

TRILOBIC VIBRANT SYSTEMS

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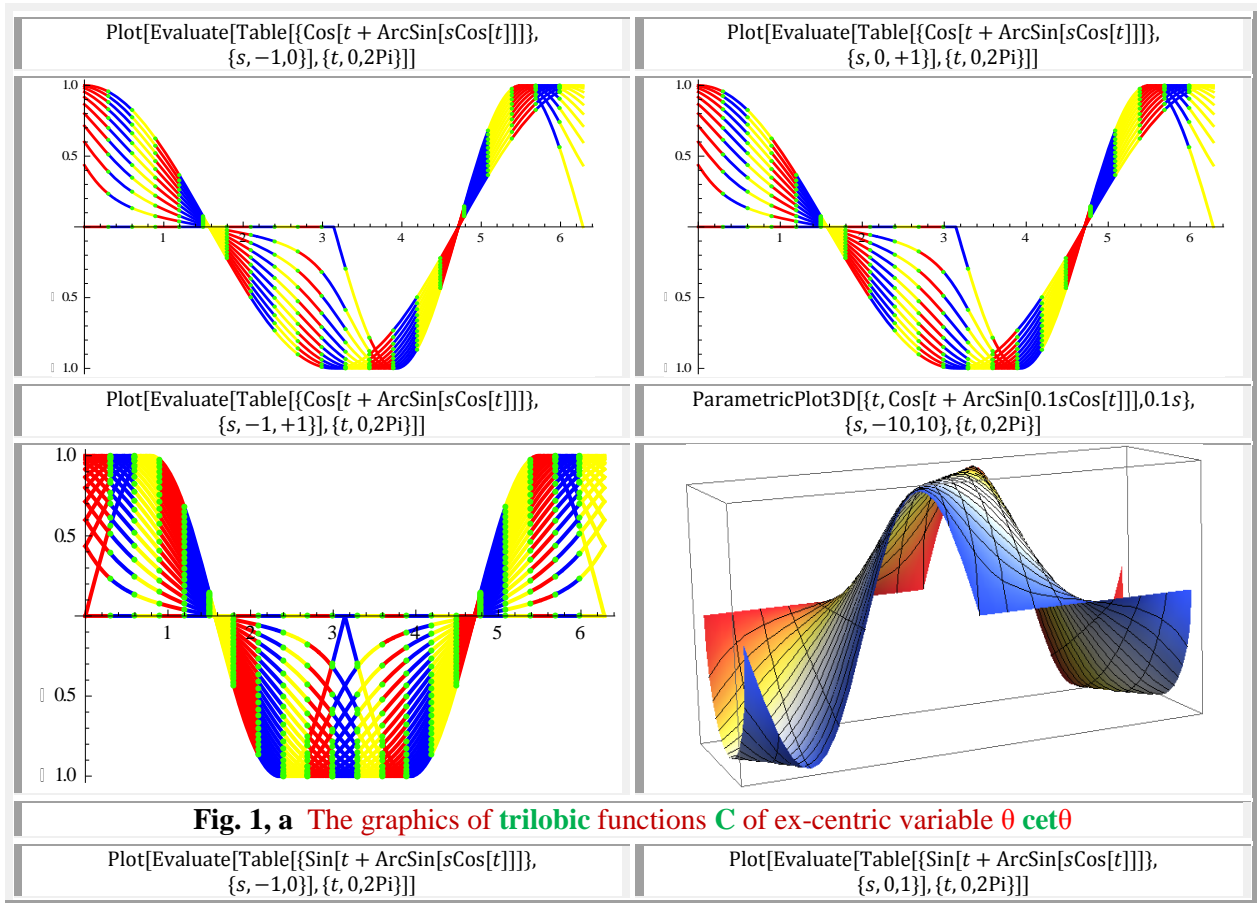
1. INTRODUCTION

The trilobes are **ex-centric circular supermathematics functions (EC-SMF)** of angular ex-centricity $\varepsilon = \frac{\pi}{2}$, with notations **cet** θ and **set** θ , for the **trilobic ex-centric cosine** and, respectively, the **trilobic ex-centric sine**, with equations:

$$(1) \quad \begin{cases} \mathbf{cet}\theta = \mathbf{cex} \left[\theta, S \left(s, \frac{\pi}{2} \right) \right] = \cos \left\{ \theta - \arcsin \left[s \cdot \sin \left(\theta - \frac{\pi}{2} \right) \right] \right\} = \mathbf{cos} \left[\theta + \arcsin \left[s \cdot \mathbf{cos}\theta \right] \right] \\ \mathbf{set}\theta = \mathbf{sex} \left[\theta, S \left(s, \frac{\pi}{2} \right) \right] = \sin \left\{ \theta - \arcsin \left[s \cdot \sin \left(\theta - \frac{\pi}{2} \right) \right] \right\} = \mathbf{sin} \left[\theta + \arcsin \left[s \cdot \mathbf{cos}\theta \right] \right] \end{cases}$$

where **S** is a point, called **ex-center**, from the unity-circle plan of **CU**[O(0, 0), R = 1], of polar coordinates **S**(s,ε), where **s** ∈ [-1,+1] is the **numerical linear ex-centricity**, and **e** = **Rs** is the **real linear ex-centricity**, for any circle of radius **R**, and **ε** is the **angular ex-centricity**.

The graphics of **trilobic ex-centric supermathematics functions (TE-SMF)** are shown in **Figure 1**.



