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Numeralogy (II)
or
Properties of Numbers

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1) Factorial base:

0, 1, 10, 11, 20, 21, 100, 101, 110, 111, 120, 121, 200, 201, 210, 211, 220, 221, 300, 301, 310, 311, 320, 321, 1000, 1001, 1010, 1011, 1020, 1021, 1100, 1101, 1110, 1111, 1120, 1121, 1200, ...

(Each number n written in the factorial base.)

(We define over the set of natural numbers the following infinite base: for $k \geq 1$ $f_k = k!$)

It is proved that every positive integer A may be uniquely written in the factorial base as:

$A = (\overline{a_n \ldots a_2 a_1})_{(F)} \stackrel{def}{=} \sum_{i=1}^{n} a_i f_i$, with all $a_i = 0, 1, \ldots i$ for $i \geq 1$.

in the following way:

- if $f_n \leq A < f_{n+1}$ then $A = f_n + r_1$;

- if $f_m \leq r_1 < f_{m+1}$ then $r_1 = f_m + r_2, m < n$;

and so on untill one obtains a rest $r_j = 0$.

What's very interesting: $a_1 = 0$ or 1; $a_2 = 0, 1$, or 2; $a_3 = 0, 1, 2$ or 3, and so on...

If we note by $f(A)$ the superior factorial part of A (i.e. the largest factorial less than or equal to A), then A is written in the factorial base as:

$$A = f(A) + f(A - f(A)) + f(A - f(A) - f(A - f(A))) + \ldots.$$

Rules of addition and substraction in factorial base:

for each digit a_i we add and substract in base $i+1$, for $i \geq 1$.

For examplu, addition:

base	5	4	3	2	
		2	1	0	+
		2	2	1	
	1	1	0	1	

because: $0 + 1 = 1$ (in base 2);

$1 + 2 = 10$ (in base 3); therefore we write 0 and keep 1;

$2 + 2 + 1 = 11$ (in base 4).

Now substraction:

base	5	4	3	2	
	1	0	0	1	-
		3	2	0	
			1	1	

becuase: $1 - 0 = 1$ (in base 2);

$0 - 2 = ?$ it's not possible (in base 3), go to the next left unit, which is 0 again (in base 4), go again to the next left unit, which is 1 (in base 5), therefore $1001 \to 0401 \to 0331$ and then $0331 - 320 = 11$.

Find some rules for multiplication and division.

In a general case:

if we want to design a base such that any number

$A = (\overline{a_n \dots a_2 a_1})_{(B)} \stackrel{def}{=} \sum_{i=1}^{n} a_i b_i$, with all $a_i = 0, 1, \dots t_i$ for $i \geq 1$, where all $t_i \geq 1$, then:

this base should be

$$b_1 = 1, b_{i+1} = (t_i + 1) * b_i \text{ for } i \geq 1.$$

2) More general-sequence sieve:

For $i = 1, 2, 3, \dots$, let $u_i > 1$, be a strictly increasing positive integer sequence, and $v_i < u_i$ another positive integer sequence. Then:

From the natural numbers set:

- keep the v_1-th number among $1, 2, 3, \dots, u_1 - 1$, and delete every u_1-th numbers;

- keep the v_2-th number among the next $u_2 - 1$ remaining numbers, and delete every u_2-th numbers;

... and so on, for step $k (k \geq 1)$:

-keep the v_k-th number among the next $u_k - 1$ remaining numbers, and delete every u_k-th numbers;

... .

Problem: study the relationship between sequences $u_i, v_i, i = 1, 2, 3, \dots$, and the remaining sequence resulted from the more general sieve.

u_i and v_i previously defined, are called sieve generators.

3) Mobile periodicals (I):

...000000000000000000000000000000100000000000000000000000000000000000...
...000000000000000000000000000001110000000000000000000000000000000000...
...000000000000000000000000000011011000000000000000000000000000000000...
...000000000000000000000000000001110000000000000000000000000000000000...
...000000000000000000000000000000100000000000000000000000000000000000...
...000000000000000000000000000001110000000000000000000000000000000000...
...000000000000000000000000000011011000000000000000000000000000000000...
...000000000000000000000000000110001100000000000000000000000000000000...
...000000000000000000000000000011011000000000000000000000000000000000...
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...000000000000000000000000000001110000000000000000000000000000000000...
...000000000000000000000000000000100000000000000000000000000000000000...
...000000000000000000000000000001110000000000000000000000000000000000...
...000000000000000000000000000011011000000000000000000000000000000000...
...000000000000000000000000000110001100000000000000000000000000000000...
...000000000000000000000000001100000110000000000000000000000000000000...
...000000000000000000000000011000000011000000000000000000000000000000...
...000000000000000000000000110000000001100000000000000000000000000000...
...000000000000000000000001100000000000110000000000000000000000000000...

