THE EXTENICS NORM APPLIED TO A TWO-DIMENSIONAL ROBOTIC WORKSPACES

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The paper presents an application of Extenics Engineering principles to a two-dimensional robotic workspace. This provides the mathematical basis and considerations for obtaining the trajectory tracking reference in robotic applications. A brief history and overview of the relevant theoretical concepts is provided.

Keywords: Extenics, robot workspace, reference generation, Extenics control.

1. INTRODUCTION

Extenics is a science whose stated aim is to deal with unsolvable problems. With applications in artificial intelligence, business, marketing, planning, design, control theory and image processing, to name just a few, it is one of the fastest developing new fields of study in the world today.

To be able to manipulate the outcomes of situations which represent contradictory problems, we need to have in place a representation, as well as a set of tools and an environment model in which to do so. This section will briefly explain the theoretical basis of Extenics and describe the general model of thought in an Extenics problem. The three pillars of Extenics Theory are Basic Element, Extension Set and Extension Logic.

Extenics Theory maps all components of a given problem into elements, which provides the basis for a working model of the problem. These are called Basic Elements and consist of the triplet formed by an object, action or relation, a possibly infinite number of characteristics and their corresponding value relating to the object. In mathematical form, we call:

$$B = \begin{pmatrix} O_m & c_{m_1} & v_{m_1} \\ & \vdots & \vdots \\ & c_{m_n} & v_{m_n} \end{pmatrix} = (O_m, c_m, v_m)$$

a basic element in Extenics Theory. The 'm' means this particular triplet defines a matter-element (although all basic elements are similar from a construction standpoint) [1,2].

Elements are organized together with the help of Extension Sets. These provide a means of classification for the initial problem, as well as the outcomes. Extension Sets are further processed using any number of transformations to achieve a desired result and new norms are introduced for work on them, such as Extenics Distances. Working with Extension Sets and the different classes of transformations to solve contradictory problems is at the very core of practical Extenics Theory applications.

Extension Set Theory is a new set theory which aims to describe the change of the nature of matters, thus taking both qualitative, as well as quantitative aspects into account. The theoretical definition for an extension set is as follows: supposing U to be an universe of discourse, u is any one element in U, k is a mapping of U to the real field I, T=(TU ,Tk, Tu) is given transformation, we call:

$$E(T) = \{(u, y, y') | u \in U, y = k(u) \in I, T_u u \in T_U U, y' = T_k k(T_u u) \in I\}$$

an extension set on the universe of discourse U, $y=k(u)$ the Dependent Function of $E(T)$, and $y' = T_k k(T_u u)$ the extension function of $E(T)$, wherein, T_U, T_k and T_u are transformations of the respective universe of discourse U, Dependent Function k and element u. If $T \neq e$, that is to say the transformation is not identical, four more concepts can be outlined, as follows:

- positive extensible field (or positive qualitative change field) of $E(T)$:

$$\underset{\smile}{E}_{+}(T) = \{(u, y, y') \mid u \in U, y = k(u) \leq 0; T_u u \in T_U U, y' = T_k k(T_u u) > 0\}$$

- negative extensible field (or negative qualitative change field) of $E(T)$:

$$\underset{\smile}{E}_{-}(T) = \{(u, y, y') \mid u \in U, y = k(u) \geq 0;\ T_u u \in T_U U, y' = T_k k(T_u u) < 0\}$$

- positive stable field (or positive quantitative change field) of $E(T)$:

$$E_{+}(T) = \{(u, y, y') \mid u \in U, y = k(u) > 0; T_u u \in T_U U, y' = T_k k(T_u u) > 0\}$$

- negative stable field (or negative quantitative change field)of $E(T)$:

$$E_{-}(T) = \{(u, y, y') \mid u \in U, y = k(u) < 0; T_u u \in T_U U, y' = T_k k(T_u u) < 0\}$$

- extension boundary of $E(T)$:

$$E_0(T) = \{(u, y, y') \mid u \in U, T_u u \in T_U U, y' = T_k k(T_u u) = 0\}$$

This is further illustrated in Figure 1 [1]

