A NOTE ON TESTING OF HYPOTHESIS

Rajesh Singh

School of Statistics, D.A.V.V.
Indore (M.P.)
India
e-mail: rsinghstat@yahoo.co.in

Jayant Singh

Department of Statistics
Rajasthan University
Jaipur
India
e-mail: Jayantsingh47@rediffmail.com

Florentin Smarandache

Chair of Department of Mathematics
University of New Mexico
Gallup
USA
e-mail: fsmarandache@yahoo.com

Abstract. In this paper problem of testing of hypothesis is discussed when the samples have been drawn from normal distribution. The study of hypothesis testing is also extended to Bayes set up.

AMS Subject Classification: 62F03.

Keywords: hypothesis, level of significance, Bayes rule.

Let the random variable (r.v.) X have a normal distribution $N(\theta, \sigma_2)$, where σ_2 is assumed to be known.

The hypothesis $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, $\theta_1 > \theta_0$ is to be tested.

Let $X_1, X_2, ..., X_n$ be a random sample from $N(\theta, \sigma^2)$ population.

Let $\overline{X}$ $\left(= \frac{1}{n}\sum_{n}^{i=1} X_i\right)$ be the sample mean.

By Neyman–Pearson lemma, the most powerful test rejects H_0 at α % level of significance, if $\dfrac{\sqrt{n}\,(\overline{X}-\theta_0)}{\sigma} \geq d_\alpha$, where d_α is such that

$$\int_{d_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}}\, e^{-\frac{Z^2}{2}}\, dZ = \alpha.$$

If the sample is such that H_0 is rejected, then will it imply that H_1 will be accepted?

In general, this will not be true for all values of θ_1, but will be true for some specific value of θ_1, i.e., when θ_1 is at a specific distance from θ_0.

H_0 is rejected

$$\text{(1)} \qquad \text{if } \frac{\sqrt{n}\,(\overline{X}-\theta_0)}{\sigma} \geq d_\alpha, \text{ i.e., } \overline{X} \geq \theta_0 + d_\alpha \frac{\sigma}{\sqrt{n}}.$$

Similarly, the Most Powerful Test will accept H_1 against H_0

$$\text{(2)} \qquad \text{if } \frac{\sqrt{n}\,(\overline{X}-\theta_0)}{\sigma} \geq d_\alpha, \text{ i.e., } \overline{X} \geq \theta_1 - d_\alpha \frac{\sigma}{\sqrt{n}}.$$

Rejecting H_0 will mean accepting H_1

$$\text{(3)} \qquad \begin{aligned} &\text{if} \quad (1) \Longrightarrow (2) \\ &\text{i.e.,} \quad \overline{X} \geq \theta_0 + d_\alpha \frac{\sigma}{\sqrt{n}} \Longrightarrow \overline{X} \geq \theta_1 - d_\alpha \frac{\sigma}{\sqrt{n}} \\ &\text{i.e.,} \quad \theta_1 - d_\alpha \frac{\sigma}{\sqrt{n}} \leq \theta_0 + d_\alpha \frac{\sigma}{\sqrt{n}}. \end{aligned}$$

Similarly, accepting H_1 will mean rejecting H_0

$$\text{(4)} \qquad \begin{aligned} &\text{if} \quad (2) \Longrightarrow (1) \\ &\text{i.e.,} \quad \theta_0 + d_\alpha \frac{\sigma}{\sqrt{n}} \leq \theta_1 - d_\alpha \frac{\sigma}{\sqrt{n}}. \end{aligned}$$

From (3) and (4) we have

$$\text{(5)} \qquad \theta_0 + d_\alpha \frac{\sigma}{\sqrt{n}} = \theta_1 - d_\alpha \frac{\sigma}{\sqrt{n}} \text{ i.e., } \theta_1 - \theta_0 = 2d_\alpha \frac{\sigma}{\sqrt{n}} \cdot$$

Thus,

$$d_\alpha \frac{\sigma}{\sqrt{n}} = \frac{\theta_1 - \theta_0}{2} \quad \text{and} \quad \theta_1 = \theta_0 + 2d_\alpha \frac{\sigma}{\sqrt{n}} \cdot$$

From (1),

$$\text{Reject } H_0 \text{ if } \overline{X} > \theta_0 + \frac{\theta_1 - \theta_0}{2} = \frac{\theta_0 + \theta_1}{2}$$

and from (2),

$$\text{Accept } H_1 \text{ if } \overline{X} > \theta_1 - \frac{\theta_1 - \theta_0}{2} = \frac{\theta_0 + \theta_1}{2} \cdot$$

Thus, rejecting H_0 will mean accepting H_1 when

$$\overline{X} > \frac{\theta_0 + \theta_1}{2} \cdot$$

From (5), this will be true only when

$$\theta_1 = \theta_0 + 2d_\alpha \frac{\sigma}{\sqrt{n}} \cdot$$

For other values of $\theta_1 \neq \theta_0 + 2d_\alpha \dfrac{\sigma}{\sqrt{n}}$ rejecting H_0 will not mean accepting H_1.

Therefore, it is recommended that, instead of testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, $\theta_1 > \theta_0$, it is more appropriate to test $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_0$.

In this situation, rejecting H_0 will mean $\theta >= \theta_0$ and is not equal to some given value $= \theta_1$.

But in Baye's setup, rejecting H_0 means accepting H_1 whatever may be H_0 and H_1.

In this set up, the level of significance is not a preassigned constant, but depends on H_0, H_1, σ_2 and n.

Consider $(0, 1)$ loss function and equal prior probabilities $1/2$ for θ_0 and θ_1. The Baye's test rejects H_0 (accept H_1)

$$\text{if } \overline{X} > \frac{\theta_0 + \theta_1}{2}$$

and accepts H_0 (rejects H_1)

$$\text{if } \overline{X} < \frac{\theta_0 + \theta_1}{2} \cdot$$

[See Rohatagi, p.463, Example 2].

The level of significance is given by

$$P_{H_0}\left[\overline{X} > \frac{\theta_0 + \theta_1}{2}\right] = P_{H_0}\left[\frac{(\overline{X} - \theta_0)\sqrt{n}}{\sigma} > \frac{(\theta_1 - \theta_0)\sqrt{n}}{2\sigma}\right] = 2 - \Phi\left(\frac{\sqrt{n}(\theta_1 - \theta_0)}{2\sigma}\right),$$

where

$$\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-\frac{Z^2}{2}} dZ.$$

Thus, the level of significance depends on $\theta_0, \theta_1, \sigma^2$ and n.

Acknowledgement. Author's are thankful to Prof. Jokhan Singh for suggesting this problem.

References

[1] Lehmann, E.L., *Testing Statistical Hypotheses*, Wiley Eastern Ltd., New Delhi, 1976.

[2] Rohatagi, V.K., *An introduction to probability theory and mathematical statistics*, Wiley Eastern Ltd., New Delhi, 1985.

Accepted: 26.04.2009