

CONSIDERATIONS ON NEW FUNCTIONS IN NUMBER THEORY

by Florentin Smarandache, Ph. D.
University of New Mexico
Gallup, NM 87301, USA

Abstract:

New functions are introduced in number theory, and for each one a general description, examples, connections, and references are given.

Keywords: arithmetic functions, representation of numbers.

1991 MSC: 11A25, 11A67

Introduction.

In this paper a small survey is presented on eighteen new functions and four new sequences, such as: Inferior/Superior f-Part, Fractional f-Part, Complementary function with respect with another function, S-Multiplicative, Primitive Function, Double Factorial Function, S-Prime and S-Coprime Functions, Smallest Power Function.

1) Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ be a strictly increasing function and x an element in \mathbb{R} . Then:

a) Inferior f-Part of x ,

ISf(x) is the smallest k such that $f(k) \leq x < f(k+1)$.

b) Superior f-Part of x ,

SSf(x) is the smallest k such that $f(k) < x \leq f(k+1)$.

Particular cases:

a) Inferior Prime Part:

For any positive real number n one defines ISp(n) as the largest prime number less than or equal to n .

The first values of this function are (Smarandache[6] and Sloane[5]):
2,3,3,5,5,7,7,7,7,11,11,13,13,13,13,17,17,19,19,19,19,23,23.

b) Superior Prime Part:

For any positive real number n one defines SSp(n) as the smallest prime number greater than or equal to n .

The first values of this function are (Smarandache[6] and Sloane[5]):
2,2,2,3,5,5,7,7,11,11,11,11,13,13,17,17,17,17,19,19,23,23,23.

c) Inferior Square Part:

For any positive real number n one defines ISs(n) as the largest square less than or equal to n .

- The first values of this function are (Smarandache[6] and Sloane[5]):
0,1,1,1,4,4,4,4,4,9,9,9,9,9,9,16,16,16,16,16,16,16,16,25,25.
- b) *Superior Square Part:*
For any positive real number n one defines $SSs(n)$ as the smallest square greater than or equal to n .
The first values of this function are (Smarandache[6] and Sloane[5]):
0,1,4,4,4,9,9,9,9,16,16,16,16,16,16,25,25,25,25,25,25,25,25,36.
- d) *Inferior Cubic Part:*
For any positive real number n one defines $ISc(n)$ as the largest cube less than or equal to n .
The first values of this function are (Smarandache[6] and Sloane[5]):
0,1,1,1,1,1,1,1,8,8,8,8,8,8,8,8,8,8,8,8,8,8,27,27,27,27.
- e) *Superior Cube Part:*
For any positive real number n one defines $SSs(n)$ as the smallest cube greater than or equal to n .
The first values of this function are (Smarandache[6] and Sloane[5]):
0,1,8,8,8,8,8,8,8,27,27,27,27,27,27,27,27,27,27,27,27,27,27,27.
- f) *Inferior Factorial Part:*
For any positive real number n one defines $ISf(n)$ as the largest factorial less than or equal to n .
The first values of this function are (Smarandache[6] and Sloane[5]):
1,2,2,2,2,6,6,6,6,6,6,6,6,6,6,6,6,6,6,6,6,6,24,24,24,24,24,24,24.
- g) *Superior Factorial Part:*
For any positive real number n one defines $SSf(n)$ as the smallest factorial greater than or equal to n .
The first values of this function are (Smarandache[6] and Sloane[5]):
1,2,6,6,6,6,6,24,24,24,24,24,24,24,24,24,24,24,24,24,24,24,24,120.

Remark 1: This is a generalization of the inferior/superior integer part of a number (floor function).

2) Let $f: Z \rightarrow Z$ be a strictly increasing function and x an element in R . Then:

Fractional f-Part of x,

$$FS_f(x) = x - IS_f(x),$$

where $IS_f(x)$ is the Inferior f -Part of x defined above.