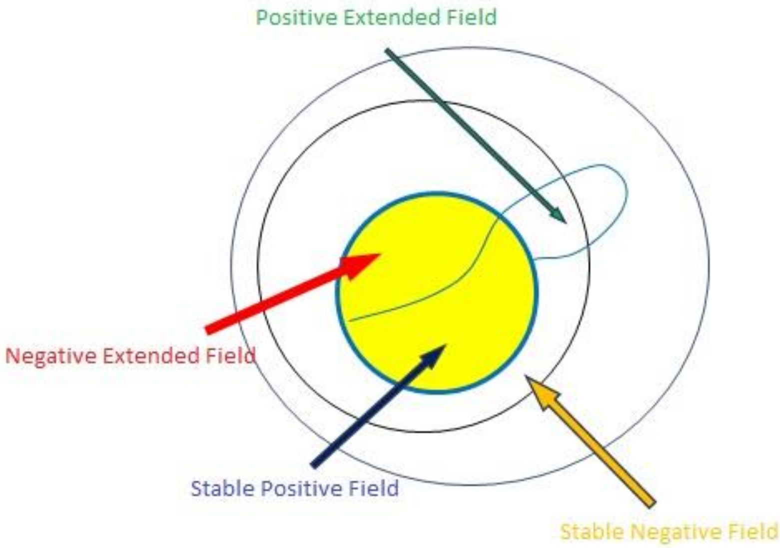

Figure 1. Universe of Discourse in an Extenics Transformation

The Extension Set, then, is defined in relation to a transformation and an existing function mapped onto the universe of discourse. Following the transformation, the Extension Set is divided into the positive and negative fields with regard to the dependent function value. Four subsets are therefore defined: the positive stable, the positive transitive, the negative stable and the negative transitive field. The stable fields are those for which the polarity of the dependent function is unaltered by the transformation, whereas transitive (also named extensible) fields are those affected by the change. This provides a useful classification and investigation tool for contradictory problem models.

2. EXTENICS WORKSPACE

Let there be a robotic application determined by a similar workspace to that presented in Figure 2. For the mechatronic mechanism within this workspace there is a two-dimensional point reference – defined exactly on the axes by the double (x,y) – which must be reached by the robot end-effector.

There is assumed an extended controller for actuator control, which implements the concepts of Extenics Theory. For this, it will be necessary to know the dependence function, calculated for the multi-dimensional case, in order to estimate the level of incompatibility, from which follows the intensity of the actuator response. This is detailed in [3-5]. For modelling the robotic workspace, it suffices to say that it is required to compute the dependent function for multi-dimensional cases.

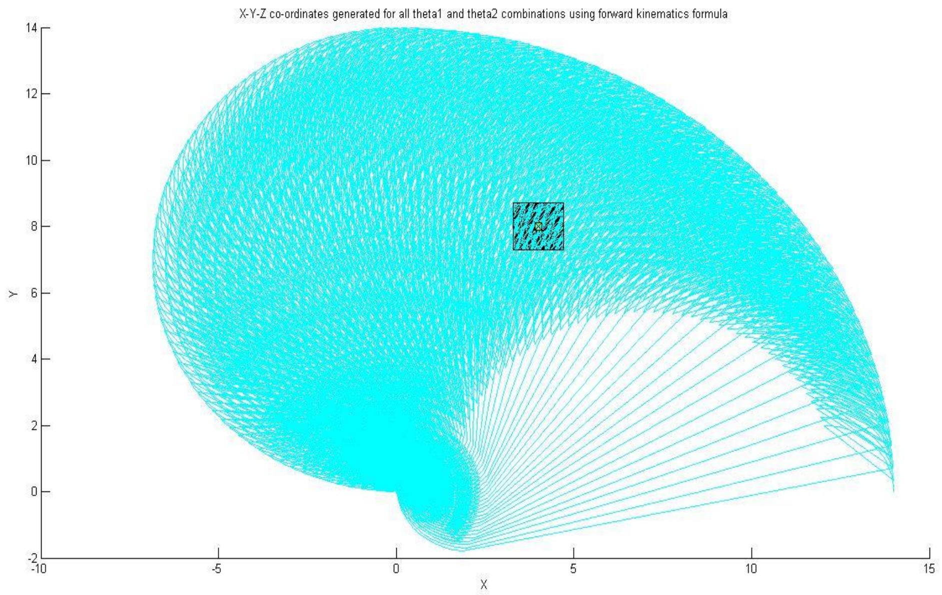

Figure 2. Robotic Workspace

Using the theories developed by Smarandache [6, 7] and Sandru [5, 8] relating to working with the dependence function in n-dimensional spaces, its point value can be obtained for the particular case.

