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On Fuzzy Soft Matrix Based on Reference Function

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Abstract**—**In this paper we study fuzzy soft matrix based on reference function.Firstly, we define some new operations such as fuzzy soft complement matrix and trace of fuzzy soft matrix based on reference function.Then, we introduced some related properties, and some examples are given. Lastly, we define a new fuzzy soft matrix decision method based on reference function.

Index Terms—Soft set, fuzzy soft set, fuzzy soft set based on reference function, fuzzy soft matrix based on reference function.

I. INTRODUCTION

Fuzzy set theory was proposed by LotfiA.Zadeh[1] in 1965,where each element (real valued) [0, 1] had a degree of membership defined on the universe of discourse X, the theory has been found extensive application in various field to handle uncertainty. Therefore,several researches were conducted on the generalization on the notions of fuzzy sets such as intuitionistic fuzzy set proposed by Atanassov[2,3], interval valued fuzzy set[5]. In the literature we found many well –known theories to describe uncertainty: rough set theory[6]..etc, but all of these theories have their inherit difficulties as pointed by Molodtsov in his pioneer work[7].The concept introduced by Molodtsov is called "soft set theory" which is set valued mapping. This new mathematical model is free from the difficulties mentioned above.Since its introduction, the concept of soft set has gained considerable attention and this concept has resulted in a series of work [8, 9,10,11,12, 13, 14].

Also as we know, matrices play an important role in science and technology. However, the classical matrix theory sometimes fails to solve the problems involving uncertainties,occurring in an imprecise environment. In [4] Thomason, introduced the fuzzy matrices to represent fuzzy relation in a system based on fuzzy set theory and discussed about the convergence of powers of fuzzy matrix. In [15,16,17],some important results on determinant of a square fuzzy matrices are discussed .Also,Ragab et al. [18,19] presented some properties of the min-max composition of fuzzy matrices. Later on, several studies and some applications of fuzzy matrices are defined in [20,21].

In 2010,Cagmanet al [13] defined soft matrix which is representation of soft set, to make operations in theoretical studies in soft set more functional. This representation has several advantages, it's easy to store and manipulate matrices and hence the soft sets represented by them in a computer.

Recently severalresearch have been studied the connection between soft set and soft matrices [13,14,22]. Later,Maji et al [9] introduced the theory of fuzzy soft set and applied it to decision making problem. In 2011, Yang and C.Ji[22],defined fuzzy soft matrix (FSM) which is very useful in representing and computing the data involving fuzzy soft sets.

The concept of fuzzy set based on reference function was first introduced by Baruah[23,24,25] in the following manner - According to him, to define a fuzzy set, two functions namely fuzzy membership function and fuzzy reference function are necessary. Fuzzy membership value is the difference between fuzzy membership function and reference function. Fuzzy membership function and fuzzy membership value are two different things. In [26, 27] M.Dhar applied this concept to fuzzy square matrix and developed some interesting properties as determinant, trace and so on. Thereafter, in [28], T.J.Neog, D. K.Sutwere extended this new concept to soft set theory, introducing a new concept called "fuzzy soft set based on fuzzy reference function". Recently,Neog. T.J, Sut D.K,M.Bora[29] combinedfuzzy soft set based on reference function with soft matrices. The paper unfolds as follows. The next section briefly introduces some definitions related tosoft set,fuzzy soft set, and fuzzy soft setbased on reference function. Section 3 presents fuzzy soft complement matrix based on reference function. Section 4presentstrace of fuzzy soft matrix based on reference function..Section5presentsnew fuzzy soft matrix theory in decision making.Conclusions appear in the last section.

II. PRELIMINARIES

In this section first we review some concepts and definitions of soft set,fuzzy soft set, and fuzzy soft set based on reference functionfrom [9,12,13,29], which will be needed in the sequel.

Remark:

For the sake of simplicity we adopt the following notation of fuzzy soft set based on reference function defined in our way as: Fuzzy soft set based on reference =$(F, A)_{rf}$

To make the difference between the notation (F, A) defined for classical soft set or its variants as fuzzy soft set.

2.1.Definition (Soft Set [13])

Suppose that U is an initial universe set and E is a set of parameters, let P(U) denotes the power set of U.A pair(F,E) is called a soft set over U where F is a mapping given by F: E→P(U).Clearly, a soft set is a mapping from parameters to P(U),and it is not a set, but a parameterized family of subsets of the universe.