Particular cases:

a) *Fractional Prime Part:*

$$FSp(x) = x - ISp(x),$$

where $ISp(x)$ is the Inferior Prime Part defined above.
Example: $FSp(12.501) = 12.501 - 11 = 1.501.$

b) *Fractional Square Part:*

$$FSs(x) = x - ISs(x),$$

where $ISs(x)$ is the Inferior Square Part defined above.
Example: $FSs(12.501) = 12.501 - 9 = 3.501.$

c) *Fractional Cubic Part*:

$$FSc(x) = x - ISc(x),$$

where $ISc(x)$ is the Inferior Cubic Part defined above.

$$\text{Example: } FSc(12.501) = 12.501 - 8 = 4.501.$$

d) *Fractional Factorial Part*:

$$FSf(x) = x - ISf(x),$$

where $ISf(x)$ is the Inferior Factorial Part defined above.

$$\text{Example: } FSf(12.501) = 12.501 - 6 = 6.501.$$

Remark 2.1: This is a generalization of the fractional part of a number.

Remark 2.2: In a similar way one defines:

- the Inferior Fractional f-Part:

$$IFSf(x) = x - ISf(x) = FSf(x);$$

- and the Superior Fractional f-Part:

$$SFSf(x) = SSf(x) - x;$$

$$\begin{aligned} \text{for example: Superior Fractional Cubic Part of } 12.501 \\ = 27 - 12.501 = 14.499. \end{aligned}$$

3) Let $g: A \rightarrow A$ be a strictly increasing function, and let " \sim " be a given internal law on A . Then we say that

$f: A \rightarrow A$ is complementary with respect to the

function g and the internal law " \sim " if:

$f(x)$ is the smallest k such that there exists a z in A so that $x \sim k = g(z)$.

Particular cases:

a) *Square Complementary Function*:

$f: N \rightarrow N$, $f(x)$ = the smallest k such that xk is a perfect square.

The first values of this function are (Smarandache[6] and Sloane[5]):
1,2,3,1,5,6,7,2,1,10,11,3,14,15,1,17,2,19,5,21,22,23,6,1,26,3,7.

b) *Cubic Complementary Function*:

$f: N \rightarrow N$, $f(x)$ = the smallest k such that xk is a perfect cube.

The first values of this function are (Smarandache[6] and Sloane[5]):
1,4,9,2,25,36,49,1,3,100,121,18,169,196,225,4,289,12,361,50.

More generally:

c) *m-power Complementary Function*:

$f: N \rightarrow N$, $f(x)$ = the smallest k such that xk is a perfect m -power.

d) *Prime Complementary Function*:

$f: N \rightarrow N$, $f(x)$ = the smallest k such that $x+k$ is a prime.

The first values of this function are (Smarandache[6] and Sloane[5]):
1,0,0,1,0,1,0,3,2,1,0,1,0,3,2,1,0,1,0,3,2,1,0,5,4,3,2,1,0,1,0,5.

4) S-Multiplicative Function:

* *

A function $f : \mathbb{N} \rightarrow \mathbb{N}$ which,
 for any $(a, b) = 1$, verifies $f(ab) = \max \{f(a), f(b)\}$;
 (i.e. it reflects the main property of the Smarandache function[8]).

References:

[1] Castillo, Jose, "Other Smarandache Type Functions",
<http://www.gallup.unm.edu/~smarandache/funct2.txt>

[2] Dumitrescu, C., Seleacu, V., "Some Notions and Questions in Number Theory", Xiquan Publ. Hse., Phoenix-Chicago, 1994.

[3] Popescu, Marcela, Nicolescu, Mariana, "About the Smarandache Complementary Cubic Function", <Smarandache Notions Journal>, Vol. 7, no. 1-2-3, 54-62, 1996.

[4] Popescu, Marcela, Seleacu, Vasile, "About the Smarandache Complementary Prime Function", <Smarandache Notions Journal>, Vol. 7, no. 1-2-3, 12-22, 1996.

[5] Sloane, N.J.A.S, Plouffe, S., "The Encyclopedia of Integer Sequences", online, email: superseeker@research.att.com (SUPERSEEKER by N. J. A. Sloane, S. Plouffe, B. Salvy, ATT Bell Labs, Murray Hill, NJ 07974, USA).