....

This sequence has the form

$$\underbrace{1,111,11011,111,}_{5}\underbrace{1,111,11011,1100011,11011,111,}_{7}\underbrace{1,111,11011,1100011,110000011,\dots}_{9}$$

4) Mobile periodicals (II):

...00000000000000000000000000000000100000000000000000000000000000000000...
...00000000000000000000000000000001110000000000000000000000000000000000...
...00000000000000000000000000000011211000000000000000000000000000000000...
...00000000000000000000000000000001110000000000000000000000000000000000...
...00000000000000000000000000000000100000000000000000000000000000000000...
...00000000000000000000000000000001110000000000000000000000000000000000...
...00000000000000000000000000000011211000000000000000000000000000000000...
...00000000000000000000000000000112321100000000000000000000000000000000...
...00000000000000000000000000000011211000000000000000000000000000000000...
...00000000000000000000000000000001110000000000000000000000000000000000...
...00000000000000000000000000000000100000000000000000000000000000000000...
...00000000000000000000000000000001110000000000000000000000000000000000...
...00000000000000000000000000000011211000000000000000000000000000000000...
...00000000000000000000000000000112321100000000000000000000000000000000...
...00000000000000000000000000001123432110000000000000000000000000000000...
...00000000000000000000000000000112321100000000000000000000000000000000...
...00000000000000000000000000000011211000000000000000000000000000000000...
...00000000000000000000000000000001110000000000000000000000000000000000...
...00000000000000000000000000000000100000000000000000000000000000000000...
...00000000000000000000000000000001110000000000000000000000000000000000...
...00000000000000000000000000000011211000000000000000000000000000000000...
...00000000000000000000000000000112321100000000000000000000000000000000...
...00000000000000000000000000001123432110000000000000000000000000000000...
...00000000000000000000000000011234543211000000000000000000000000000000...
...00000000000000000000000000001123432110000000000000000000000000000000...
...00000000000000000000000000000112321100000000000000000000000000000000...
...00000000000000000000000000000011211000000000000000000000000000000000...
...00000000000000000000000000000001110000000000000000000000000000000000...
...00000000000000000000000000000000100000000000000000000000000000000000...
...00000000000000000000000000000001110000000000000000000000000000000000...
...00000000000000000000000000000011211000000000000000000000000000000000...
...00000000000000000000000000000112321100000000000000000000000000000000...
...00000000000000000000000000001123432110000000000000000000000000000000...
...00000000000000000000000000011234543211000000000000000000000000000000...
...00000000000000000000000000112345654321100000000000000000000000000000...
...00000000000000000000000000011234543211000000000000000000000000000000...
...00000000000000000000000000001123432110000000000000000000000000000000...
...00000000000000000000000000000112321100000000000000000000000000000000...
...00000000000000000000000000000011211000000000000000000000000000000000...
...00000000000000000000000000000001110000000000000000000000000000000000...
...00000000000000000000000000000000100000000000000000000000000000000000...
...00000000000000000000000000000001110000000000000000000000000000000000...
...00000000000000000000000000000011211000000000000000000000000000000000...
...00000000000000000000000000000112321100000000000000000000000000000000...
...00000000000000000000000000001123432110000000000000000000000000000000...
...00000000000000000000000000011234543211000000000000000000000000000000...
...00000000000000000000000000112345654321100000000000000000000000000000...
...00000000000000000000000001123456765432110000000000000000000000000000...

....

This sequence has the form

$$\underbrace{1, 111, 11211, 111,}_{5}\underbrace{1, 111, 11211, 1123211, 11211, 111,}_{7}\underbrace{1, 111, 11211, 1123211, 112343211, \ldots}_{9}$$

5) Infinite numbers (I):