2.2. Example.

Suppose that U={s1,s2,s3,s4} is a set of students and E={e1,e2,e3} is a set of parameters, which stand for result, conduct and sports performances respectively. Consider the mapping from parameters set E to the set of all subsets of power set U.Then soft set (F,E) describes the character of the students with respect to the given parameters, for finding the best student of an academic year.

(F, E) = {{result = s1, s3, s4} {conduct = s1,s2 } {sports performances = s2,s3,s4 }}

2.3. Definition (FuzzySoft Set [9, 12])

Let U be an initial universe set and E be the set of parameters. Let A⊆E .A pair (F,A) is called fuzzy soft set over U where F is a mapping given by F: $A \rightarrow F^u$,whereF^udenotes the collection of all fuzzy subsets of U.

2.4.Example.

Consider the example2.2,in soft set(F,E),if s1 is medium in studies, we cannot expressed with only the two numbers 0 and 1,we can characterize it by a membership function instead of the crisp number 0 and 1,which associates with each element a real number in the interval [0,1].Then fuzzy soft set can describe as

(F, A)={F(e1) = {(s1,0.9), (s2,0.3), (s3,0.8), (s4,0.9)}, F(e2) = {(s1,0.8), (s2,0.9), (s3,0.4), (s4,0.3)}},where A={ e1,e2}.

In the following, Neog et al. [29] showed by an example that this definition sometimes gives degenerate cases and revised the above definition as follows:

2.5 .Definition [29]

Let A (μ_1,μ_2)={x,$\mu_1(x)$, $\mu_2(x)$; x ∈ U } and B (μ_3,μ_4) ={x, $\mu_3(x)$, $\mu_4(x)$; x ∈ U } be two fuzzy sets defined over the same universe U.

Then the operations intersection and union are defined asA (μ_1, μ_2) ∩ B (μ_3, μ_4)= {x, min($\mu_1(x)$,$\mu_3(x)$) ,max($\mu_2(x)$,$\mu_4(x)$) ; x ∈ U } and A (μ_1, μ_2) ∪ B (μ_3, μ_4)= {x, max($\mu_1(x)$,$\mu_3(x)$) ,min($\mu_2(x)$,$\mu_4(x)$) ; x ∈ U }

2.6.Definition [29]

Let A (μ_1,μ_2)={x, $\mu_1(x)$, $\mu_2(x)$; x ∈ U } and B (μ_3,μ_4)={x, $\mu_3(x)$, $\mu_4(x)$; x ∈ U } be two fuzzy sets defined over the same universe U.To avoid degenerate cases we assume that min($\mu_1(x)$, $\mu_3(x)$) ≥ max($\mu_2(x)$,$\mu_4(x)$) for all x∈ U.

Then the operations intersection and union are defined as A (μ_1, μ_2) ∩ B (μ_3, μ_4)= {x, min($\mu_1(x)$,$\mu_3(x)$) ,max($\mu_2(x)$,$\mu_4(x)$) ; x ∈ U } and A (μ_1, μ_2) ∪ B (μ_3, μ_4)= {x, max($\mu_1(x)$,$\mu_3(x)$) ,min($\mu_2(x)$,$\mu_4(x)$) ; x ∈ U }

2.7. Definition[29]

For usual fuzzy setsA $(\mu, 0)$={x, $\mu(x)$,0 ; x ∈ U }and B $(1, \mu)$={x, 1 , $\mu(x)$; x ∈ U } defined over the same universe U, we have A $(\mu, 0)$ ∩ B $(1, \mu)$= {x, min((x), 1) ,max(0 ,$\mu(x)$) ; x ∈ U }= {x, $\mu(x)$, $\mu(x)$; x ∈ U }, which is nothing but the null setφ and A $(\mu, 0)$ ∪ B $(1, \mu)$= {x, max($\mu(x)$, 1) ,min(0, $\mu(x)$) ; x ∈ U } = {x,1, 0 ; x ∈ U }, which is nothing but the universal set U.

This means if we define a fuzzy set$(A(\mu, 0))^c$ ={x, 1, $\mu(x)$; x ∈ U } it is nothing but the complement ofA $(\mu, 0)$={x, $\mu(x)$,0 ; x ∈ U }.

2.8. Definition[29]

Let A (μ_1,μ_2)={x, $\mu_1(x)$, $\mu_2(x)$; x ∈ U } and B (μ_3,μ_4)={x, $\mu_3(x)$, $\mu_4(x)$; x ∈ U } be two fuzzy sets defined over the same universe U.The fuzzy setA $(\mu_1,\mu_2)$is a subset of the fuzzy set B (μ_3,μ_4)if for all x ∈ U , $\mu_1(x) \leq \mu_3(x)$ and $\mu_4(x) \leq \mu_2(x)$.