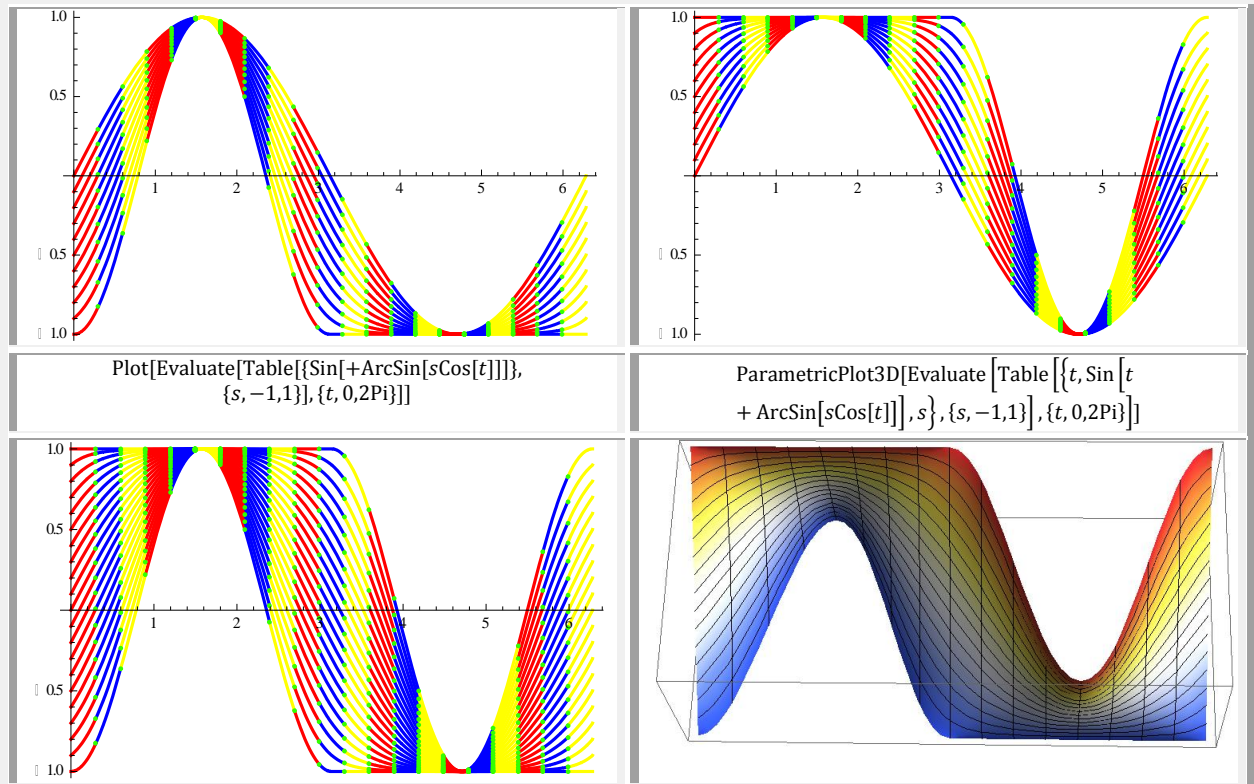


Fig. 1, b The graphics of trilobes **S** of ex-centric variable θ set θ

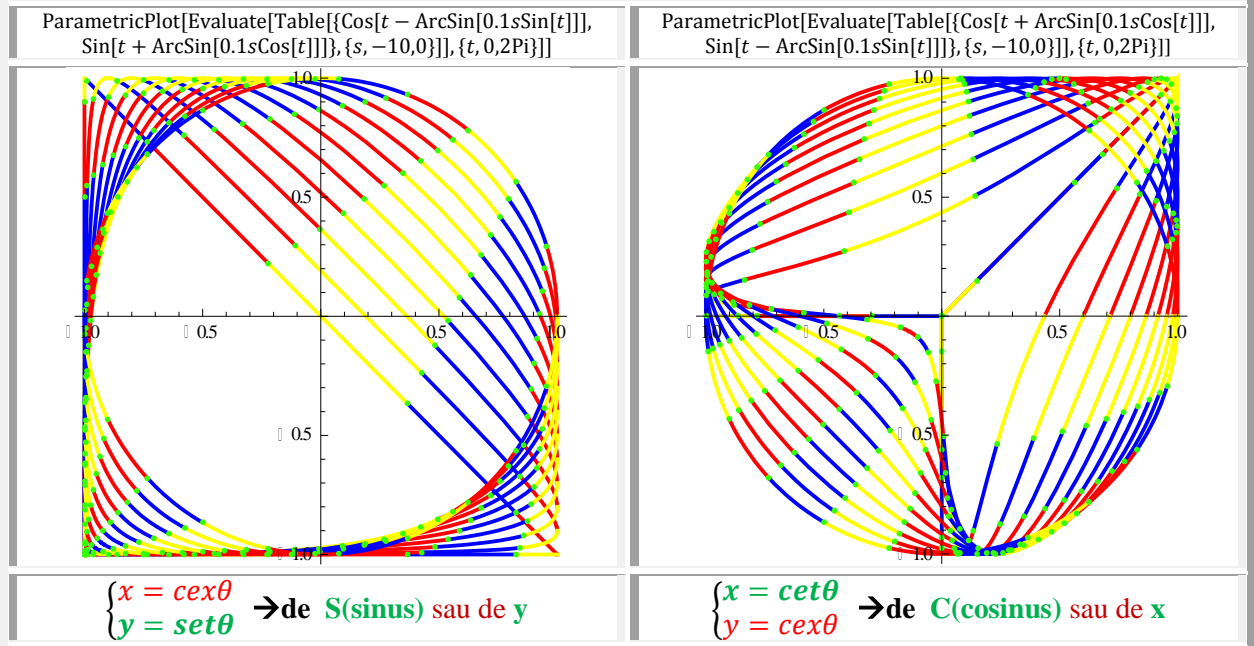


Fig. 2, a The graphics of trilobes **S** (TS) ◀ and of trilobes **C** (TC) ▶ of ex-centric variable θ in 2D

The trilobic ex-centric supermathematics functions are abbreviated as (TE-SMF).

It follows that, for a numerical linear ex-centricity $s = 0$, TE-SMF degenerates into central circular functions (CC-SMF) or circular functions / ordinary Euler trigonometric functions $\cos\alpha$ and $\sin\alpha$ ($s = 0 \rightarrow \alpha \equiv \theta$), and for an angular ex-centricity $\varepsilon = 0$ and $s \neq 0$, it degenerates in EC-SMF $cex\theta$ and, respectively, $sex\theta$.

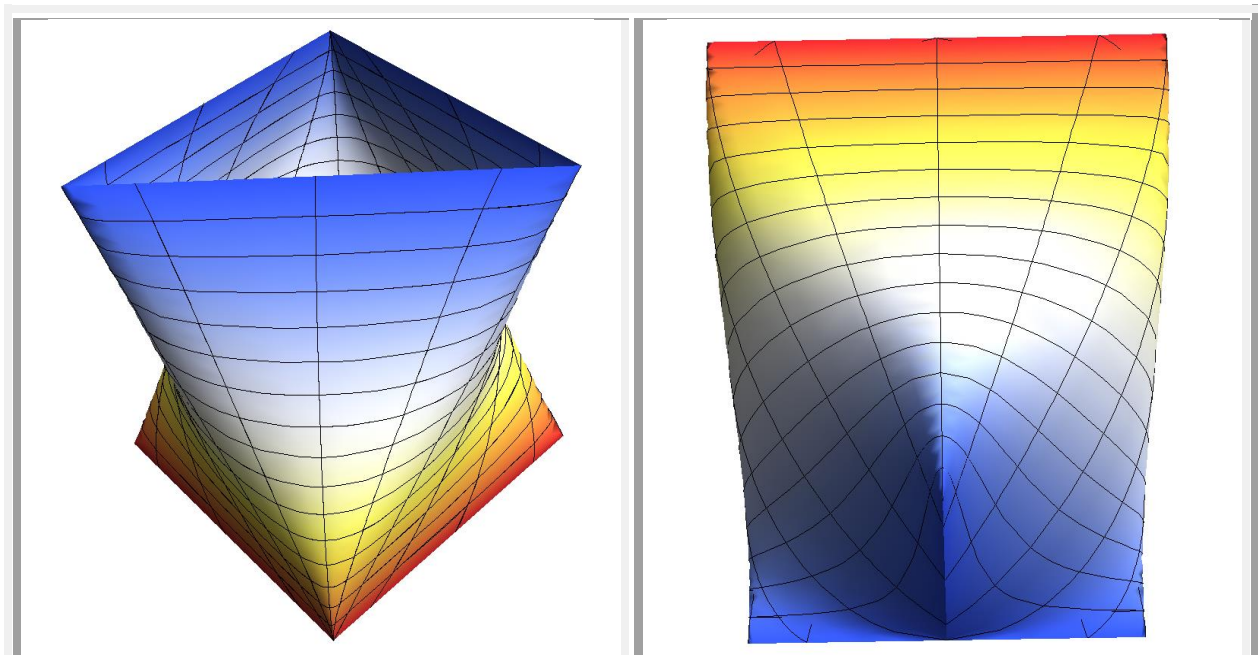


Fig. 2, b The graphics of trilobes $S \blacktriangleleft$ and of trilobes $C \blacktriangleright$ of ex-centric variable θ in 3D