[6] Smarandache, Florentin, "Only Problems, not Solutions!", Xiquan Publishing House, Phoenix-Chicago, 1990, 1991, 1993; ISBN: 1-879585-00-6.
 (reviewed in <Zentralblatt fur Mathematik> by P. Kiss: 11002, 744, 1992;
 and in <The American Mathematical Monthly>, Aug.-Sept. 1991);

[7] "The Florentin Smarandache papers" Special Collection, Arizona State University, Hayden Library, Tempe, Box 871006, AZ 85287-1006, USA; (Carol Moore & Marilyn Wurzbarger: librarians).

[8] Tabirca, Sabin, "About S-Multiplicative Functions", <Octogon>, Brasov, Vol. 7, No. 1, 169-170, 1999.

5) Smarandache-Kurepa Function:

For p prime, $SK(p)$ is the smallest integer such that $!SK(p)$ is divisible by p , where $!SK(p) = 0! + 1! + 2! + \dots + (p-1)!$

For example:

p	2	3	7	11	17	19	23	31	37	41	61	71	73	89
SK(p)	2	4	6	6	5	7	7	12	22	16	55	54	42	24

References:

[1] Ashbacher, C., "Some Properties of the Smarandache-Kurepa and Smarandache-Wagstaff Functions", in <Mathematics and Informatics Quarterly>, Vol. 7, No. 3, pp. 114-116, September 1997.

[2] Weisstein, Eric W., "Concise Encyclopedia of Mathematics", CRC Press, Boca Raton, Florida, 1998.

6) Smarandache-Wagstaff Function:

For p prime, $SW(p)$ is the smallest integer such that $W(SW(p))$ is divisible by p , where $W(p) = 1! + 2! + \dots + (p)!$

For example:

p	3	11	17	23	29	37	41	43	53	67	73	79	97
$SW(p)$	2	4	5	12	19	24	32	19	20	20	7	57	6

Reference:

- [1] Ashbacher, C., "Some Properties of the Smarandache-Kurepa and Smarandache-Wagstaff Functions", in <Mathematics and Informatics Quarterly>, Vol. 7, No. 3, pp. 114-116, September 1997.
- [2] Weisstein, Eric W., "Concise Encyclopedia of Mathematics", CRC Press, Boca Raton, Florida, 1998.

7) Smarandache Ceil Functions of n-th Order:

$Sk(n)$ is the smallest integer for which n divides $Sk(n)^k$.

For example, for $k=2$, we have:

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$S2(n)$	2	4	3	6	10	12	5	9	14	8	6	20	22	15	12	7

References:

- [1] Ibstedt, H., "Surfing on the Ocean of Numbers -- A Few Smarandache Notions and Similar Topics", Erhus University Press, Vail, USA, 1997; pp. 27-30.
- [2] Begay, A., "Smarandache Ceil Functions", in <Bulletin of Pure and Applied Sciences>, India, Vol. 16E, No. 2, 1997, pp. 227-229.
- [3] Weisstein, Eric W., "Concise Encyclopedia of Mathematics", CRC Press, Boca Raton, Florida, 1998.

8) Pseudo-Smarandache Function:

$Z(n)$ is the smallest integer such that $1 + 2 + \dots + Z(n)$ is divisible by n .

For example:

n	1	2	3	4	5	6	7
$Z(n)$	1	3	2	3	4	3	6

Reference:

- [1] Kashihara, K., "Comments and Topics on Smarandache Notions and Problems", Erhus University Press, Vail, USA, 1996.
- [2] Weisstein, Eric W., "Concise Encyclopedia of Mathematics", CRC Press, Boca Raton, Florida, 1998.

9) Smarandache Near-To-Primordial Function:

* * *

SNTP(n) is the smallest prime such that either $p - 1$, p , or $p + 1$ is divisible by n ,

*

where p , of a prime number p , is the product of all primes less than or equal to p .

For example:

n	1	2	3	4	5	6	7	8	9	10	11	...	59	...
SNTP(n)	2	2	2	5	3	3	3	5	?	5	11	...	13	...

References:

- [1] Mudge, Mike, "The Smarandache Near-To-Primordial (S.N.T.P.) Function", <Smarandache Notions Journal>, Vol. 7, No. 1-2-3, August 1996, p. 45.
- [2] Ashbacher, Charles, "A Note on the Smarandache Near-To-Primordial Function", <Smarandache Notions Journal>, Vol. 7, No. 1-2-3, August 1996, pp. 46-49.
- [3] Weisstein, Eric W., "Concise Encyclopedia of Mathematics", CRC Press, Boca Raton, Florida, 1998.