Two fuzzy setsC={x, $\mu_C(x)$x ∈ U }and D={x, $\mu_D(x)$; x ∈ U } in the usual definition would be expressed asC$(\mu_C, 0)$={x, $\mu_C(x)$, 0; x ∈ U }and D$(\mu_D, 0)$={x, $\mu_D(x)$,0; x ∈ U }

Accordingly, we have C$(\mu_C, 0)$ ⊆ D$(\mu_D, 0)$ if for all x ∈ U, $\mu_C(x) \leq \mu_D(x)$, which can be obtained by putting$\mu_2(x) = \mu_4(x)$=0 in the new definition.

2.9 .Defintion [29] (Fuzzy soft matrices (FSMs) based on reference function)

Let U be an initial universe, E be the set of parameters and A ⊆ E. Let $(f_A$, E) be fuzzy soft set (FS) over U. Then a subset of U ×E is uniquely defined by R_A = {(u, e); e∈ A, u∈ $f_A(e)$} which is called a relation form of $(f_A$, E).

2.10. Example

Assume that U ={u_1 ,u_2 ,u_3 ,u_4 } is a universal setand E ={ e_1 , e ,e_3 ,e_4 ,e_5 } be the set of parameters and A={e_1 ,e_2 ,e_3 } ⊆ E
and

$f_A(e_1)$ ={ $u_1/(0.7,0)$, $u_2/(0.1,0)$, $u_3/(0.2,0)$, $u_4/(0.6,0)$ }
$f_A(e_2)$ ={ $u_1/(0.8,0)$, $u_2/(0.6,0)$, $u_3/(0.1,0)$, $u_4/(0.5,0)$ }
$f_A(e_3)$ ={ $u_1/(0.1,0)$, $u_2/(0.2,0)$, $u_3/(0.7,0)$, $u_4/(0.3,0)$ }

Then the fuzzy soft set (f_A , E) is a parameterized family { $f_A(e_1)$, $f_A(e_2)$,$f_A(e_3)$ } of all fuzzy soft sets over U. Then the relation form of (f_A , E) is written as

TABLE 1.The relation form of (f_A , E)

R_A	e_1	e_2	e_3	e_4
u_1	(0.7, 0)	(0.8, 0)	(0.1, 0)	(0, 0)
u_2	(0.1, 0)	(0.6, 0)	(0.2, 0)	(0, 0)
u_3	(0.2, 0)	(0.1, 0)	(0.7, 0)	(0, 0)
u_4	(0.6, 0)	(0.5, 0)	(0.3, 0)	(0, 0)

Hence,the fuzzy soft matrix representing this fuzzy soft set would be represented as

$$A = \begin{bmatrix} (0.7,0)(0.8,0)(0.1,0)(0,0) \\ (0.1,0)(0.6,0)(0.2,0)(0,0) \\ (0.2,0)(0.1,0)(0.7,0)(0,0) \\ (0.6,0)(0.5,0)(0.3,0)(0,0) \end{bmatrix}$$

2.11. Definition[29]

We define the membership value matrix corresponding to the matrix A as MV(A) =[$\delta_{ji}(c_i)$] Where $\delta_{(A)ij}$ = $\mu_{j1}(c_i)$- $\mu_{j2}(c_i)$ i= 1,2,3,…,m and j =1,2,3…..,n ,where $\mu_{j1}(c_i)$ and $\mu_{j2}(c_i)$ represent the fuzzy membership function and fuzzy reference function respectively ofc_i in the fuzzy set F(e_j).

2.12. Definition [29]

Let the fuzzy soft matrices corresponding to the fuzzy soft sets (F,E), and (G,E) beA=[a_{ij}] ∈ $FSM_{m\times n}$, B=[b_{ij}]where a_{ij} = ($\mu_{j1}(c_i)$, $\mu_{j2}(c_i)$) and b_{ij} =($\chi_{j1}(c_i)$,$\chi_{j2}(c_i)$),i =1,2,3,…, m ;j =1,2,3,…,n ;Then Aand Bare called fuzzy soft equal matrices denoted byA=B, if $\mu_{j1}(c_i) = \chi_{j1}(c_i)$ and $\mu_{j1}(c_i) = \chi_{j1}(c_i)$ for all i,j.