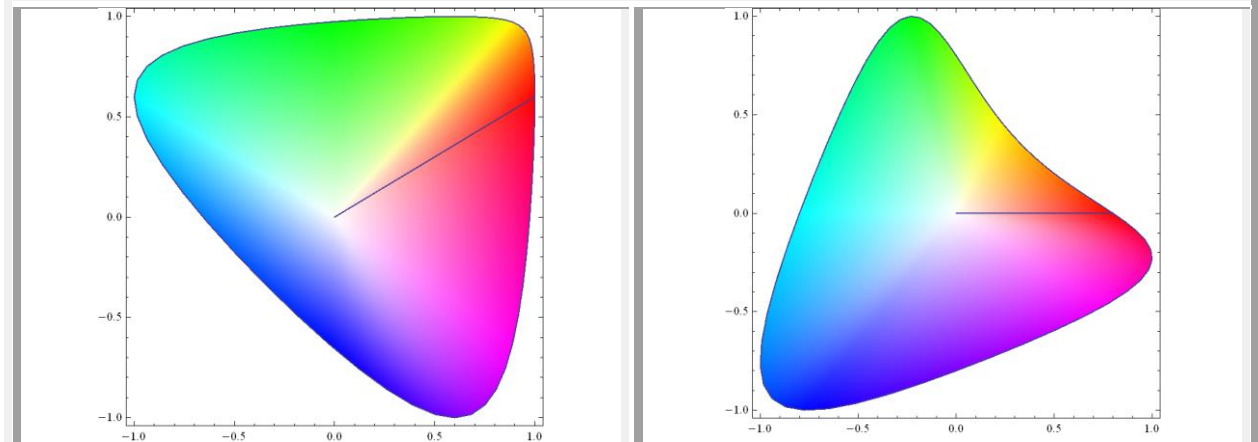


Fig. 2, c Trilobic disks $S \blacktriangleleft$ and $C \blacktriangleright$ of $s = 0,6$

The term of **TE-SMF** derives from the fact that, for $s \in (0, 1)$, the parametric equations, consisting of a combination of **EC-SMF** and **TE-SMF**, express closed plane curves of **3 lobes**, which, for $s = 0$, degenerates in a perfect circle and for $s = \pm 1$ in a **rectangular isosceles triangle (TS)** or in an **ex-centric rectangular isosceles triangle (TC)** \blacktriangleleft , a figure in the shape of **inclined Y**, visible in the graphs of **Figures 2, a** \blacktriangleright .

2. DIFFERENTIAL EQUATION OF TRILOBIC VIBRANT SYSTEMS

Let us have the functions $x(t), y(t) : \mathbb{R} \rightarrow [-1, +1]$ and $\theta = \Omega.t$

$$(1) \quad \begin{cases} x(t) = \text{cet}[\Omega t, S(s, \varepsilon)] \\ y(t) = \text{set}[\Omega t, S(s, \varepsilon)] \end{cases}$$

of the same ex-center $S(s, \varepsilon)$, where s is the polar radius and ε – the polar angle, in a unity-circle of radius $R = 1 \rightarrow CU(0,1)$.

Their derivatives, for $\theta = \Omega.t$ and $\frac{d\theta}{dt} = \Omega = 1$, are:

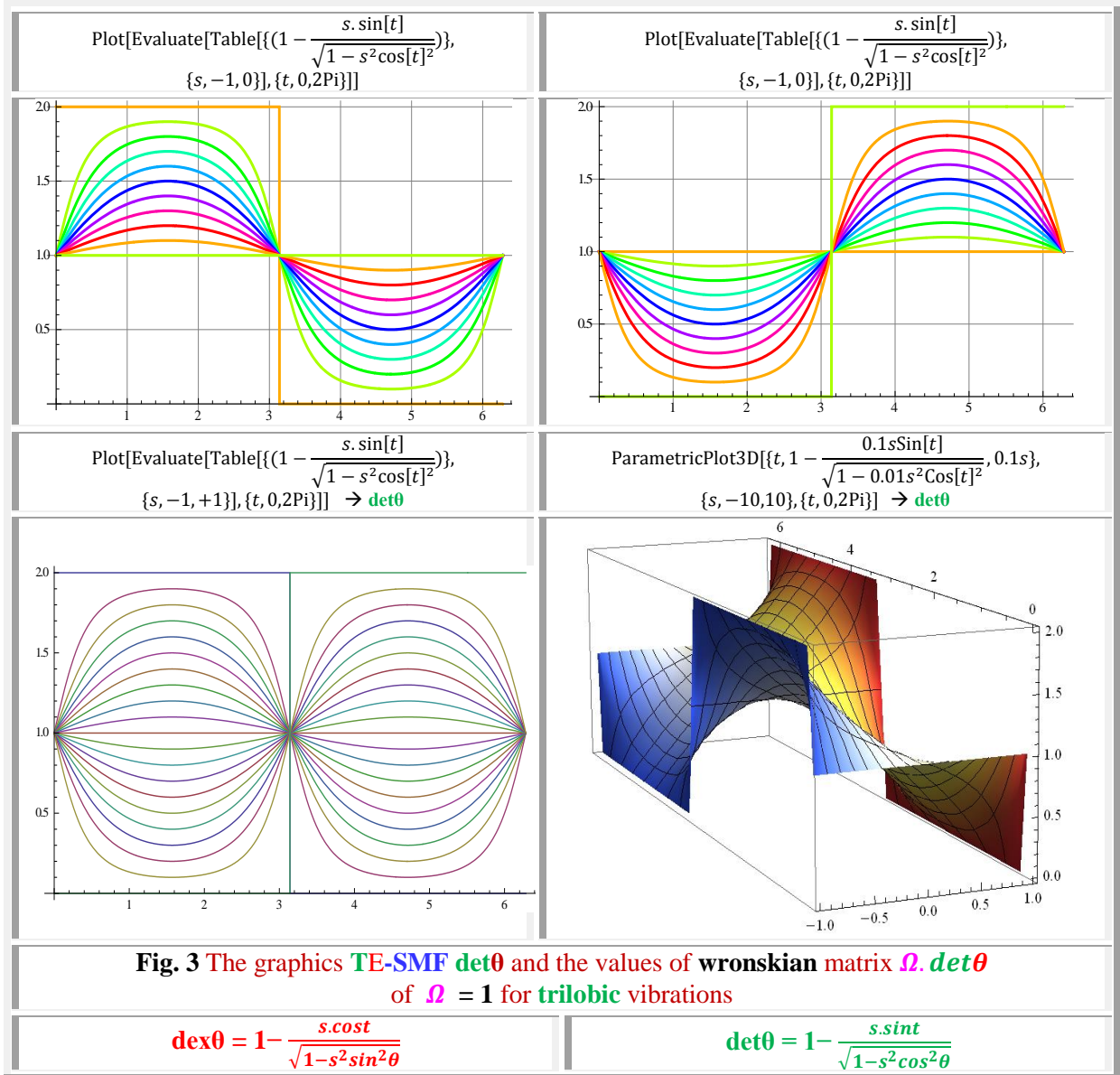
$$(2) \quad \begin{cases} \dot{x}(t) = \frac{d}{dt} \text{cet}[\Omega t, S(s, \varepsilon)] = \frac{d\theta}{dt} \frac{d}{d\theta} \text{cet}[\theta, S(s, \varepsilon)] = \Omega \cdot \frac{d}{d\theta} \text{cet}[\theta, S(s, \varepsilon)] = -\Omega \cdot \text{det}\theta \cdot \text{set}\theta \\ \dot{y}(t) = \frac{d}{dt} \text{set}[\Omega t, S(s, \varepsilon)] = \frac{d\theta}{dt} \frac{d}{d\theta} \text{set}[\theta, S(s, \varepsilon)] = \Omega \cdot \frac{d}{d\theta} \text{set}[\theta, S(s, \varepsilon)] = +\Omega \cdot \text{det}\theta \cdot \text{cet}\theta \end{cases}$$

where $\Omega \cdot \text{det}\theta = \omega$ and, explicitly:

$$(3) \quad \begin{cases} \dot{x}(t) = -\Omega \cdot \left(1 - \frac{s \cdot \sin\theta}{\sqrt{1-s^2 \cos^2\theta}}\right) \sin[\theta + \arcsin(s \cdot \cos\theta)] = -\omega \cdot \text{set}\theta \\ \dot{y}(t) = \Omega \cdot \left(1 + \frac{s \cdot \sin\theta}{\sqrt{1-s^2 \cos^2\theta}}\right) \cos[\theta + \arcsin(s \cdot \sin\theta)] = \omega \cdot \text{cet}\theta \end{cases}$$

from where we get the expression **TE-SMF trilobc ex-centric derivative** of ex-centric variable θ :

$$(4) \quad \text{det}\theta = 1 - \frac{s \cdot \sin\theta}{\sqrt{1-s^2 \cos^2\theta}} = \frac{d\alpha(\theta)}{d\theta} = \frac{\text{aet}\theta}{d\theta}, \text{ with graphics in Figure 3.}$$



