degree of hesitation and applicable in various fields of research .For example , in decision making problems, particularly in the case of medial of medical diagnosis ,sales analysis ,new product marketing , financial services, etc. Atanassov et.al [2,3] have widely applied theory of intuitionistic sets in logic programming, Szmidt and Kacprzyk [4] in group decision making, De et al [5] in medical diagnosis etc. Therefore in various engineering application, intuitionstic fuzzy sets techniques have been more popular than fuzzy sets techniques in recent years. After defining a lot of operations over Intuitionstic fuzzy sets during last ten years [6] ,in 2011, Atanassov [7, 8] constructed two new operations based on the First Zadeh's IF-implication which are the first Zadeh's conjunction and disjounction, after that, in 2013, Atanassov[9] introduced the second type of Zadeh 's conjunction and disjunction based on the Second Zadeh's IF-implication.

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Another important concept that addresses uncertain information is the soft set theory originated by Molodtsov [10]. This concept is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Molodtsov has successfully applied the soft set theory in many different fields such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, and probability. In recent years, soft set theory has been received much attention since its appearance. There are many papers devoted to fuzzify the concept of soft set theory which leads to a series of mathematical models such as fuzzy soft set [11,12,13,14,15], generalized fuzzy soft set [16,17], possibility fuzzy soft set [18] and so on. Thereafter, Maji and his coworker [19] introduced the notion of intuitionstic fuzzy soft set which is based on a combination of the intuitionistic fuzzy sets and soft set models and studied the properties of intuitionistic fuzzy soft set. Later, a lot of extentions of intuitionistic fuzzy soft are appeared such as generalized intuitionistic fuzzy soft set [20], possibility Intuitionistic fuzzy soft set [21] etc.

In this paper, our aim is to extend the three new operations introduced by Atanassov to the case of intuitionistic fuzzy soft and study its properties. This paper is arranged in the following manner. In Section 2, some definitions and notion about soft set, fuzzy soft set and intuitionistic fuzzy soft set and some properties of its. These definitions will help us in later section . In Section 3, we discusses the three operations of intuitionistic fuzzy soft such as first Zadeh's implication, First Zadeh's intuitionistic fuzzy conjunction and first Zadeh intuitionistic fuzzy disjunction. Section 4 concludes the paper.

2. Preliminaries

In this section, some definitions and notions about soft sets and intutionistic fuzzy soft set are given. These will be useful in later sections

Let U be an initial universe, and E be the set of all possible parameters under consideration with respect to U. The set of all subsets of U, i.e. the power set of U is denoted by P(U) and the set of all intuitionistic fuzzy subsets of U is denoted by IFU . Let A be a subset of E.

Definition 2.1 .A pair (F , A) is called a soft set over U , where F is a mapping given by F : A P (U).

In other words, a soft set over U is a parameterized family of subsets of the universe U . For e ∈ A, F (e) may be considered as the set of e-approximate elements of the soft set (F , A).

Definition 2.2. Let U be an initial universe set and E be the set of parameters. Let IF^U denote the collection of all intuitionistic fuzzy subsets of U. Let . A ⊆ E pair (F, A) is called an intuitionistic fuzzy soft set over U where F is a mapping given by F: A→ IF^U .

Definition 2.3. Let F: A→ IF^U then F is a function defined as

$$F(\varepsilon) = \{ x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) : x \, \varepsilon \, U \}$$

where μ , ν denote the degree of membership and degree of non-membership respectively.

Definition 2.4 . For two intuitionistic fuzzy soft sets (F , A) and (G, B) over a common universe U , we say that (F , A) is an intuitionistic fuzzy soft subset of (G, B) if

(1) A ⊆ B and
(2) F (ε) ⊆G(ε) for all $\varepsilon \in$ A. i.e $\mu_{F(\varepsilon)}(x) \leq \mu_{G(\varepsilon)}(x)$, $\nu_{F(\varepsilon)}(x) \geq \nu_{G(\varepsilon)}(x)$ for all $\varepsilon \in$ E and

We write (F,A) ⊆ (G, B).

In this case (G, B) is said to be a soft super set of (F , A).