In [13], the 'addition (+)' operation between two fuzzy soft matrices is defined as follows

2.13. Definition [29]

Let U={c_1,c_2,c_3,……,c_m} be the universal set and Ebe the set of parameters given by E={e_1,c_2,e_3,……,e_n}.Let the set of all m× n fuzzy soft matrices over U be $FSM_{m\times n}$.

Let A , B ∈ $FSM_{m\times n}$,where A= $[a_{ij}]_{m\times n}$, a_{ij} = ($\mu_{j1}(c_i)$, $\mu_{j2}(c_i)$) and B= $[b_{ij}]_{m\times n}$, b_{ij} =($\chi_{j1}(c_i)$,$\chi_{j2}(c_i)$).To avoid degenerate cases we assume that**min** ($\mu_{j1}(c_i)$,$\chi_{j1}(c_i)$) ≥**max** ($\mu_{j2}(c_i)$,$\chi_{j2}(c_i)$) for all i and j .The operation of 'addition (+)' between A and B is defined as **A+ B=C ,where** C= $[c_{ij}]_{m\times n}$,c_{ij} =(max ($\mu_{j1}(c_i)$,$\chi_{j1}(c_i)$), min ($\mu_{j2}(c_i)$, $\chi_{j2}(c_i)$))

2.14. Example

Let U={c_1,c_2,c_3,c_4 } be the universal set and Ebe the set of parameters given by E={e_1,e_2,e_3 }
We consider the fuzzy soft sets based on reference function.

(F,E)={F(e_1)={(c_1,0.3, 0)),(c_2,0.5, 0), (c_3, 0.6, 0), (c_4,0.5,0)},F(e_2)={(c_1,0.7, 0)),(c_2,0.9, 0), (c_3, 0.7, 0), (c_4,0.8,0)},F(e_3)={(c_1,0.6, 0)),(c_2,0.7, 0), (c_3, 0.7, 0), (c_4,0.3,0)}}.

(G,E)={G(e_1)={(c_1,0.8, 0)),(c_2,0.7, 0), (c_3, 0.5, 0), (c_4,0.4,0)}, G(e_2)={(c_1,0.9, 0)),(c_2,0.9, 0), (c_3, 0.8, 0), (c_4,0.7,0)},G(e_3)={(c_1,0.5, 0)),(c_2,0.9, 0), (c_3, 0.6, 0), (c_4,0.8,0)}}.

The fuzzy soft matrices based on reference function representing these two fuzzy soft sets are respectively

$$A = \begin{bmatrix} (0.3,0)(0.7,0)(0.6,0) \\ (0.5,0)(0.9,0)(0.7,0) \\ (0.6,0)(0.7,0)(0.7,0) \\ (0.5,0)(0.8,0)(0.3,0) \end{bmatrix}, B = \begin{bmatrix} (0.8,0)(0.9,0)(0.5,0) \\ (0.7,0)(0.9,0)(0.9,0) \\ (0.5,0)(0.8,0)(0.6,0) \\ (0.4,0)(0.7,0)(0.8,0) \end{bmatrix}$$

$$\text{HereA+B} = \begin{bmatrix} (0.8,0)(0.9,0)(0.6,0) \\ (0.7,0)(0.9,0)(0.9,0) \\ (0.6,0)(0.8,0)(0.7,0) \\ (0.5,0)(0.8,0)(0.8,0) \end{bmatrix}$$

III. FUZZY SOFT COMPLEMENT MATRIX BASED ON REFERENCE FUNCTION

In this section ,westart by introducing the notion of the fuzzy soft complement matrix based on reference function,and we prove some formal properties.

3.1. Definition

Let A= $[(a_{ij},0)]_{m\times n}$ ∈ $FSM_{m\times n}$ according to the definition in [26], then A^c is calledfuzzy soft complement matrix if $A^c = [(1, a_{ij})]_{m\times n}$ for all a_{ij} ∈ [0 , 1].

3.2 .Example

Let A =$\begin{bmatrix} (0.7,0)(0.8,0) \\ (0.1,0)(0.6,0) \end{bmatrix}$ be fuzzy soft matrix based on reference function, then the complement of this matrix is $A^c = \begin{bmatrix} (1,0.7)(1,0.8) \\ (1,0.1)(1,0.6) \end{bmatrix}$.