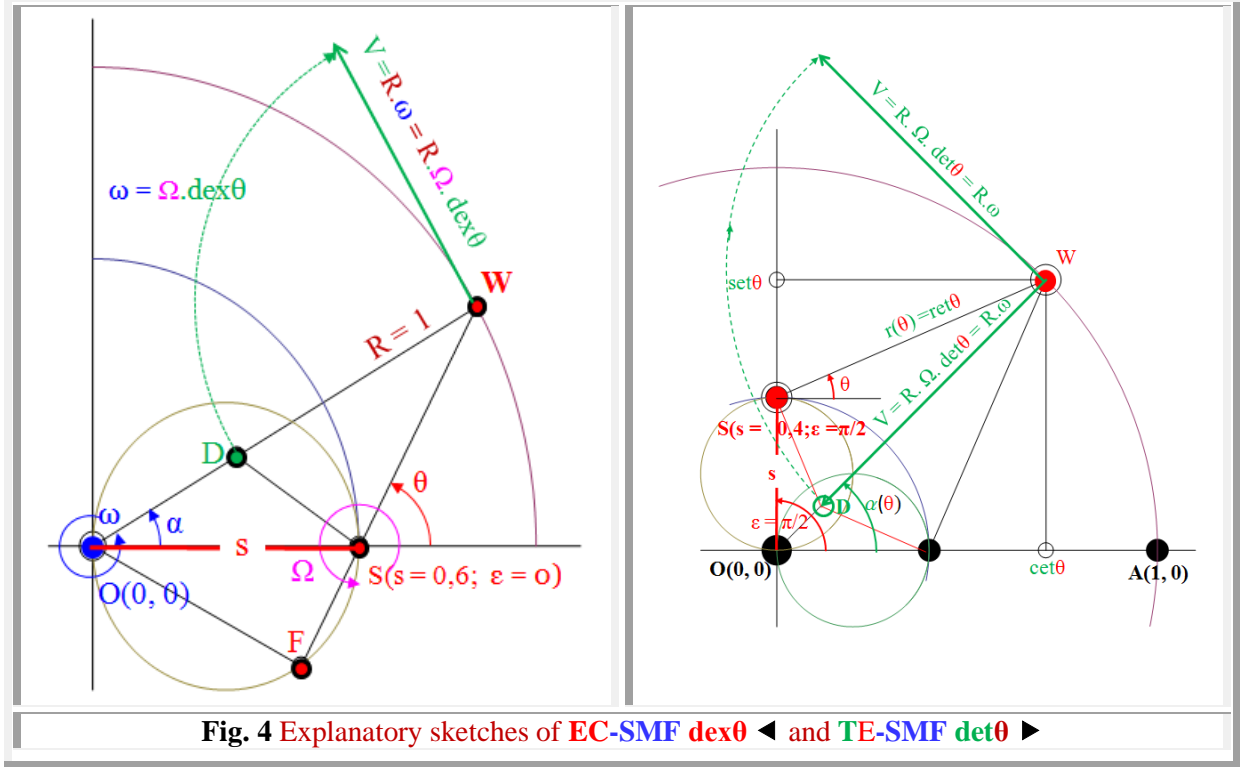


Fig. 4 Explanatory sketches of EC-SMF $\text{dex}\theta$ ◀ and TE-SMF $\text{det}\theta$ ▶

The function **trilobic ex-centric amplitude** $\text{act}\theta = \alpha(\theta)$ is represented by the angle $\alpha(\theta)$ or by the **centric variable** α , to the center $O(0, 0)$, as a function of angle θ to the ex-center $S(s, \frac{\pi}{2})$ or of **ex-centric variable** θ (Fig. 4 ▶), and $\Omega. \text{det}\theta = \omega(t)$, so that the second derivative of TE-SMF (3) is:

$$(5) \quad \begin{cases} \ddot{x} = \frac{d}{dt}(-\omega. \text{set}\theta) = -3. \text{set}\theta - \omega^2. \text{cet}\theta \\ \ddot{y} = \frac{d}{dt}(\omega. \text{cet}\theta) = 3. \text{cet}\theta - \omega^2. \text{set}\theta \end{cases}$$

where we denoted by $3 = \frac{d\Omega}{dt}$ the angular acceleration of a moving point on the circle of parametric equations expressed by the relations (1), with variable angular speed (4), as it can be observed in Figure 5, following the angular distribution of colors.

The **wronskian** matrix of trilobic system of vibrations is:

$$(6) \quad \begin{vmatrix} x(t) & y(t) \\ \dot{x}(t) & \dot{y}(t) \end{vmatrix} = \begin{vmatrix} \text{cet}[\Omega t, S(s, \varepsilon)] & \text{set}[\Omega t, S(s, \varepsilon)] \\ -\Omega. \text{det}\theta. \text{set}\theta & \Omega. \text{det}\theta. \text{cet}\theta \end{vmatrix} = \Omega. \text{det}\theta [\text{cet}^2\theta. + \text{set}^2\theta] = \Omega. \text{det}\theta$$

because $\text{cet}^2\theta + \text{set}^2\theta = 1$, as well as their counterparts $\text{cex}^2\theta + \text{sex}^2\theta = 1$, as well as their **ancestor / precursor** archaics $\cos^2\alpha + \sin^2\alpha = 1$.

The graphics of **wronskian** matrix, for $\Omega = 1$, are shown in Figure 3, from where it can be inferred that the values are strictly positive for $|s| < 1$ and, by consequence, there is a **linear differential equation**, of a dynamic technical system, of nonlinear elastic property, admitting these functions as fundamental system of solutions.