...111111111111111111111111111111110111111111111111111111111111111111111...
...111111111111111111111111111111100011111111111111111111111111111111111...
...11111111111111111111111111111100100111111111111111111111111111111111...
...111111111111111111111111111111100011111111111111111111111111111111111...
...111111111111111111111111111111110111111111111111111111111111111111111...
...111111111111111111111111111111100011111111111111111111111111111111111...
...11111111111111111111111111111100100111111111111111111111111111111111...
...1111111111111111111111111111100111001111111111111111111111111111111...
...11111111111111111111111111111100100111111111111111111111111111111111...
...111111111111111111111111111111100011111111111111111111111111111111111...
...111111111111111111111111111111110111111111111111111111111111111111111...
...111111111111111111111111111111100011111111111111111111111111111111111...
...11111111111111111111111111111100100111111111111111111111111111111111...
...1111111111111111111111111111100111001111111111111111111111111111111...
...111111111111111111111111111100111110011111111111111111111111111111...
...1111111111111111111111111111100111001111111111111111111111111111111...
...11111111111111111111111111111100100111111111111111111111111111111111...
...111111111111111111111111111111100011111111111111111111111111111111111...
...111111111111111111111111111111110111111111111111111111111111111111111...
...111111111111111111111111111111100011111111111111111111111111111111111...
...11111111111111111111111111111100100111111111111111111111111111111111...
...1111111111111111111111111111100111001111111111111111111111111111111...
...111111111111111111111111111100111110011111111111111111111111111111...
...11111111111111111111111111100111111100111111111111111111111111111...
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...1111111111111111111111111111100111001111111111111111111111111111111...
...11111111111111111111111111111100100111111111111111111111111111111111...
...111111111111111111111111111111100011111111111111111111111111111111111...
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...1111111111111111111111111100111111111001111111111111111111111111...
...111111111111111111111111100111111111110011111111111111111111111111...

....

6) Infinite numbers (II):

...1111111111111111111111111111111211111111111111111111111111111111111...
...111111111111111111111111111111222111111111111111111111111111111111...
...11111111111111111111111111111223221111111111111111111111111111111111...
...111111111111111111111111111111222111111111111111111111111111111111...
...1111111111111111111111111111111211111111111111111111111111111111111...
...111111111111111111111111111111222111111111111111111111111111111111...
...11111111111111111111111111111223221111111111111111111111111111111111...
...111111111111111111111111111122343221111111111111111111111111111111...
...11111111111111111111111111111223221111111111111111111111111111111111...
...111111111111111111111111111111222111111111111111111111111111111111...
...1111111111111111111111111111111211111111111111111111111111111111111...
...111111111111111111111111111111222111111111111111111111111111111111...
...11111111111111111111111111111223221111111111111111111111111111111111...
...111111111111111111111111111122343221111111111111111111111111111111...
...111111111111111111111111111223454322111111111111111111111111111111...
...111111111111111111111111111122343221111111111111111111111111111111...
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...111111111111111111111111112234565432211111111111111111111111111111...
...111111111111111111111111122345676543221111111111111111111111111111...
...111111111111111111111111223456787654322111111111111111111111111111...

....

7) Car:

...000000000000000000000000000000000000000000000000000000000000000000...
...0000000000000000011111111111111111111111110000000000000000000000000...
...0000000000000000111111111111111111111111111000000000000000000000000...
...0000000000000000110000000000000000000000000110000000000000000000000...
...0000000000000001100000000000000000000000000011000000000000000000000...
...0000000111111111000000000000000000000000000000111111111000000000000...
...0000001111111110000000000000000000000000000000011111111100000000000...
...0000001100000000000000000000000000000000000000000000011000000000000...
...0000001100000000000000000000000000000000000000000000011000000000000...
...0000001100000444000000000000000000000000000000000444000110000000000...
...0000001111114444411111111111111111111111111111114444411110000000000...
...0000001111144444441111111111111111111111111111144444441110000000000...
...0000000000004444400000000000000000000000000000044444000000000000000...
...0000000000000444000000000000000000000000000000004440000000000000000...
...000000000000000000000000000000000000000000000000000000000000000000...
....

8) Finite lattice:

...0000000000000000000000000000000000000000000000000000000000000000000...
...0777000000000000700000007777777700777777770077007777777700777777770...
...0777000000000007770000007777777700777777770077007777777700777777770...
...0777000000000077077000000000770000000007700000770077700000000770000000...
...0777000000000770007700000007700000000077000007700777000000007777770000...
...0777000000007777777770000007700000000077000007700777000000007700000000...
...0777777000770000000770000077000000000770000077007777777770077777777770...
...0777777007700000000077000077000000000770000077007777777770077777777770...
...0000000000000000000000000000000000000000000000000000000000000000000...
....

9) Infinite lattice:

...1111111111111111111111111111111111111111111111111111111111111111111...
...1777111111111111711111117777777711777777771177117777777711777777771...
...1777111111111117771111117777777711777777771177117777777711777777771...
...1777111111111177177111111111771111111117711111771177711111111771111111...
...1777111111111771117711111117711111111177111117711777111111117777771111...
...1777111111117777777777111111771111111117711111771177711111111771111111...
...1777777111771111111177111117711111111177111117711777777777117777777771...
...1777777117711111111117711117711111111177111117711777777777117777777771...
...1111111111111111111111111111111111111111111111111111111111111111111...
....