3.3. Proposition

Let A, B be two fuzzy soft matrix based on fuzzy reference function .Then

(i)$(A^c)^T = (A^T)^c$ (1)

(ii) $(A^c + B^c)^T = (A^T)^c + (B^T)^c$ (2)

Proof:

To show (i)

$(A^c)^T = (A^T)^c$

We have, let A ∈ $FSM_{m\times n}$, then

$$A = [(\mu_{j1}(c_i), \mu_{j2}(c_i)]$$
$$A^c = [1, \mu_{j1}(c_i)]$$
$$(A^c)^T = [1, \mu_{i1}(c_j)]$$
$$\text{For } A^T = [(\mu_{i1}(c_j), \mu_{i2}(c_j)],$$

we have

$$(A^T)^c = [1, \mu_{i1}(c_j)]$$
$$\text{Hence } (A^c)^T = (A^T)^c$$

The proof of (ii) follows similar lines as above.

3.4. Example

$$\text{Let } A = \begin{bmatrix} (0.2,0)(0.3,0) \\ (0.1,0)(0.4,0) \end{bmatrix}, B = \begin{bmatrix} (0.5,0)(0.4,0) \\ (0.6,0)(0.2,0) \end{bmatrix}$$

$$A^c = \begin{bmatrix} (1,0.2)(1,0.3) \\ (1,0.1)(1,0.4) \end{bmatrix}, B^c = \begin{bmatrix} (1,0.5)(1,0.4) \\ (1,0.6)(1,0.2) \end{bmatrix}$$

$$(A^c)^T = \begin{bmatrix} (1,0.2)(1,0.1) \\ (1,0.3)(1,0.4) \end{bmatrix}, (B^c)^T = \begin{bmatrix} (1,0.5)(1,0.6) \\ (1,0.4)(1,0.2) \end{bmatrix}, (A^T)^c + (B^T)^c = \begin{bmatrix} (1,0.2)(1,0.1) \\ (1,0.3)(1,0.2) \end{bmatrix}$$

$$A^c + B^c = \begin{bmatrix} (1,0.2)(1,0.3) \\ (1,0.1)(1,0.2) \end{bmatrix}, (A^c + B^c)^T = \begin{bmatrix} (1,0.2)(1,0.1) \\ (1,0.3)(1,0.2) \end{bmatrix}$$

Then

$$(A^c + B^c)^T = (A^T)^c + (B^T)^c.$$

IV. TRACE OF FUZZY SOFT MATRIXBASED ON REFERENCE FUNCTION

In this section we extend the concept of trace of fuzzy square matrix proposed M. Dhar[26] to fuzzy soft square matrix based on reference function, and we prove some formal properties.

4.1. Definition

Let A be a square matrix. Then the trace ofthe matrix A is denoted by tr A and is defined as:

$$\text{trA} = (\max(\mu_{ii}), \min(r_{ii})) \quad (3)$$

where μ_{ii} stands for the membership functions lying along the principal diagonal and r_{ii} refers to the reference function of the corresponding membership functions.

4.2. Proposition

Let A and B be two fuzzy softsquare matrices each of order n.

Then

$$\text{tr}(A+B) = \text{trA} + \text{trB} \quad (4)$$

proof.

We have from the proposed definition of trace of fuzzy soft matrices

$$\text{trA} = (\max a_{ii}, \min r_{ii})$$

and

$$\text{trB} = (\max b_{ii}, \min r'_{ii})$$

then

$$A+B = C \text{ where } C = [c_{ij}]$$

Following the definition of addition of twofuzzy soft matrices, we have

$$C_{ij} = (\max(a_{ii}, b_{ii}), \min(r_{ii}, r'_{ii}))$$

According to definition 4.1 the trace of fuzzy soft matrixbased on reference function would be:

tr (C) $= [\max\{\max(a_{ii}, b_{ii})\}, \min\{\min(r_{ii}, r'_{ii})\}]$

$= [\max\{\max(a_{ii}), \max(b_{ii})\}, \min\{\min(r_{ii}), \min(r'_{ii})\}]$

= trA+trB,

Conversely,

trA+trB $= [\max\{\max(a_{ii}), \max(b_{ii})\}, \min\{(\min(r_{ii}), \min(r'_{ii}))\}]$

$= [\max\{\max(a_{ii}, b_{ii}), \min(\min(r_{ii}, r'_{ii})\}]$

=tr(A+B)

hence the resulttrA+trB =tr(A+B)