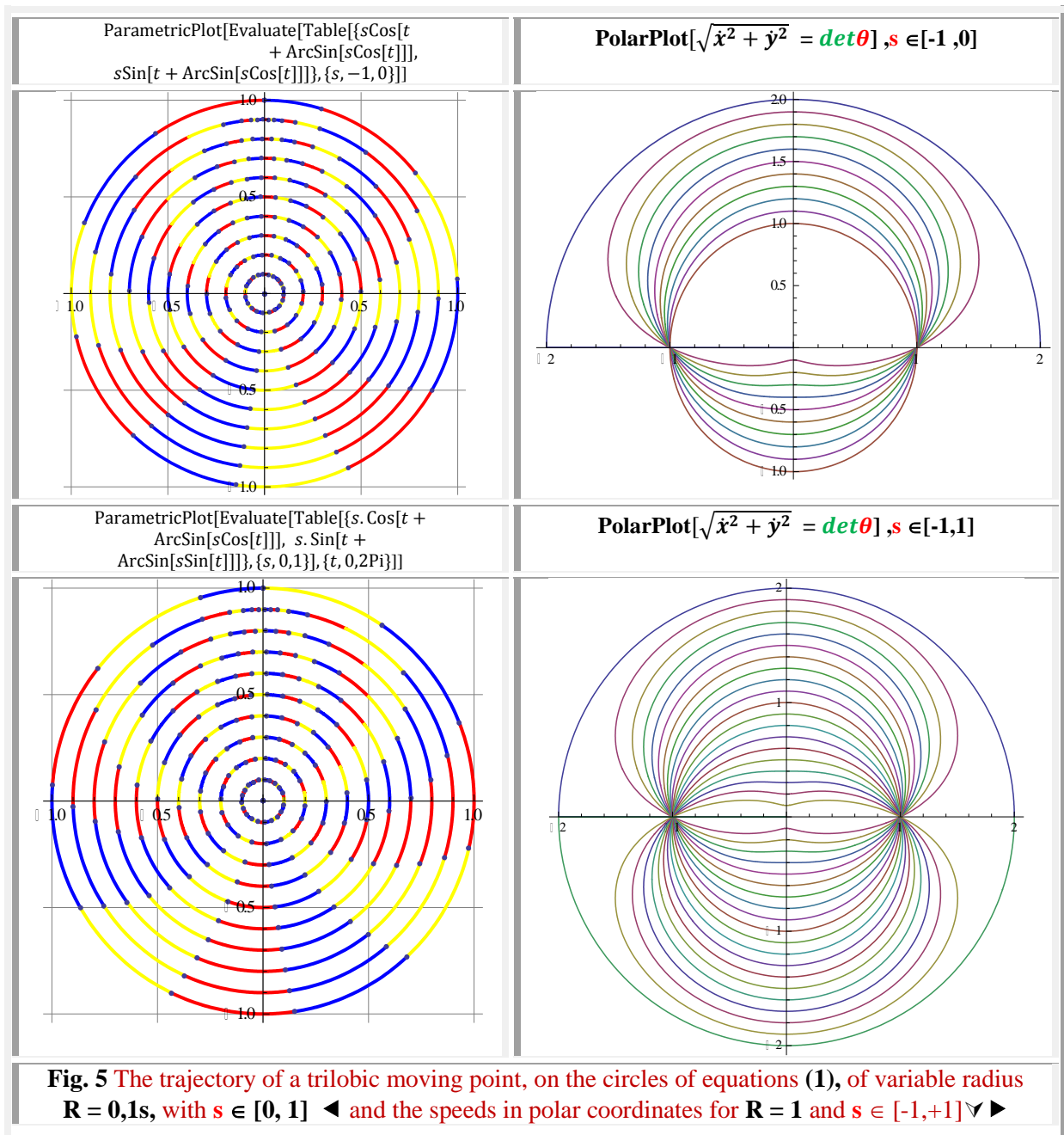
The fundamental system of solutions is:

$$(7) \quad Z = C_1 \text{cet}\Omega t + C_2 \text{set}\Omega t,$$

where $C_1, C_2 \in \mathbb{R}$ are constant and (7) is the general solution of the following differential equation.

The equations is:

$$(8) \quad \begin{vmatrix} z & x & y \\ \dot{z} & \dot{x} & \dot{y} \\ \ddot{z} & \ddot{x} & \ddot{y} \end{vmatrix} = 0$$



$$(9) \quad \ddot{z} \begin{vmatrix} x & y \\ \dot{x} & \dot{y} \end{vmatrix} - \dot{z} \begin{vmatrix} x & y \\ \ddot{x} & \ddot{y} \end{vmatrix} + z \begin{vmatrix} \dot{x} & \dot{y} \\ \ddot{x} & \ddot{y} \end{vmatrix} = 0$$

$$(8') \quad \begin{vmatrix} z & \text{cet}\theta & \text{set}\theta \\ \dot{z} & -\omega.\text{set}\theta & \omega.\text{cet}\theta \\ \ddot{z} & -3.\text{set}\theta - \omega^2.\text{cet}\theta & 3.\text{cet}\theta - \omega^2.\text{set}\theta \end{vmatrix} = 0$$

$$(8'') \quad \ddot{z} \begin{vmatrix} \text{cet}\theta & \text{set}\theta \\ -\omega.\text{set}\theta & \omega.\text{cet}\theta \end{vmatrix} - \dot{z} \begin{vmatrix} \text{cet}\theta & \text{set}\theta \\ -3.\text{set}\theta - \omega^2.\text{cet}\theta & 3.\text{cet}\theta - \omega^2.\text{set}\theta \end{vmatrix} +$$

$$z \left| \begin{array}{cc} -\omega \cdot \text{set}\theta & \omega \cdot \text{cet}\theta \\ -3 \cdot \text{set}\theta - \omega^2 \cdot \text{cet}\theta & 3 \cdot \text{cet}\theta - \omega^2 \cdot \text{set}\theta \end{array} \right| = 0$$

$$(10) \quad \ddot{z} \cdot \omega (\text{cet}^2\theta + \text{set}^2\theta) - \dot{z} \cdot [3 \cdot \text{cet}^2\theta - \omega^2 \cdot \text{cet}\theta \text{set}\theta + 3 \cdot \text{set}^2\theta + \omega^2 \cdot \text{cet}\theta \cdot \text{set}\theta] + z[-3 \cdot \omega \cdot \text{cet}\theta \text{set}\theta + \omega^3 \cdot \text{set}^2\theta + 3 \cdot \omega \text{set}\theta \cdot \text{cet}\theta + \omega^3 \cdot \text{cet}^2\theta] = 0$$

$$(10') \quad \ddot{z} \cdot \omega - \dot{z} \cdot 3 + z \cdot \omega^3 = 0$$

or

$$(10'') \quad \ddot{z} - \dot{z} \frac{3}{\omega} + z \omega^2 = 0$$

which is the differential equation of open, unamortized vibrations, of **trilobic mechanical systems**, identical equation, **in form**, with the equation of open, unamortized vibrations of **ex-centric** systems, and with the equation of **quadrilobic** ones.

3. INTEGRAL CURVES IN THE PHASE PLANE

The integral curves are the plane curves described by the speed points rotating on the unity-circle of $R = 1$, or on another circle of radius equally to the maximum amplitude $R = A$, according to their projection position on the axis Ox , i.e. $V(x)$ and are sunt shown in **Figure 6**.

Their parametric equations are:

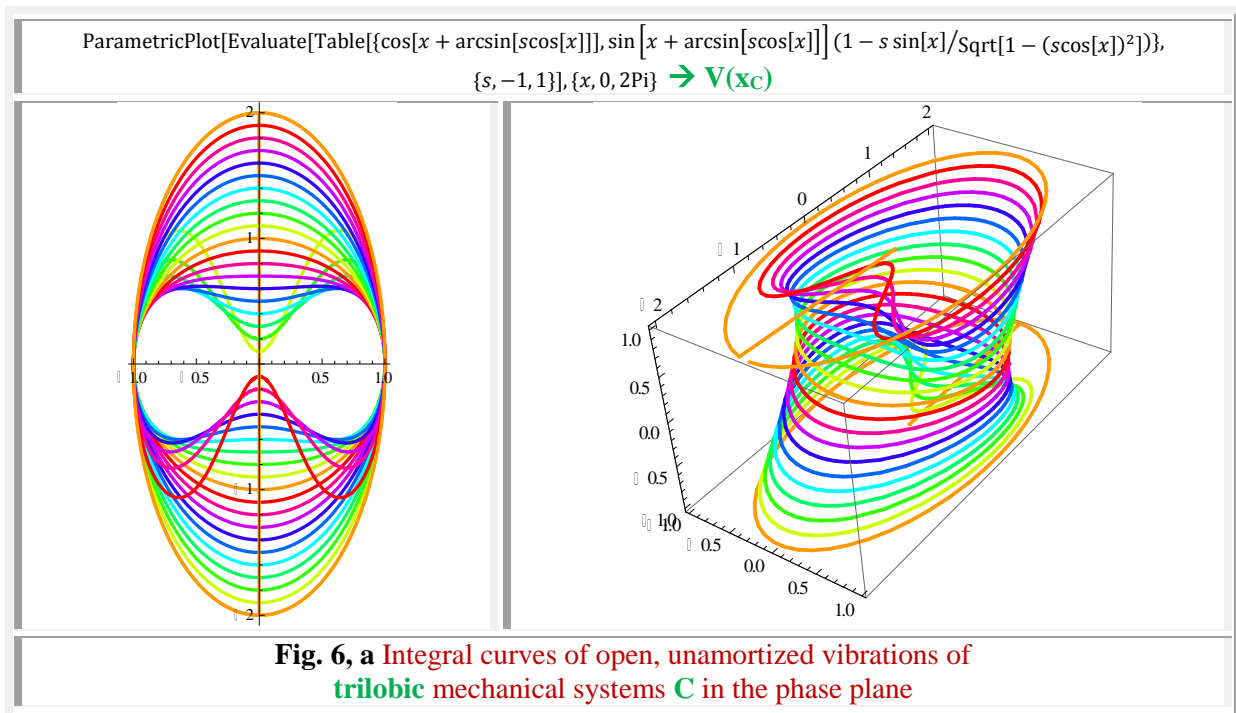
- for trilobes C

$$(11) \quad \begin{cases} x = \text{cet}\theta \\ y = -\Omega \cdot \text{det}\theta \cdot \text{set}\theta \end{cases}$$

- for trilobes S

$$(12) \quad \begin{cases} x = \text{set}\theta \\ y = \Omega \cdot \text{det}\theta \cdot \text{cet}\theta \end{cases}$$

with graphics in **Figure 6, a** and, respectively, **6, b**.