Definition 2.5. Two soft sets (F , A) and (G, B) over a common universe U are said to be soft equal if (F , A) is a soft subset of (G, B) and (G, B) is a soft subset of (F , A).

Definition 2.6. Let U be an initial universe, E be the set of parameters, and A ⊆ E .

(a) (F , A) is called a relative null soft set (with respect to the parameter set A), denoted by $\emptyset_A$, if F (a) = ∅ for all a ∈ A.

(b) (G, A) is called a relative whole soft set (with respect to the parameter set A), denoted by U_A ,if G(e) = U for all e ∈ A.

Definition 2.7. Let (F, A) and (G, B) be two IFSSs over the same universe U. Then the union of (F,A) and (G,B) is denoted by '(F,A)∪(G,B)' and is defined by (F,A) ∪ (G,B)=(H,C), where C=A∪B and the truth-membership, falsity-membership of (H,C) are as follows:

$$H(\varepsilon) = \begin{cases} \{(\mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) : x \; U\} & \text{, if } \varepsilon \in \text{A} - \text{B}, \\ \{(\mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x) : x \; U\} & \text{, if } \varepsilon \in \text{B} - \text{A} \\ \{\max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \min(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) : x \; U\} & \text{if } \varepsilon \in \text{A} \cap \text{B} \end{cases}$$

Where $\mu_{H(\varepsilon)}(x) = \max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))$ and $\nu_{H(\varepsilon)}(x) = \min(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))$

Definition 2.8. Let (F, A) and (G, B) be two IFSS over the same universe U such that A ∩ B ≠ 0. Then the intersection of (F, A) and (G, B) is denoted by '(F, A) ∩ (G, B)' and is defined by (F, A) ∩(G, B) = (K, C),where C =A ∩B and the truth-membership, falsity-membership of (K, C) are related to those of (F, A) and (G, B) by:

$$K(\varepsilon) = \begin{cases} \{(\mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) : x \; U\} & \text{, if } \varepsilon \in \text{A} - \text{B}, \\ \{(\mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x) : x \; U\} & \text{, if } \varepsilon \in \text{B} - \text{A} \\ \{\min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) : x \; U\} & \text{if } \varepsilon \in \text{A} \cap \text{B} \end{cases}$$

Where $\mu_{K(\varepsilon)}(x) = \min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))$ and $\nu_{K(\varepsilon)}(x) = \max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))$

3. New Operations on Intuitionstic Fuzzy Soft Sets Based on First Zadeh's Logical Operators

3.1 First Zadeh's Implication of Intuitionistic Fuzzy Soft Sets

Definition 3.1.1. Let (F, A) and (G, B) are two intuitionistic fuzzy soft set s over (U,E) .We define the First Zadeh's intuitionistic fuzzy soft set implication (F, A) $\underset{z,1}{\rightarrow}$(G,B) is defined by

(F, A) $\underset{z,1}{\rightarrow}$ (G,B) = [max $\{\nu_{F(\varepsilon)}(x)$, min $(\mu_{F(\varepsilon)}(x)$, $\mu_{G(\varepsilon)}(x))\}$, min $(\mu_{F(\varepsilon)}(x)$, $\nu_{G(\varepsilon)}(x))$

Proposition 3.1.2. Let (F, A) ,(G, B) and (H, C) are three intuitionistic fuzzy soft set s over (U,E). Then the following results hold

(i) (F, A) ∩ (G,B) $\underset{z,1}{\rightarrow}$ (H, C) ⊇ [(F , A) $\underset{z,1}{\rightarrow}$ (H, C)] ∩ [(G , B) $\underset{z,1}{\rightarrow}$ (H, C)]

(ii) (F, A) ∪ (G,B) $\underset{z,1}{\rightarrow}$ (H, C) ⊇ [(F , A) $\underset{z,1}{\rightarrow}$ (H, C)] ∪ [(G , B) $\underset{z,1}{\rightarrow}$ (H, C)]

(iii) (F, A) ∩(G,B) $\underset{z,1}{\rightarrow}$ (H, C) ⊇ [(F , A) $\underset{z,1}{\rightarrow}$ (H, C)] ∪ [(G , B) $\underset{z,1}{\rightarrow}$ (H, C)]

(iv) (F, A) $\underset{z,1}{\rightarrow}$ $(\mathrm{F},\mathrm{A})^{c}$ = $(\mathrm{F},\mathrm{A})^{c}$

(v) (F, A) $\underset{z,1}{\rightarrow}$ (φ,A) =$(\mathrm{F},\mathrm{A})^{c}$

Proof.