4.3. Example:

Let us consider the following two fuzzy soft matrices A and Bbased on reference function for illustration purposes

$$A = \begin{bmatrix} (0.3,0)(0.7,0)(0.8,0) \\ (0.4,0)(0.5,0)(0.3,0) \\ (0.6,0)(0.1,0)(0.4,0) \end{bmatrix} \text{and} B = \begin{bmatrix} (1,0)(0.2,0)(0.3,0) \\ (0.8,0)(0.5,0)(0.2,0) \\ (0.5,0)(1,0)(0.8,0) \end{bmatrix}$$

The addition of two soft matrices would be:

$$A+B = \begin{bmatrix} (1,0)(0.7,0)(0.8,0) \\ (0.8,0)(0.5,0)(0.3,0) \\ (0.6,0)(1,0)(0.8,0) \end{bmatrix}$$

Using the definition of trace of fuzzy soft matrices, we see the following results:

tr A = { max(0.3, 0.5, 0.4), min (0, 0, 0)}=(0.5 , 0)

tr B = { max(1, 0.5, 0.8), min (0, 0, 0)}=(1 , 0)

Thus we have

trA+trB = { max(1, 0.5, 0.8) , min (0, 0, 0)}=(1 , 0)

And

tr (A+B) = { max(1, 0.5, 0.8) , min (0, 0, 0)}= (1, 0)

Hence the result

$$\text{trA+trB} = \text{tr(A+B)}$$

4.4.Proposition

Let A = $[a_{ij}, r_{ij}]$ ∈ $\text{FSM}_{m\times n}$ be fuzzy soft squarematrix of order n, if λ is a scalar such that $0 \le \lambda \le 1$. Then tr(λ A)= λ tr(A) (5)

proof.
to prove
tr(λ A) = λtrA $0 \leq \lambda \leq 1$
we have
tr(λ A) ={ max(λa_{ii}), min (λr_{ii})}
=λ{ max(a_{ii}), min(r_{ii})}
= λtr (A)

4.5. Example

$$\text{Let } A = \begin{bmatrix} (0.3,0)(0.7,0)(0.8,0) \\ (0.4,0)(0.5,0)(0.3,0) \\ (0.6,0)(0.1,0)(0.4,0) \end{bmatrix} \text{and } \lambda = 0.5$$

Then

$$\lambda A = \begin{bmatrix} (0.15,0)(0.35,0)(0.40,0) \\ (0.20,0)(0.25,0)(0.15,0) \\ (0.30,0)(0.05,0)(0.20,0) \end{bmatrix}$$

tr(λ A) = { max(0.15, 0.25, 0.20), min ((0, 0, 0)}
= (0.25,0)
Again
tr A=(0.5,0)
and hence
tr(λ A) =0.5 (0.5, 0) = (0.25,0)

4.6. Proposition:

Let A=[a_{ij} , r_{ij}] $\in \mathrm{FSM}_{m \times n}$ be fuzzy soft square matrices each of order n .
Then
trA=tr(A^t),where A^t is the transpose of A

4.7.Example

$$\text{Let } A^t = \begin{bmatrix} (0.3,0)(0.4,0)(0.6,0) \\ (0.7,0)(0.5,0)(0.1,0) \\ (0.8,0)(0.3,0)(0.4,0) \end{bmatrix}$$

Then
tr(A^t)= { max(0.3, 0.5, 0.4) , min (0, 0, 0)}
=(0.5, 0)
HencetrA=tr(A^t)

The same result will hold if we consider the complements of fuzzy soft square matrices.

$$A^c = \begin{bmatrix} (1,0.3)(1,0.7)(1,0.8) \\ (1,0.4)(1,0.5)(1,0.3) \\ (1,0.6)(1,0.1)(1,0.4) \end{bmatrix}$$

tr A^c ={max (1, 1, 1), min (0.3, 0.5, 0.4)}=(1, 0.3)
If we consider another fuzzy soft matrice B:

$$B = \begin{bmatrix} (1,0)(0.2,0)(0.3,0) \\ (0.8,0)(0.5,0)(0.2,0) \\ (0.5,0)(1,0)(0.8,0) \end{bmatrix}$$

$$B^c = \begin{bmatrix} (1,1)(1,0.2)(1,0.3) \\ (1,0.8)(1,0.5)(1,0.2) \\ (1,0.5)(1,1)(1,0.8) \end{bmatrix}$$

Then the trace of B^c will be the following:
tr(B^c)={max(1,1,1),min (1, 0.5, 0.8)}=(1, 0.5)