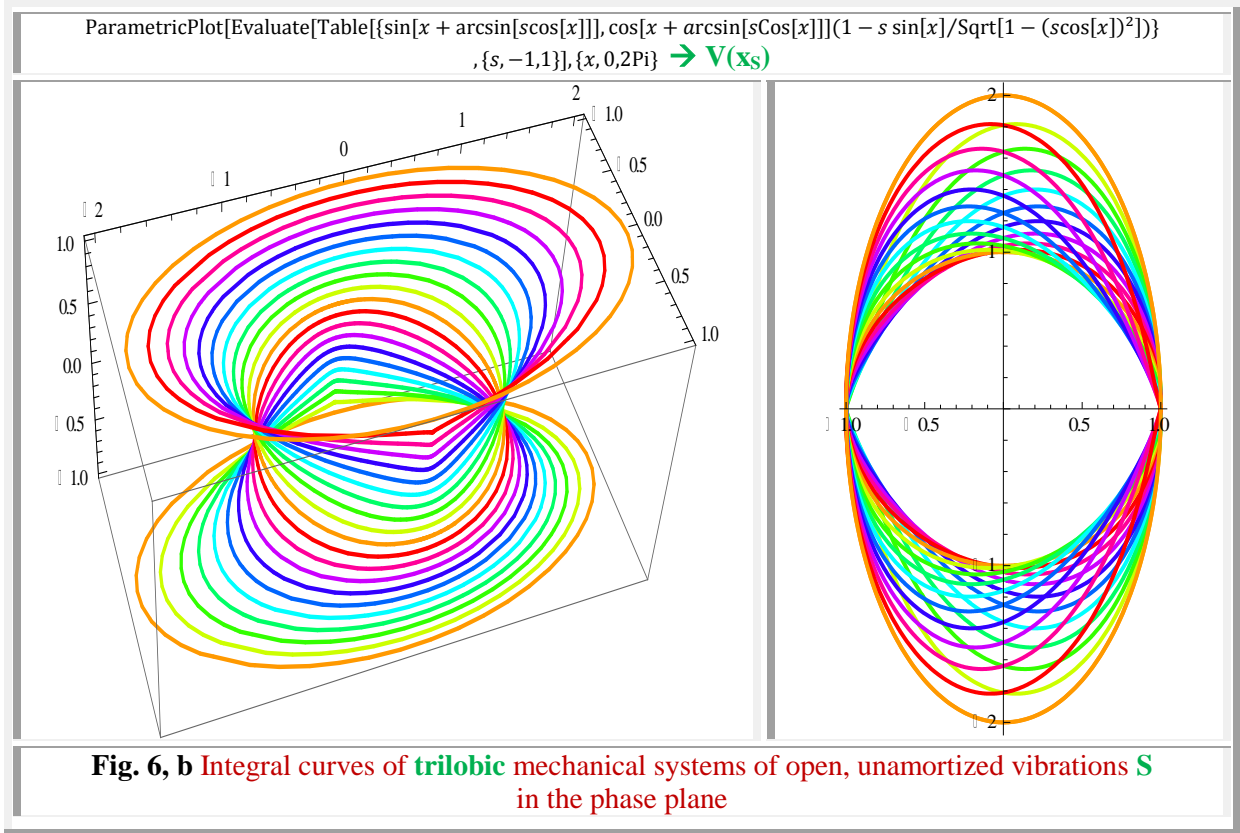


In the case of open, unamortized vibrations, there are only two forces in the system:

- the force exerted by the elastic element of the system, proportional to the movement x , namely

$$(13) \quad \mathbf{F}_{el} = \mathbf{k} \cdot \mathbf{x} = \begin{cases} \mathbf{k} \cdot \mathbf{c} \cdot \omega t \\ \mathbf{k} \cdot \mathbf{s} \cdot \omega t \end{cases}$$

where \mathbf{k} is an elastic constant of the element and the acceleration force, proportional to the **mass** \mathbf{m} of the oscillating system and to the mass acceleration of the system, i.e.



$$(14) \quad \mathbf{F}_{acc} = \mathbf{m} \cdot \ddot{\mathbf{x}} = \begin{cases} \mathbf{m}(-3 \cdot \mathbf{s} \cdot \omega^2 \cdot \mathbf{c} \cdot \omega t) \\ \mathbf{m}(+3 \cdot \mathbf{c} \cdot \omega^2 \cdot \mathbf{s} \cdot \omega t) \end{cases}$$

4. STATIC ELASTIC PROPERTIES (SEP) OF TRILOBIC OSCILLATING SYSTEMS

Since there are only two forces in the considered system, under condition of dynamic equilibrium, they must be equal and of contrary signs / directions, meaning that

$$(15) \quad \mathbf{F}_{el} + \mathbf{F}_{acc} = 0, \quad \rightarrow \quad \mathbf{F}_{el} = -\mathbf{F}_{acc}$$

and, as a result, the **static elastic properties (SEP)** of **trilobed systems** are expressed by the parametric equations:

$$(16) \quad \begin{cases} x = \mathbf{c} \cdot \omega t \\ y = \ddot{x} = -(-3 \cdot \mathbf{s} \cdot \omega^2 \cdot \mathbf{c} \cdot \omega t) \end{cases}$$

and, explicitly, for **trilobed systems C**:

$$(16') \quad \begin{cases} x = \cos[\theta + \arcsin(s \cdot \cos\theta)] \\ y = -(-\cos[x + \arcsin[s \cdot \cos[x]]] \left(1 - \frac{s \cdot \sin[x]}{\sqrt{1-s^2 \cos^2[x]}}\right)^2 - \left(-\frac{s \cdot \cos[x]}{\sqrt{1-s^2 \cos^2[x]}} + \frac{\cos[x] \sin[x]^2}{(1-s^2 \cos^2[x])^{3/2}}\right) \sin[x + \arcsin[s \cdot \cos[x]]]) \end{cases}$$