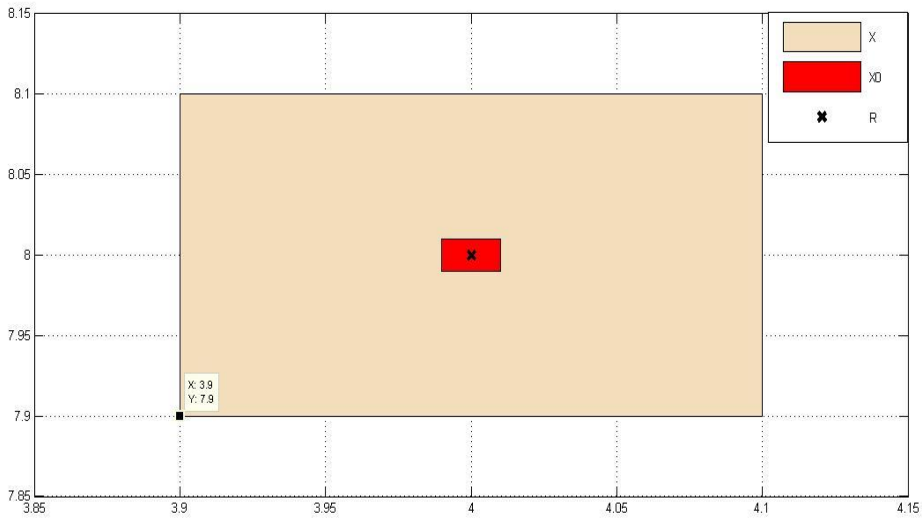

Figure 3. Nested extended interval space relative to reference R

Starting from the given reference point $R(x_r,y_r)$, which is the optimum for the actuator – controlled position, there is an accepted reference interval X_0 and acceptable interval X, the optimum for both being the singular reference R.

In order to find the values of the extended indicators, two regions of the two-dimensional space are considered, as can be seen in Figures 4 and 6. The two zone correspond to the field variations on x and y.

For any point $P(x_P,y_P)$ in the first region (shown in Figure 4) can be calculated [6, 7] (cf. Smarandache) the 2D extended distance to the existing intervals:

$$\rho(x,X) = |PP_2|$$
$$\rho(x,X_0) = |PP_1| \qquad (6.1)$$

This will yield the dependent function as:

$$k(P) = \frac{\rho(x,X)}{D(x,X_0,X)} = \frac{\rho(x,X)}{\rho(x,X)-\rho(x,X_0)} = \frac{|PP_2|}{|PP_2|-|PP_1|} \qquad (6.2)$$

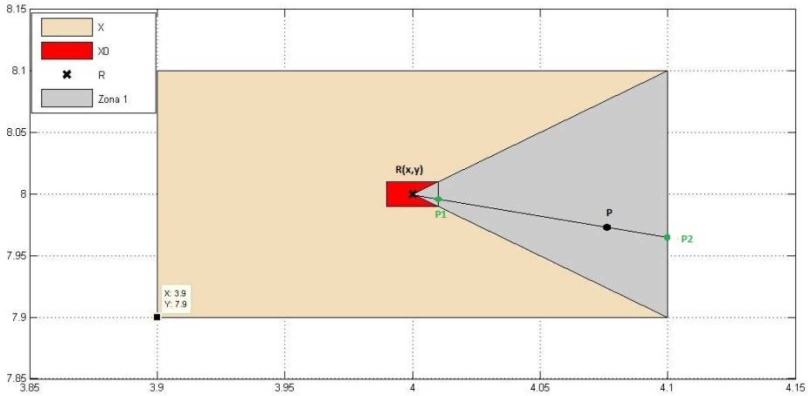

Figure 4. Vertical region of the extended field

In the figure, point P is chosen outside X_0, but inside X, which will lead to a negative sub-unitary value for the dependence function (the function denominator is negative). Had P belonged to X_0, k(Q) would have been similarly computed, with the result being positive and, had P been chosen outside X, the dependence function value would have been lower than -1.

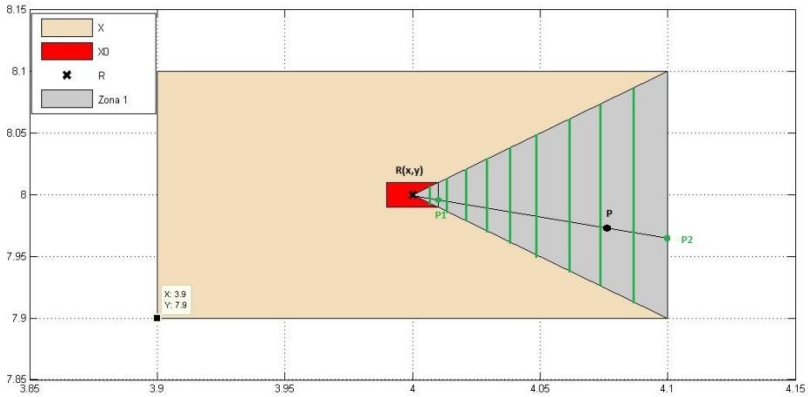

Figure 5. Vertical point classes having the same dependence function value

The dependence function value in point P will thus be the 2D extended distance between the point and the closest frontier of the larger interval, divided by the difference of the 2D extended distance between the point and the larger interval, and the point and the smaller interval. All of these distances are considered along the line defined by the optimum point R(x,y) and the chosen point $P(x_P,y_P)$.