(i) (F, A) ∩ (G,B) $\underset{z,1}{\rightarrow}$ (H, C)

$$= \left\{ min\left(\mu_{F(\varepsilon)}(x),\mu_{G(\varepsilon)}(x)\right), \max\left(\nu_{F(\varepsilon)}(x),\nu_{G(\varepsilon)}(x)\right) \right\} \underset{z,1}{\rightarrow} (\mu_{H(\varepsilon)}(x), \nu_{H(\varepsilon)}(x))$$

$$= \begin{bmatrix} \text{MAX}\left\{ \max\left(\nu_{F(\varepsilon)}(x),\nu_{G(\varepsilon)}(x)\right), \min\left(min\left(\mu_{F(\varepsilon)}(x),\mu_{G(\varepsilon)}(x)\right), \mu_{H(\varepsilon)}(x)\right)\right\}, \\ \text{MIN}\left\{min\left(\mu_{F(\varepsilon)}(x),\mu_{G(\varepsilon)}(x)\right), \nu_{H(\varepsilon)}(x)\right\} \end{bmatrix} \quad (1)$$

[(F , A) $\underset{z,1}{\rightarrow}$ (H, C)] ∩ [(G , B) $\underset{z,1}{\rightarrow}$ (H, C)]

= [max $\{\nu_{F(\varepsilon)}$, min $(\mu_{F(\varepsilon)}$, $\mu_{H(\varepsilon)})\}$, min $(\mu_{F(\varepsilon)}$, $\nu_{H(\varepsilon)})$] ∩

[max $\{\nu_{G(\varepsilon)}$, min $(\mu_{G(\varepsilon)}$, $\mu_{H(\varepsilon)})\}$, min $(\mu_{G(\varepsilon)}$, $\nu_{H(\varepsilon)})$]

$$= \begin{bmatrix} \text{MIN}\left\{\max\left(\nu_{F(\varepsilon)}(x), min\left(\mu_{F(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)\right)\right), \max\left(\nu_{G(\varepsilon)}(x), min\left(\mu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)\right)\right)\right\}, \\ \text{MAX}\left\{min\left(\mu_{F(\varepsilon)}(x),\nu_{H(\varepsilon)}(x)\right), min\left(\mu_{G(\varepsilon)}(x),\nu_{H(\varepsilon)}(x)\right)\right\} \end{bmatrix} \quad (2)$$

From (1) and (2) it is clear that (F, A) ∩ (G,B) $\underset{z,1}{\rightarrow}$ (H, C) ⊇ [(F , A) $\underset{z,1}{\rightarrow}$ (H, C)] ∩ [(G , B) $\underset{z,1}{\rightarrow}$ (H, C)]

(ii) And (iii) the proof is similar to (i)

(iv) (F, A) $\underset{z,1}{\rightarrow}$ $(\mathrm{F},\mathrm{A})^{c}$ = $(\mathrm{F},\mathrm{A})^{c}$

$$= \begin{bmatrix} \text{Max}\left\{\nu_{F(\varepsilon)}(x), min\left(\mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\right)\right\}, \\ \text{MIN}\left\{\mu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x)\right\} \end{bmatrix}$$

= $(\nu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x))$

It is shown that the first Zadeh's intuitionistic fuzzy soft implication generate the complement of intuitionistic fuzzy soft set.

(v) The proof is straightforward .