Remark: of course, it's interesting to "design" a large variety of numerical <object sequences> in the same way. Their numbers may be infinte if the picture's background is zeroed, or infinite if the picture's background is not zeroed - as for the previous examples.

10) Multiplication:

Another way to multiply two integer numbers, A and B:

- let k be an integer ≥ 2;
- write A and B on two different vertical columns: $c(A)$, respectively $c(B)$;
- multiply A by k and write the product A_1 on the column $c(A)$;

- divide B by k, and write the integer part of the quotient B_1 on the column $c(B)$;

... and so on with the new numbers A_1 and B_1, until we get a $B_i < k$ on the column $c(B)$;

Then:

- write another column $c(r)$, on the right side of $c(B)$, such that:

  for each number of column $c(B)$, which may be a multiple of k plus the rest r (where $r = 0, 1, 2, \ldots, k-1$), the corresponding number on $c(r)$ will be r;

- multiply each number of column A by its corresponding r of $c(r)$, and put the new products on another column $c(P)$ on the right side of $c(r)$;

- finally add all numbers of column $c(P)$.

$A \times B =$ the sum of all numbers of $c(P)$.

Remark that any multiplication of integer numbers can be done only by multiplication with $2, 3, \ldots, k$, division by k, and additions.

This is a generalization of Russian multiplication (where $k = 2$).

This multiplication is usefull when k is very small, the best values being for $k = 2$ (Russian multiplication - known since Egyptian time), or $k = 3$. If k is greater than or equal to $\min\{10, B\}$, tjis multiplication is trivial (the obvious multiplication).

Example 1. *(if we choose $k = 3$):*

$73 \times 97 = ?$

x_3	/3		
$c(A)$	$c(B)$	$c(r)$	$c(P)$
73	*97*	*1*	*73*
219	*32*	*2*	*438*
657	*10*	*1*	*657*
1971	*3*	*0*	*0*
5913	*1*	*1*	*5913*
			7081 total

therefore: $73 \times 97 = 7081$.

Remark that any multiplication of integer numbers can be done only by multiplication with $2, 3$, divisions by 3, and additions.

Example 2. *(if we choose $k = 4$):*

$73 \times 97 = ?$

x_4	/4		
$c(A)$	$c(B)$	$c(r)$	$c(P)$
73	*97*	*1*	*73*
292	*24*	*0*	*0*
1168	*6*	*2*	*2336*
4672	*1*	*1*	*4672*
			7081 total

therefore: $73 \times 97 = 7081$.

Remark that any multiplication of integer numbers can be done only by multiplication with $2, 3, 4$, divisions by 4, and additions.

Example 3. *(if we choose $k = 5$):*

$73 \times 97 = ?$

x_5	/5		
$c(A)$	$c(B)$	$c(r)$	$c(P)$
73	*97*	*2*	*146*
365	*19*	*4*	*1460*
1825	*3*	*3*	*5475*
			7081 total

therefore: $73 \times 97 = 7081$.

Remark that any multiplication of integer numbers can be done only by multiplication with $2, 3, 4, 5$, divisions by 5, and additions.

This multiplication becomes less usefull when k increases.

Look at another example (4), what happens when $k = 10$:

Example 4. $73 \times 97 = ?$

$x_1 0$	/10		
$c(A)$	$c(B)$	$c(r)$	$c(P)$
73	*97*	*7*	*511* $(= 73 \times 7)$
730	*9*	*9*	*6570* $(= 73 \times 9)$
			7081 total

therefore: $73 \times 97 = 7081$.

Remark that any multiplication of integer numbers can be done only by multiplication with $2, 3, \ldots, 10$, divisions by 10, and additions - hence we obtain just the obvious multiplication!

11) Division by $k\hat{}n$:

Another way to devide an integer numbers A by $k\hat{}n$, where k, n are integers ≥ 2:

- write A and $k\hat{}n$ on two different vertical columns: $c(A)$, respectively $c(k\hat{}n)$;
- devide A by k, and write the integer quotient A_1 on the column $c(A)$;
- devide $k\hat{}n$ by k, and write the quotient $q_1 = k\hat{}(n-1)$ on the column $c(k\hat{}n)$;

... and so on with the new numbers A_1 and q_1, untill we get $q_n = 1 (= k\hat{}0)$ on the column $c(k\hat{}n)$;

Then:

- write another column $c(r)$, on the left side of $c(A)$, such that:

  for each number of column $c(A)$, which may be multiple of k plus the rest r (where $r = 0, 1, 2, \ldots, k-1$), the corresponding number on $c(r)$ will be r;
- write another column $c(P)$, on the left side of $c(r)$, in the following way: the element on line i (except the last line which is 0) will be $k\hat{}(i-1)$;
- multiply each number of column $c(P)$ by its corresponding r of $c(r)$, and put the new products on another column $c(R)$ on the left side of $c(P)$;
- finally add all numbers of column $c(R)$ to get the final rest R, while the final quotient will be stated in front of $c(k\hat{}n)$'s 1.