As explained in [9] (Smarandache, Vlădăreanu, 2012) this will determine the dependent function within the region, as the final expression does not depend on the value of the y-coordinate in the chosen point. This concept is illustrated in Figure 5. For every point $Q(x_Q,y_Q)$ in the second region (see Figure 6), a similar final expression is reached, where:

$$k(Q) = \frac{\rho(x,X)}{D(x,X_0,X)} = \frac{\rho(x,X)}{\rho(x,X)-\rho(x,X_0)} = \frac{|QQ_2|}{|QQ_2|-|QQ_1|} \quad (6.3)$$

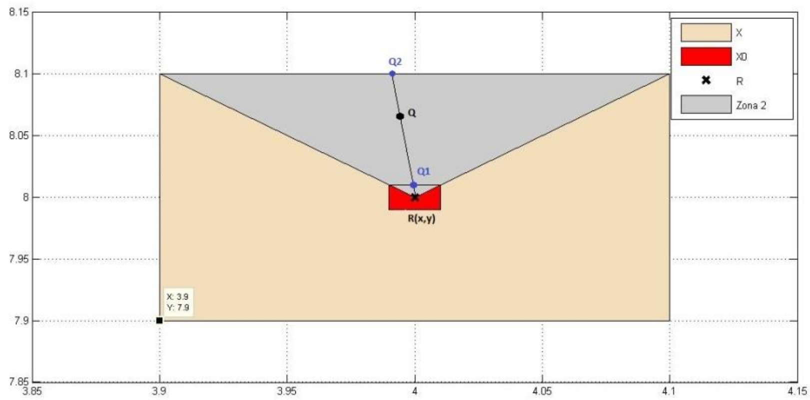

Figure 6. Horizontal region of the extended field

This will determine classes of horizontal points having the same value of the dependent function, as is shown in Figure 7.

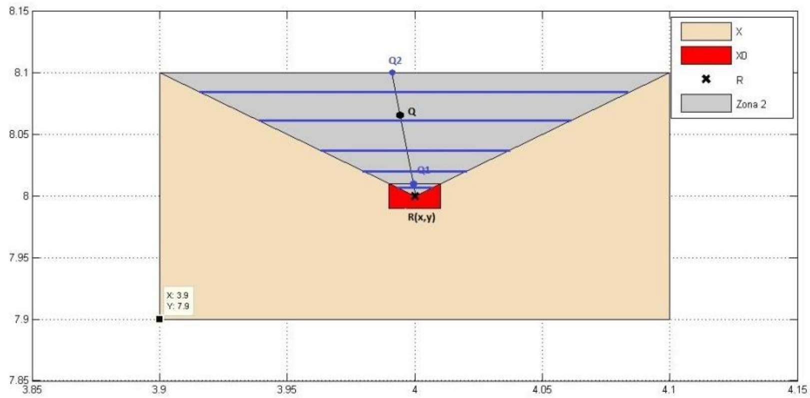

Figure 7. Classes of horizontal points having the same value of the dependent function

Thus, it can be seen that for such a distribution of the extended intervals upon which the dependent function is based, the two-dimensional problem can be separated into two distinct one-dimensional problems. It should be noted that this characteristic in not necessarily present in all applications, for which different distributions of extended intervals may exist. Beyond the scope of this paper, the subject can be further investigated in [6, 10]. These results are the basis for the design, implementation and simulation of Extenics Control concepts presented in various papers, and in modelling the robotic workspace and the reference and control system for a humanoid walking robot [11].

3. CONCLUSIONS

Modelling a robotic workspace using concepts from Extenics Theory contributes to the development of a new type of innovative control for robot actuators. The advantages of extended control are remarkable through the lack of added complexity in design or implementation. The controller architecture is very straightforward, once the function interpreter is established. While the place and limits of the extended sets need to be specified and may involve some fine tuning, their optimization is not vital, and perfectly fine results can be obtained with simple and intuitive values (such as setting the accepted interval to be ±2% of the reference value).

Extenics control, as discussed in this paper, benefits greatly from being a novelty approach to controller design. While this paper proves a working model can be established with basic parameters, the possibilities for tweaking and optimizing in the hopes of obtaining improved performance are virtually limitless.

Perhaps most importantly, it represents a shift in the paradigm of controller structure. While the controllers themselves have evolved greatly over the years, changes in the way one looks at controllers and controller structures have not been frequent. By way of being an implementation of a more generalized theory, whose aim is precisely to formalize the process of innovation, there is virtually no end to the possibilities for further research. Also, as Extenics Theory continues to grow and mature as a discipline in itself, the theoretical advances made are sure to have a favourable impact upon this field of research.

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