Example 3.1.3.
(F ,A) = {F(e_1) = (a , 0.3 , 0.2)}
(G ,B) = {G(e_1) = (a , 0.4 , 0.5)}
(H ,C) = {H(e_1) = (a , 0.3 , 0.6)}

(F, A) ∩ (G,B) $\underset{z,1}{\rightarrow}$ (H, C) =
[max { (max (0.2, min (0.3,0.4)) , 0.3 } , min **{** min (0.3,0.5), 0.6))} = (0.5, 0.3)
(F, A) ∩ (G,B) ={(a ,0.3 ,0.5)}

3.2. First Zadeh's Intuitionistic Fuzzy Conjunction of Intuitionistic Fuzzy Soft Set

Definition 3.2.1. Let (F, A) and (G, B) are two intuitionistic fuzzy soft sets over (U,E) .We define the first Zadeh's intuitionistic fuzzy conjunction of (F, A) and (G,B) as the intuitionistic fuzzy soft set (H,C) over (U,E), written as (F, A) $\tilde{\wedge}_{z,1}$ (G,B) =(H ,C). Where C = A ∩ B ≠ ∅ and ∀ ε ∈ C, x ∈ U,

$\mu_{H(\varepsilon)}(x) = MIN\left(\mu_{F(\varepsilon)}(x)\ ,\mu_{G(\varepsilon)}(x)\right)$
$\nu_{H(\varepsilon)}(x) = \ Max\left\{\nu_{F(\varepsilon)}(x)\ ,min\left(\mu_{F(\varepsilon)}(x)\ ,\nu_{G(\varepsilon)}(x)\right)\right\}$

Example 3.2. 2.
Let U={a, b, c} and E ={ e_1 , e_2 , e_3 , e_4} , A ={ e_1 , e_2, e_4} ⊆ E, B={ e_1 , e_2 , e_3} ⊆ E

(F, A) ={ F(e_1) ={((a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)},
F(e_2) ={((a, 0.7, 0.1), (b, 0, 0.8), (c, 0.3, 0.5)},
F(e_4) ={((a, 0.6, 0.3), (b, 0.1, 0.7), (c, 0.9, 0.1)}}

(G, B) ={ G(e_1) ={((a, 0.2, 0.6), (b, 0.7, 0.1), (c, 0.8, 0.1)},
G(e_2) ={((a, 0.4, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)},
G(e_3) ={((a, 0, 0.6), (b, 0, 0.8), (c, 0.1, 0.5)}}

Let (F, A) $\tilde{\wedge}_{z}$ (G,B) =(H ,C) ,where C = A ∩ B = { e_1 , e_2 }

(H, C)={H (e_1) ={(a, min(0.5, 0.2), max(0.1, min(0.5, 0.6)))
(b, min(0.1, 0.7), max(0.8, min(0.1, 0.1)))
(c, min(0.2, 0.8), max(0.5, min(0.2, 0.1)))},
H (e_2) ={(a, min(0.7, 0.4), max(0.1, min(0.7, 0.1)))
(b, min(0, 0.5), max(0.8, min(0, 0.3)))
(c, min(0.3, 0.4), max(0.5, min(0.3, 0.5)))}}

(H, C)= { H (e_1)= {(a, min(0.5, 0.2), max(0.1, 0.5)),
(b, min(0.1, 0.7), max(0.8, 0.1)),

(c, min(0.2, 0.8), max(0.5, 0.1))},
H (e_2)= {(a, min(0.7, 0.4), max(0.1, 0.1)),
(b, min(0, 0.5), max(0, 0.8)),
(c, min(0.3, 0.4), max(0.5, 0.3))}}

(H, C)= { H (e_1)= {(a, 0.2, 0.5),(b, 0.1, 0.8), (c,0.2, 0.5)},
H (e_2)= {(a, 0.4, 0.1), (b, 0, 0), (c, 0.3, 0.5)}}

Proposition 3.2. 3. Let (F, A) ,(G, B) and (H, C) are three intuitionistic fuzzy soft set s over (U,E). Then the following result hold

$$(F, A)\ \widetilde{\wedge}_{z,1}\ (G,B) \underset{z,1}{\rightarrow} (H, C) \supseteq [(F , A) \underset{z,1}{\rightarrow} (H, C)]\ \widetilde{\wedge}_{z,1}\ [(G , B) \underset{z,1}{\rightarrow} (H, C)]$$