Following the definition 2.13 of addition of two fuzzy soft matrices based on reference function, we have.

$$A^c + B^c = \begin{bmatrix} (1,0.3)(1,0.2)(1,0.3) \\ (1,0.4)(1,0.5)(1,0.2) \\ (1,0.5)(1,0.1)(1,0.4) \end{bmatrix}$$

tr (A^c +B^c) ={max(1,1,1),min (0.3, 0.5, 0.4)}
= (1, 0.3)

V. NEW FUZZY SOFT MATRIX THEORY IN DECISION MAKING

In this section we adopted the definition of fuzzy soft matrix decision method proposed by P. Rajarajeswari,P. Dhanalakshmi in [30] to the case of fuzzy soft matrix based on reference function in order to define a new fuzzy soft matrix decision method based on reference function.

5.1. Definition: (Value Matrix)

Let A=[a_{ij} , 0] $\in [\mathrm{FSM}]_{m \times n}$ .Then we define the value matrix of fuzzy soft matrix A based on reference function as V(A)=[a_{ij}] =[a_{ij} - r_{ij}], i=1, 2, ,….,m , j= 1,2,3,…, n, where r_{ij} = $[0]_{m \times n}$.

5.2. Definition:(Score Matrix)

If A=[a_{ij}] $\in$ FSM,B=[b_{ij}] $\in [\mathrm{FSM}]_{m \times n}$ .Then we define score matrix of A and B as:

$s_{A,B}$=[d_{ij}]$_{mxn}$where [d_{ij}]=V(A)-V(B)

5.3. Definition:(Total Score)

If A=[a_{ij} , 0] $\in [\mathrm{FSM}]_{m \times n}$,B=[b_{ij} , 0] $\in [\mathrm{FSM}]_{m \times n}$.Let the corresponding value matrices be V(A),V(B) and their score matrix is $s_{A,B}$ =[d_{ij}]$_{mxn}$then we define total score for each c_i in U as $s_i = \sum_{j=1}^{n} d_{ij}$.

Methodology and algorithm

Assume that there is a set of candidates(programmer), U={c_1 , c_2, ,…,c_n} is a set of candidates to be recruited by software development organization in programmer post.Let E is a set of parameters related to innovative attitude of the programmer. We construct fuzzy soft set (F,E)over U represent the selection of candidate by field expert X,where F is a mapping F:E→ F^u,F^u is the collection of all fuzzy subsets of U. We further construct another fuzzy soft set (G,E)over Urepresent the selection of candidate by field expert Y,where G is a mapping G:E → F^u ,F^u is the collection of all fuzzy subsets of U.The matricesA and B corresponding to the fuzzy softsets (F,E) and (G,E) are constructed,we compute the complementsand theirmatrices A^c and B^c corresponding to (F, E) ^{c} and (G, E) ^{c} respectively. Compute A+B which is themaximum membership of selection of candidates by the judges. Compute $A^c + B^c$ which is the maximum membership of non selection of candidates by the judges. usingdef (5.1) ,Compute V(A+B),V($A^c + B^c$) $s_{((A+B),(A^c+B^c))}$ and the total score S_i for each candidate in U.Finally find S_j = max(S_i),then conclude that the candidate c_j has selected by the judges.If S_j has more than one value the process is repeated by reassessing the parameters.

Now, using definitions 5-1, 5-2 and 5-3 we can construct a fuzzy soft matrix decision making method based on reference function by the following algorithm.

Algorithm

Step1:Input the fuzzy soft set (F, E), (G,E) and obtain the fuzzy soft matrices A,B corresponding to (F,E) and (G, E) respectively.

Step2:Write the fuzzy soft complement set $(F,E)^c$, $(G,E)^c$ and obtain the fuzzy soft matrices A^c , B^c corresponding to $(F,E)^c$,and $(G,E)^c$ respectively.

Step3:Compute (A+B),(A^c+B^c), V(A+B),V(A^c+B^c) and $s_{((A+B),(A^c+B^c))}$.

Step4:Compute the total score S_i for each c_i in U.

Step5:Find c_i for which max (S_i) .

Then we conclude that the candidate c_i is selected for the post.

Incase max S_i occurs for more than one value, then repeatthe process by reassessing the parameters.

Case Study

Let (F,E) and (G,E) be two fuzzy soft set based on reference function representing the selection of four candidates from the universal set U= {c_1,c_2,c_3,c_4} by the experts X,and Y.Let E = {e_1, e_2,e_3} be the set of parameters which stand for intelligence,innovative and analysis .