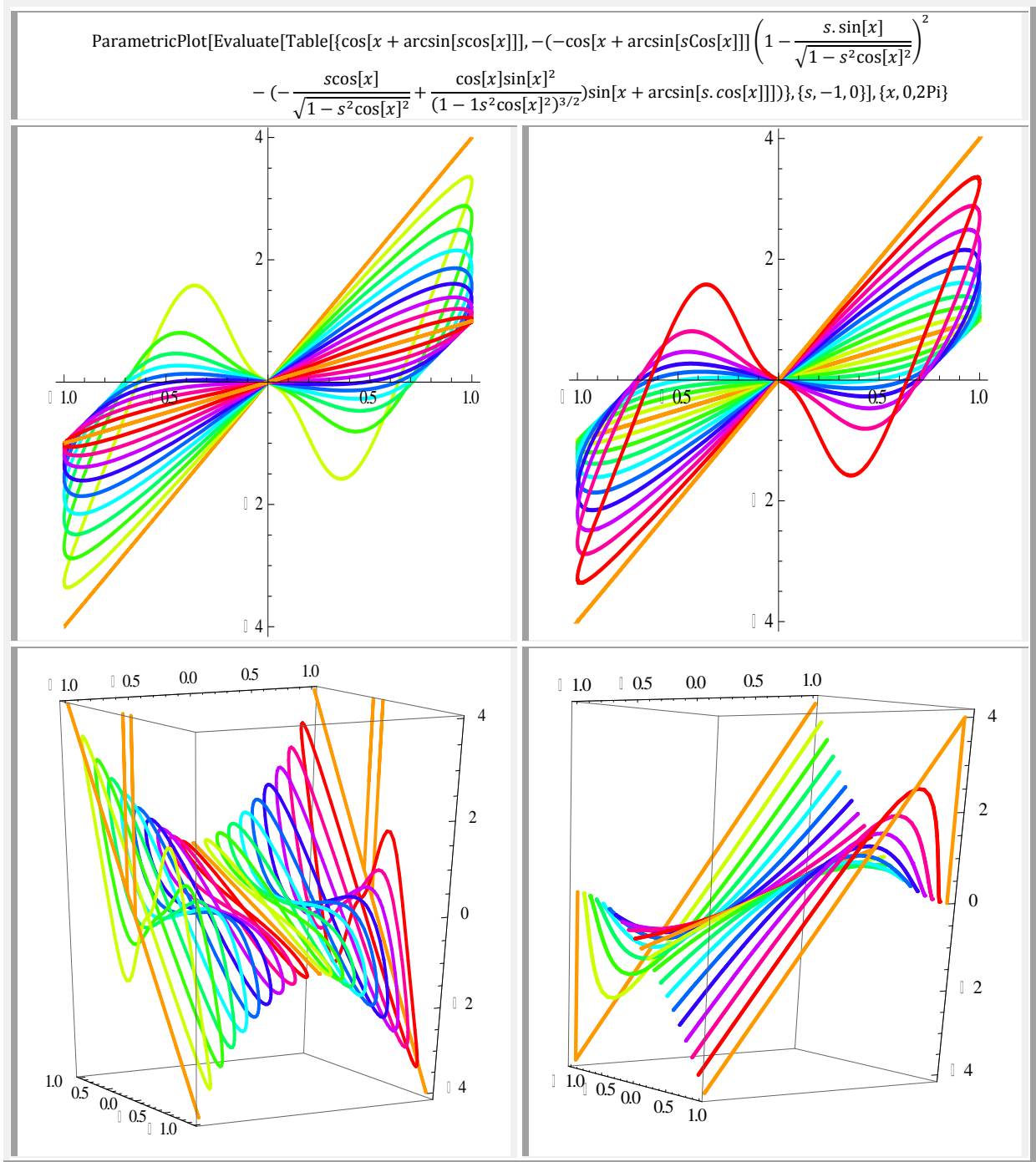
with graphics in **Figure 7** \blacktriangle , while for **trilobed systems S**, the parametric equations are:

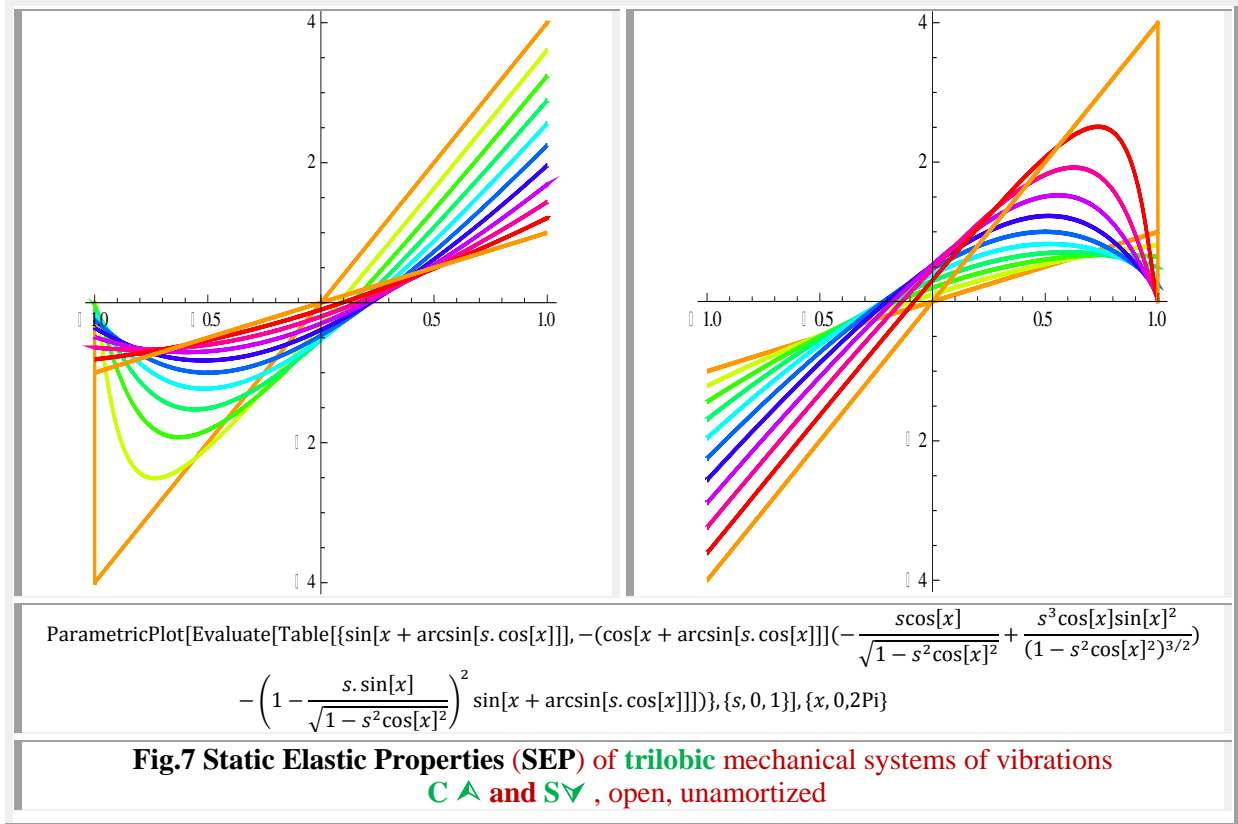
$$(17) \quad \begin{cases} x = \mathbf{s} \cdot \omega t \\ y = \ddot{y} = -(3 \cdot \mathbf{c} \cdot \omega^2 \cdot \mathbf{s} \cdot \omega t) \end{cases}$$

and, explicitly:

$$(17') \quad \begin{cases} x = \sin[\theta + \arcsin(s \cdot \cos\theta)] \\ y = -(\cos[x + \arcsin[s \cdot \cos[x]]]) \left(-\frac{s \cos[x]}{\sqrt{1-s^2 \cos[x]^2}} + \frac{s^3 \cos[x] \sin[x]^2}{(1-s^2 \cos[x]^2)^{3/2}} \right) - \left(1 - \frac{s \cdot \sin[x]}{\sqrt{1-s^2 \cos[x]^2}} \right)^2 \sin[x + \arcsin[s \cdot \cos[x]]] \end{cases}$$

with graphics in **Figure 7**.