Proof. Let (F, A) ,(G, B) and (H,C) are three intuitionistic fuzzy soft set ,then
$(F, A)\ \widetilde{\wedge}_{z,1}\ (G,B) \underset{z,1}{\rightarrow} (H, C)$ =

$$\begin{bmatrix} \text{Max}\left\{\max\left(\nu_{F(\varepsilon)}(x), min\left(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right)\right), \min\left(min\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right), \mu_{H(\varepsilon)}(x)\right)\right\}, \\ \text{MIN}\left\{min\left(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right), \nu_{H(\varepsilon)}(x)\right\} \end{bmatrix} \quad (1)$$

Let $[(F , A) \underset{z,1}{\rightarrow} (H, C)]\ \widetilde{\wedge}_{z,1}\ [(G , B) \underset{z,1}{\rightarrow} (H, C)]$

$$(F , A) \underset{z,1}{\rightarrow} (H, C) = \begin{bmatrix} \text{MAX}\left\{\nu_{F(\varepsilon)}(x), min\left(\mu_{F(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)\right)\right\}, \\ \text{MIN}\left\{\mu_{F(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)\right\} \end{bmatrix}$$

$$[(G , B) \underset{z,1}{\rightarrow} (H, C)] = \begin{bmatrix} \text{MAX}\left\{\nu_{G(\varepsilon)}(x), min\left(\mu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)\right)\right\}, \\ \text{MIN}\left\{\mu_{G(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)\right\} \end{bmatrix}$$

Then $[(F , A) \underset{z,1}{\rightarrow} (H, C)]\ \widetilde{\wedge}_{z,1}\ [(G , B) \underset{z,1}{\rightarrow} (H, C)]$ =

$$\begin{bmatrix} \text{MIN}\left(max\left\{\nu_{F(\varepsilon)}(x), min\left(\mu_{F(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)\right)\right\}, max\left\{\nu_{G(\varepsilon)}(x), min\left(\mu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)\right)\right\}\right), \\ \text{MAX}\left(\min\{\mu_{F(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)\}, \min\left\{max\left(\nu_{G(\varepsilon)}(x), min\left(\mu_{G(\varepsilon)}(x), \mu_{H(\varepsilon)}(x)\right)\right), min\left(\mu_{G(\varepsilon)}(x), \nu_{H(\varepsilon)}(x)\right)\right\}\right) \end{bmatrix} \quad (2)$$

From (1) and (2) it is clear that

$$(F, A)\ \widetilde{\wedge}_{z,1}(G,B) \underset{z,1}{\rightarrow} (H, C) \supseteq [(F , A) \underset{z,1}{\rightarrow} (H, C)]\ \widetilde{\wedge}_{z,1}\ [(G , B) \underset{z,1}{\rightarrow} (H, C)]$$

3. 3. The First Zadeh's Intuitionistic Fuzzy Disjunction of Intuitionstic Fuzzy Soft Set

Definition 3.3.1. Let (F, A) and (G, B) are two intuitionistic fuzzy soft set s over (U,E) .We define the first Zadeh's intuitionistic fuzzy disjunction of (F, A) and (G,B) as the intuitionistic fuzzy soft set (H,C) over (U,E), written as $(F, A)\ \widetilde{\vee}_{z,1}\ (G,B)$ =(H ,C). Where C = A ∩ B ≠ ∅ and ∀ ε ∈ A , x ∈ U

$$\mu_{H(\varepsilon)}(x) = Max\left\{\mu_{F(\varepsilon)}(x), min\left(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right)\right\}$$

$\nu_{H(\varepsilon)}(x)= \ Min\big(\nu_{F(\varepsilon)}(x)\ , \nu_{G(\varepsilon)}(x))\ \big)$

Example 3.3.2. Let U={a, b,c} and E ={ e_1 , e_2 , e_3 , e_4} , A ={ e_1 , e_2, e_4} ⊆ E, B={ e_1 , e_2 , e_3} ⊆ E

(F, A) ={ F(e_1) ={((a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)},
F(e_2) ={((a, 0.7, 0.1), (b, 0, 0.8), (c, 0.3, 0.5)},
F(e_4) ={((a, 0.6, 0.3), (b, 0.1, 0.7), (c, 0.9, 0.1)}}