Therefore:

$A/(k\hat{}n) = A_n$ and rest R_n.

Remark that any division of an integer number by $k\hat{}n$ can done only by divisions to k, calculations of powers of k, multiplications with $1, 2, \ldots, k-1$, additions.

This division is usefull when k is small, the best values being when k is an one-digit number, and n large. If k is very big and n very small, this division becomes useless.

Example 1. $1357/(2^7) =?$

			/2	/2	
c(R)	c(P)	c(r)	c(A)	c(2^7)	
1	2^0	*1*	*1357*	2^7	*line*$_1$
0	2^1	*0*	*678*	2^6	*line*$_2$
4	2^2	*1*	*339*	2^5	*line*$_3$
8	2^3	*1*	*169*	2^4	*line*$_4$
0	2^4	*0*	*84*	2^3	*line*$_5$
0	2^5	*0*	*42*	2^2	*line*$_6$
64	2^6	*1*	*21*	2^1	*line*$_7$
			10	2^0	*last_line*
77					

Therefore: $1357/(2^7) = 10$ *and rest* 77.

Remark that the division of an integer number by any power of 2 can be done only by divisions to 2, calculations of power of 2, multiplications and additions.

Example 2. $19495/(3^8) =?$

			/3	/3	
c(R)	c(P)	c(r)	c(A)	c(3^8)	
1	3^0	*1*	*19495*	3^8	*line*$_1$
0	3^1	*0*	*6498*	3^7	*line*$_2$
0	3^2	*0*	*2166*	3^6	*line*$_3$
54	3^3	*2*	*722*	3^5	*line*$_4$
0	3^4	*0*	*240*	3^4	*line*$_5$
486	3^5	*2*	*80*	3^3	*line*$_6$
1458	3^6	*2*	*26*	3^2	*line*$_7$
4374	3^7	*2*	*8*	3^1	*line*$_8$
			2	3^0	*last_line*
6373					

Therefore: $19495/(3^8) = 2$ *and rest* 6373.

Remark that the division of an integer number by any power of 3 can be done only by divisions to 3, calculations of power of 3, multiplications and additions.

References

[1] Alain Bouvier et Michel George, sous la deriction de Francois Le Lionnais, "Dictionnaire des Mathematiques", Presses Universitaires de France, Paris, 1979, p. 659;

"The Florentin Smarandache papers" special collection, Arizona State University, Tempe, AZ 85287.

12) Almost prime of first kind:

$a_1 \geq 2$, and for $n \geq 1$ a_{n+1} is the smallest number that is not divisible by any of the previous terms (of the sequence) $a_1, a_2, \ldots, a_n$.

Example for $a_1 = 10$:

10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 25, 27, 29, 31, 35, 37, 41, 43, 47, 49, 53, 57, 61, 67, 71, 73, ...

If one starts by $a_1 = 2$, it obtains the complete prime sequence and only it.

If one starts by $a_2 > 2$, it obtains after a rank r, where $a_r = p(a_1)^2$ with $p(x)$ the strictly superior prime part of x, i.e. the largest prime strictly less than x, the prime sequence:

- between a_1 and a_r, the sequence contains all prime numbers of this interval and some composite numbers;

- from a_{r+1} and up, the sequence contains all prime numbers greater than a_r and no composite numbers.

13) Almost primes of second kind:

$a_1 \geq 2$, and for $n \geq 1$ a_{n+1} is the smallest number that is coprime with all of the previous terms (of the sequence) $a_1, a_2, \ldots, a_n$.

This second kind sequence merges faster to prime numbers than the first kind sequence.

Example for $a_1 = 10$:

10, 11, 13, 17, 19, 21, 23, 29, 31, 37, 41, 43, 47, 53, 57, 61, 67, 71, 73, ...

If one starts by $a_1 = 2$, it obtains the complete prime sequence and only it.

If one starts by $a_2 > 2$, it obtains after a rank r, where $a_r = p_i p_j$ with p_i and p_j prime number strictly less than and not dividing a_1, the prime sequence:

- between a_1 and a_r, the sequence contains all prime numbers of this interval and some composite numbers;

- from a_{r+1} and up, the sequence contains all prime numbers greater than a_r and no composite numbers.