(F,E)={F(e_1)={(c_1,0.1, 0)),(c_2,0.5, 0), (c_3, 0.1, 0), (c_4,0.4,0)},F(e_2)={(c_1,0.6, 0)),(c_2,0.4, 0), (c_3, 0.5, 0), (c_4,0.7,0)},F(e_3)={(c_1,0.5, 0)),(c_2,0.7, 0), (c_3, 0.6, 0), (c_4, 0.5, 0)}}.

(G,E)={G(e_1)={(c_1,0.2, 0)),(c_2,0.6, 0), (c_3, 0.2, 0), (c_4,0.3,0)}, G(e_2)={(c_1,0.6, 0)),(c_2,0.5, 0), (c_3, 0.6, 0), (c_4,0.8,0)}, ,G(e_3)={(c_1,0.5, 0)),(c_2,0.8, 0), (c_3, 0.7, 0), (c_4,0.5,0)}}.

These two fuzzy soft sets based on reference function are represented by the following fuzzy soft matrices based on reference function respectively

$$A = \begin{bmatrix} (0.1,0) & (0.6,0) & (0.5,0) \\ (0.5,0) & (0.4,0) & (0.7,0) \\ (0.1,0) & (0.5,0) & (0.6,0) \\ (0.4,0) & (0.7,0) & (0.5,0) \end{bmatrix} B = \begin{bmatrix} (0.2,0.) & (0.6,0) & (0.5,0) \\ (0.6,0) & (0.5,0) & (0.8,0) \\ (0.2,0) & (0.6,0) & (0.7,0) \\ (0.3,0) & (0.8,0) & (0.5,0) \end{bmatrix}$$

Then, the fuzzy soft complement matricesbased on reference function are

$$A^c = \begin{bmatrix} (1,0.1) & (1,0.6) & (1,0.5) \\ (1,0.5) & (1,0.4) & (1,0.7) \\ (1,0.1) & (1,0.5) & (1,0.6) \\ (1,0.4) & (1,0.7) & (1,0.5) \end{bmatrix}, B^c = \begin{bmatrix} (1,0.2) & (1,0.6) & (1,0.5) \\ (1,0.6) & (1,0.5) & (1,0.8) \\ (1,0.2) & (1,0.6) & (1,0.7) \\ (1,0.3) & (1,0.8) & (1,0.5) \end{bmatrix}$$

Then the addition matrices are

$$A+B = \begin{bmatrix} (0.2,0) & (0.6,0) & (0.5,0) \\ (0.6,0) & (0.5,0) & (0.8,0) \\ (0.2,0) & (0.6,0) & (0.7,0) \\ (0.4,0) & (0.8,0) & (0.5,0) \end{bmatrix}, A^c+B^c = \begin{bmatrix} (1,0.1) & (1,0.6) & (1,0.5) \\ (1,0.5) & (1,0.4) & (1,0.7) \\ (1,0.1) & (1,0.5) & (1,0.6) \\ (1,0.3) & (1,0.7) & (1,0.5) \end{bmatrix}$$

$$V(A+B) = \begin{bmatrix} 0.2 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.8 \\ 0.2 & 0.6 & 0.7 \\ 0.4 & 0.8 & 0.5 \end{bmatrix}, V(A^c+B^c) = \begin{bmatrix} 0.9 & 0.4 & 0.5 \\ 0.5 & 0.6 & 0.3 \\ 0.9 & 0.5 & 0.4 \\ 0.7 & 0.3 & 0.5 \end{bmatrix}$$

Calculate the score matrix and the total score for selection

$$s_{((A+B),(A^c+B^c))} = \begin{bmatrix} -0.7 & 0.2 & 0 \\ 0.1 & -0.1 & 0.5 \\ -0.7 & 0.1 & 0.3 \\ -0.3 & 0.5 & 0 \end{bmatrix}$$

$$\text{Total score} = \begin{bmatrix} -0.5 \\ 0.5 \\ -0.3 \\ 0.2 \end{bmatrix}$$

We see that the second candidate has the maximum value and thus conclude that from both the expert's opinion, candidate$c_2$is selected for the post.

VI. CONCLUSIONS

In our work, we have put forward some new concepts such as complement, trace of fuzzy soft matrix based on reference function. Some related properties have been established with example. Finally an application of fuzzy soft matrix based on reference function in decision making problem is given. It 's hoped that our work will enhance this study in fuzzy soft matrix.

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