(G, A) ={ G(e_1) ={((a, 0.2, 0.6), (b, 0.7, 0.1), (c, 0.8, 0.1)},
G(e_2) ={((a, 0.4, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)},
G(e_3) ={((a, 0, 0.6), (b, 0, 0.8), (c, 0.1, 0.5)}}

Let (F, A) $\widetilde{\vee}_{z,1}$ (G,B) =(H ,C),where C = A ∩ B = { e_1 , e_2 }

(H, C)={H (e_1) ={(a, max(0.5, min(0.1, 0.2)), min(0.1, 0.6))
(b, max(0.1, min(0.8, 0.7)), min(0.8, 0.1))
(c, max(0.2, min(0.5, 0.8)), min(0.5, 0.1)) },
H (e_2) ={(a, max(0.7, min(0.1, 0.4)), min(0.1, 0.1))
(b, max(0, min(0.8, 0.5)), min(0.8, 0.3))
(c, max(0.3, min(0.5, 0.4)), min(0.5, 0.5))}}

(H, C)= { H (e_1)= {(a, max(0.5, 0.1), min(0.1, 0.6)),
(b, max(0.1, 0.7), min(0.8, 0.1)),
(c, max(0.2, 0.5), min(0.5, 0.1))},
H (e_2)= {(a, max(0.7, 0.1), min(0.1, 0.1)),
(b, max(0, 0.5), min(0.8, 0.3)),
(c, max(0.3, 0.4), min(0.5, 0.5))}}

(H, C)= { H (e_1)= {(a, 0.5, 0.1),(b, 0.7, 0.1), (c,0.5, 0.1)},
H (e_2)= {(a, 0.7, 0.1),(b, 0.5, 0.3), (c,0.4, 0.5)}}

Proposition 3.3.3.
(i) (φ ,A) $\widetilde{\wedge}_{z,1}$ (U, A) = (φ ,A)
(ii) (φ ,A) $\widetilde{\vee}_{z,1}$ (U, A) = (U, A)
(iii) (F, A) $\widetilde{\vee}_{z,1}$ (φ ,A) = (F,A)

Proof.
(i) Let (φ ,A) $\widetilde{\wedge}_{z,1}$ (U, A) =(H, A) ,where For all ε ∈ A , x ∈ U, we have

$\mu_{H(\varepsilon)}(x)$ =min (0 ,1) = 0
$\nu_{H(\varepsilon)}(x)$= max (1 ,min (0, 0)) =max (1 , 0)= 1

Therefore (H, A)= (0 ,1) , For all ε ∈ A , x ∈ U

It follows that ((φ ,A) $\widetilde{\wedge}_{z,1}$ (U, A) = (φ ,A)

(ii) Let (φ ,A) $\widetilde{\vee}_{z,1}$ (U, A) =(H, A) ,where For all ε ∈ A , x ∈ U, we have

$\mu_{H(\varepsilon)}(x)$ = max (0 ,min (1, 1)) =max (0 ,1)= 1
$\nu_{H(\varepsilon)}(x)$= min (1 ,0) = 0

Therefore (H, A) = (1,0) , For all $\varepsilon \in$ A , x ∈ U

It follows that ((φ ,A) $\widetilde{\wedge}_{z,1}$ (U, A) = (U, A)

(iii) Let (F, A) $\widetilde{\vee}_{z,1}$ (φ ,A) =(H, A) ,where For all $\varepsilon \in$ A , x ∈ U, we have

$\mu_{H(\varepsilon)}(x)$ = max ($\mu_{F(\varepsilon)}(x)$,min ($\nu_{F(\varepsilon)}(x)$, 0)) = max ($\mu_{F(\varepsilon)}(x)$, 0) = $\mu_{F(\varepsilon)}(x)$
$\nu_{H(\varepsilon)}(x)$= min ($\nu_{H(\varepsilon)}(x)$,1) = $\nu_{H(\varepsilon)}(x)$

Therefore (H, A) = ($\mu_{F(\varepsilon)}(x)$, $\nu_{H(\varepsilon)}(x)$) , For all $\varepsilon \in$ A , x ∈ U