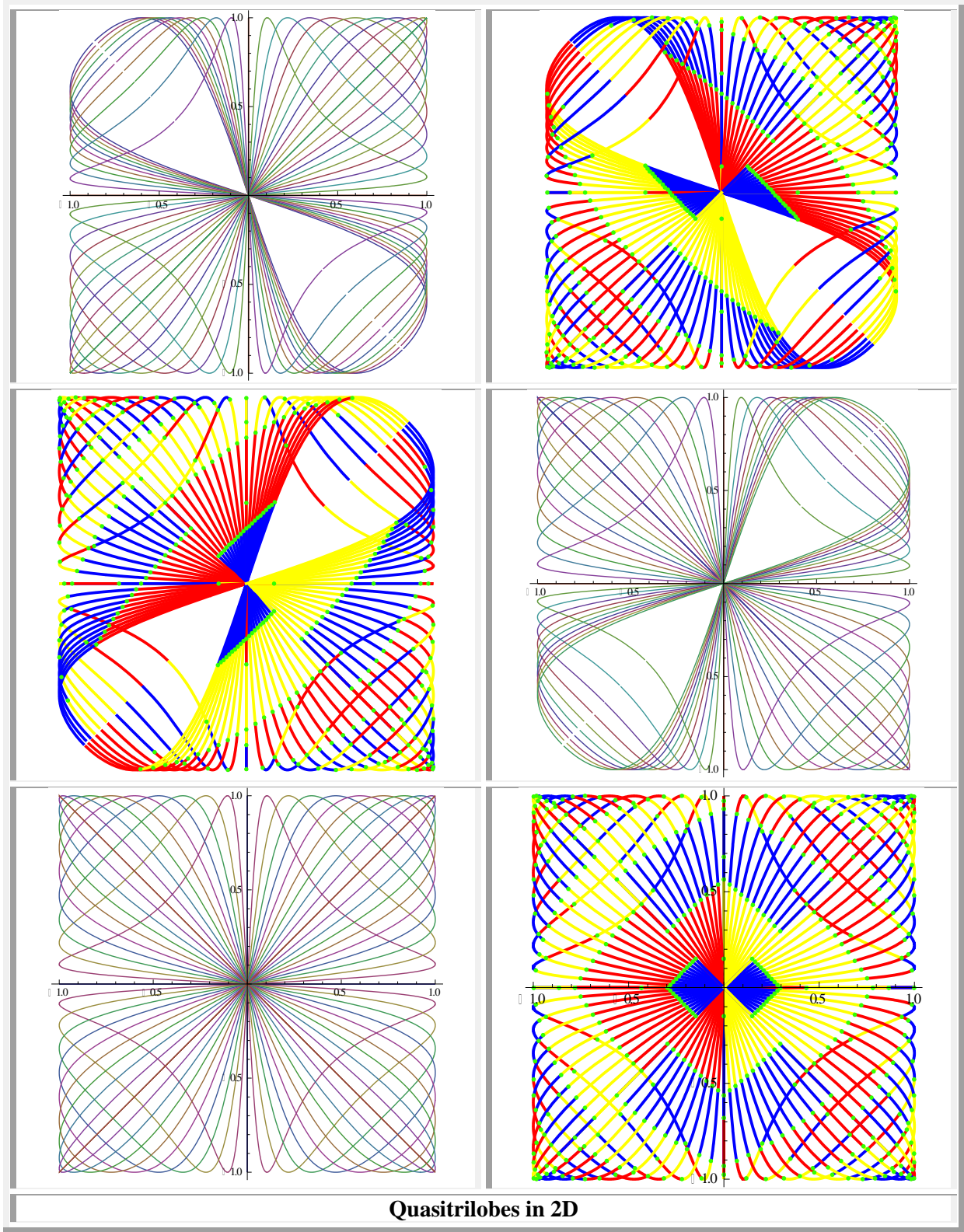
From graphics in **Figure 7**, it follows that **SEP linear** for **s = 0**, as it was expected, since we are in this case in the **centric field**, of classical linear systems of vibrations, expressed by circular centric functions **cosα** and **sinα**, but also for **s = ± 1**, which is a less expected result, even a surprise, which also appeared with the other systems expressed by **supermathematic functions** mentioned above (**quadrilobes**, expressed by **quadrilobical functions coq0** and **siq0**, but also by **SMF- ex-centric circular**, by functions **cex0** and **sex0**).

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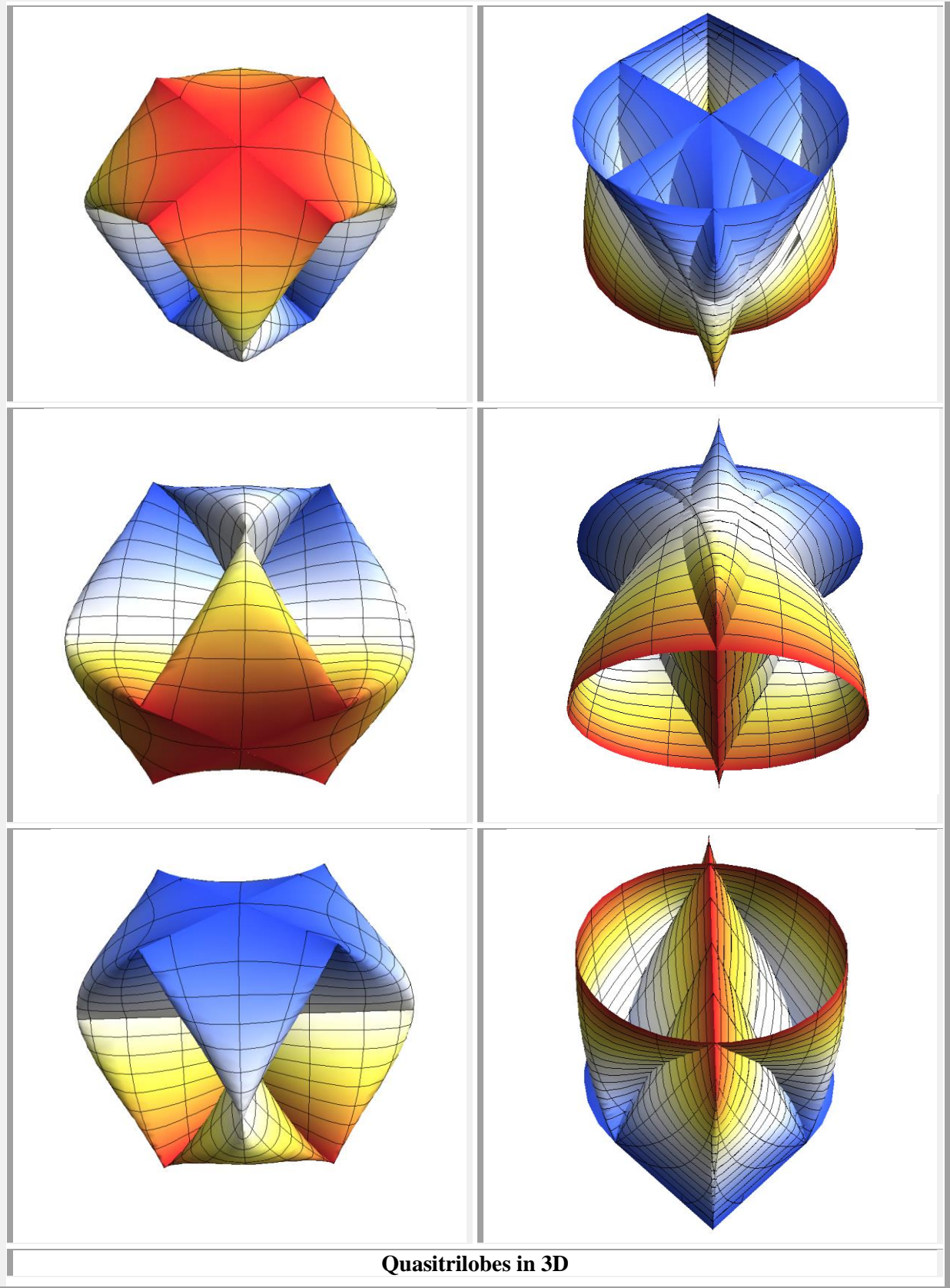
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ANEXA 1



Quasitrilobes in 2D



Quasitrilobes in 3D