It follows that (F, A) $\widetilde{\vee}_{z,1}$ (φ ,A) =(F, A)

Proposition 3.3.4.
(F, A) $\widetilde{\vee}_{z,1}$ (G,B) $\underset{z,1}{\rightarrow}$ (H, C) ⊇ [(F , A) $\underset{z,1}{\rightarrow}$ (H, C)] $\widetilde{\vee}_{z,1}$ [(G , B) $\underset{z,1}{\rightarrow}$ (H, C)]

Proof. The proof is similar as in proposition 3.2.3

Proposition 3.3.5.
(i) [(F, A) $\widetilde{\wedge}_{z,1}$ (G, B)]^c =(F, A) ^{c} $\widetilde{\vee}_{z,1}$ (G , B)^c
(ii) [(F, A) $\widetilde{\vee}_{z,1}$ (G, B)]^c =(F, A) ^{c} $\widetilde{\wedge}_{z,1}$ (G , B)^c
(iii) [(F, A) ^{c} $\widetilde{\wedge}_{z,1}$ (G , B) ^{c}]^c = (F, A) $\widetilde{\vee}_{z,1}$(G, B)

Proof.
(i) Let [(F ,A) $\widetilde{\wedge}_{z,1}$ (G, B)]^c =(H, C) ,where For all $\varepsilon \in$ C , x ∈ U, we have

$$[(F\ ,A)\ \widetilde{\wedge}_{z,1}\ (G, B)]^c = \begin{bmatrix} \text{MIN}\{\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\}, \\ \text{MAX}\left\{\nu_{F(\varepsilon)}(x), min\left(\ \mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right)\right\} \end{bmatrix}^c$$

$$= \begin{bmatrix} \text{MAX}\left\{\nu_{F(\varepsilon)}(x), min\left(\ \mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\right)\right\}, \\ \text{MIN}\{\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\} \end{bmatrix}$$

$$= (F\ , A)\ ^c\ \widetilde{\vee}_{z,1}\ (G\ , B)\ ^c$$

(ii) Let [(F ,A) $\widetilde{\vee}_{z,1}$(G, B)]^c =(H, C) ,where For all $\varepsilon \in$ C , x ∈ U , we have

$$[(F\ ,A)\ \widetilde{\vee}_{z,1}(G, B)]^c = \begin{bmatrix} \text{MAX}\left\{\mu_{F(\varepsilon)}(x), min\left(\ \nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right)\right\}, \\ \text{MIN}\{\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\} \end{bmatrix}$$

$$= \begin{bmatrix} \text{MIN}\{\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\}, \\ \text{MAX}\left\{\mu_{F(\varepsilon)}(x), min\left(\ \nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\right)\right\} \end{bmatrix}^c$$

$$= (F\ , A)\ ^c\ \widetilde{\wedge}_{z,1}\ \ (G\ , B)\ ^c$$

(iii) The proof is straightforward.

The following equalities are not valid.

$(F$,A) $\widetilde{\vee}_{z,1}$(G, B) = (G ,B) $\widetilde{\vee}_{z,1}$(F, A)
$(F$,A) $\widetilde{\wedge}_{z,1}$(G, B) = (G ,B) $\widetilde{\wedge}_{z,1}$(F, A)
[$(F$,A) $\widetilde{\wedge}_{z,1}$(G, B)] $\widetilde{\wedge}_{z,1}$(K, C) = (F ,A) $\widetilde{\wedge}_{z,1}$ [(G, B) $\widetilde{\wedge}_{z,1}$(K, C)]
[$(F$,A) $\widetilde{\vee}_{z,1}$(G, B)] $\widetilde{\vee}_{z,1}$(K, C) = (F ,A) $\widetilde{\vee}_{z,1}$ [(G, B) $\widetilde{\vee}_{z,1}$(K, C)]
[$(F$,A) $\widetilde{\wedge}_{z,1}$(G, B)] $\widetilde{\vee}_{z,1}$(K, C) = [(F ,A) $\widetilde{\vee}_{z,1}$ (G, B)] $\widetilde{\wedge}_{z,1}$ [(G, B) $\widetilde{\vee}_{z,1}$ (K, C)]
[$(F$,A) $\widetilde{\vee}_{z,1}$(G, B)] $\widetilde{\wedge}_{z,1}$(K, C) = [(F ,A) $\widetilde{\wedge}_{z,1}$ (G, B)] $\widetilde{\vee}_{z,1}$ [(G, B) $\widetilde{\wedge}_{z,1}$ (K, C)]

Example 3.3.6. Let U={a, b,c} and E ={ e_1 , e_2 , e_3 , e_4} , A ={ e_1 , e_2, e_4} ⊆ E,
B={ e_1 , e_2 , e_3} ⊆ E

(F, A) ={ F(e_1) ={((a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)},
F(e_2) ={((a, 0.7, 0.1), (b, 0, 0.8), (c, 0.3, 0.5)},
F(e_4) ={((a, 0.6, 0.3), (b, 0.1, 0.7), (c, 0.9, 0.1)}}

(G, A) ={ G(e_1) ={((a, 0.2, 0.6), (b, 0.7, 0.1), (c, 0.8, 0.1)},
G(e_2) ={((a, 0.4, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)},
G(e_3) ={((a, 0, 0.6), (b, 0, 0.8), (c, 0.1, 0.5)}}

Let (F, A) $\widetilde{\wedge}_{z,1}$ (G,B) =(H ,C) ,where C = A ∩ B = { e_1 , e_2 }

Then (F, A) $\widetilde{\wedge}_{z,1}$ (G,B) = (H, C)= { H (e_1) = {(a, 0.2, 0.5), (b, 0.1, 0.8), (c,0.2, 0.5)},
H (e_2)= {(a, 0.4, 0.1), (b, 0, 0), (c,0.3, 0.5)}}

For (G, B) $\widetilde{\wedge}_{z,1}$ (F, A) = (K, C) ,where K = A ∩ B = { e_1 , e_2 }

(K, C)={K (e_1) ={(a, min (0.2, 0.5), max (0.6, min (0.2, 0.1)))
(b, min (0.7, 0.1), max (0.1, min(0.7, 0.8)))
(c, min (0.8, 0.2), max (0.1, min (0.8, 0.5)))},
K (e_2) ={(a, min (0.7 0.4), max(0.1, min (0.4, 0.1)))
(b, min (0.5, 0.), max(0.3, min (0.5, 0.8)))
(c, min (0.4, 0.3), max(0.5, min (0.4, 0.5)))}}

(K, C)= { K (e_1)= {(a, min (0.2, 0.5), max (0.6, 0.1)),
(b, min (0.7, 0.1), max (0.1, 0.7)),
(c, min (0.8, 0.2), max (0.1, 0.5))},
K (e_2)= {(a, min (0.4, 0.7), max (0.1, 0.1)),
(b, min (0.5, 0), max (0.3, 0.5)),
(c, min (0.4, 0.3), max (0.5, 0.4))}}

(K, C)= { K (e_1)= {(a, 0.2, 0.6),(b, 0.1, 0.7), (c,0.2, 0.5)},
K (e_2)= {(a, 0.4, 0.1),(b, 0, 0.5), (c,0.3, 0.5)}}

Then (G, B) $\widetilde{\wedge}_{z,1}$ (F, A) = (K, C) = { K (e_1)= {(a, 0.2, 0.6),(b, 0.1, 0.7), (c,0.2, 0.5)},
K (e_2)= {(a, 0.4, 0.1),(b, 0, 0.5), (c,0.3, 0.5)}}

It is obviously that (F, A) $\widetilde{\wedge}_{z,1}$ (G,B) $\neq$ (G, B) $\widetilde{\wedge}_{z,1}$ (F, A)

Conclusion

In this paper, three new operations have been introduced on intuitionistic fuzzy soft sets. They are based on First Zadeh's implication, conjunction and disjunction operations on intuitionistic fuzzy sets. Some examples of these operations were given and a few important properties were also studied. In our following papers, we will extended the following three operations such as second zadeh's IF-implication, second zadeh's conjunction and second zadeh's disjunction to the intuitionistic fuzzy soft set. We hope that the findings, in this paper will help researcher enhance the study on the intuitionistic soft set theory.

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