10) Double-Factorial Function:

SDF(n) is the smallest number such that $SDF(n)!!$ is divisible by n , where the double factorial

$m!! = 1 \times 3 \times 5 \times \dots \times m$, if m is odd;
and $m!! = 2 \times 4 \times 6 \times \dots \times m$, if m is even.

For example:

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
SDF(n)	1	2	3	4	5	6	7	4	9	10	11	6	13	14	5	6

Reference:

- [1] Dumitrescu, C., Seleacu, V., "Some notions and questions in number theory", Erhus Univ. Press, Glendale, 1994, Section #54 ("Smarandache Double Factorial Numbers").

11) Primitive Functions:

Let p be a positive prime.

$S_p : \mathbb{N} \rightarrow \mathbb{N}$, having the property that $(S_p(n))!$ is divisible by p^n ,

and it is the smallest integer with this property.

For example:

$S_3(4) = 9$, because $9!$ is divisible by 3^4 , and it is the smallest one with this property.

These functions help computing the Smarandache Function.

Reference:

- [1] Smarandache, Florentin, "A function in number theory", <Analele Universitatii Timisoara>, Seria St. Mat., Vol. XVIII, fasc. 1, 1980, pp. 79-88.

12) Smarandache Function:

$S : \mathbb{N} \rightarrow \mathbb{N}$, $S(n)$ is the smallest integer such that $S(n)!$ is divisible by n .

Reference:

- [1] Smarandache, Florentin, "A function in number theory", <Analele Universitatii Timisoara>, Seria St. Mat., Vol. XVIII, fasc. 1, 1980, pp. 79-88.

13) Smarandache Functions of the First Kind:

$S : \mathbb{N} \rightarrow \mathbb{N}$
 n

i) If $n = u^r$ (with $u = 1$, or $u = p$ prime number), then

$S(a) = k$, where k is the smallest positive integer such that

$k!$ is a multiple of u^{ra} ;

ii) If $n = p_1^{r_1} \cdot p_2^{r_2} \cdot \dots \cdot p_t^{r_t}$, then $S(a) = \max_{1 \leq j \leq t} \{ S_{p_j^{r_j}}(a) \}$.

14) Smarandache Functions of the Second Kind:

$S : \mathbb{N} \rightarrow \mathbb{N}$, $S(n) = S(k)$ for k in \mathbb{N} ,

where S are the Smarandache functions of the first kind.

15) Smarandache Function of the Third Kind:

$S(n) = S_a(b)$, where S_a is the Smarandache function of the

first kind, and the sequences (a_n) and (b_n) are different from

the following situations:

- i) $a_n = 1$ and $b_n = n$, for n in \mathbb{N} ; *
- ii) $a_n = n$ and $b_n = 1$, for n in \mathbb{N} . *

Reference:

[1] Balacenoiu, Ion, "Smarandache Numerical Functions", <Bulletin of Pure and Applied Sciences>, Vol. 14E, No. 2, 1995, pp. 95-100.

16) S. Prime Functions are defined as follows:

$P : \mathbb{N} \rightarrow \{0, 1\}$, with

$$P(n) = \begin{cases} 0, & \text{if } n \text{ is prime;} \\ 1, & \text{otherwise.} \end{cases}$$

For example $P(2) = P(3) = P(5) = P(7) = P(11) = 0$, whereas $P(0) = P(1) = P(4) = P(6) = \dots = 1$.

More general:

$P_k : \mathbb{N} \rightarrow \{0, 1\}$, where k is an integer ≥ 2 , and

$$P_k(n_1, n_2, \dots, n_k) = \begin{cases} 0, & \text{if } n_1, n_2, \dots, n_k \text{ are all prime numbers;} \\ 1, & \text{otherwise.} \end{cases}$$

17) S. Coprime Functions are similarly defined:

$C_k : \mathbb{N} \rightarrow \{0, 1\}$, where k is an integer ≥ 2 , and

$$C_k(n_1, n_2, \dots, n_k) = \begin{cases} 0, & \text{if } n_1, n_2, \dots, n_k \text{ are coprime numbers;} \\ 1, & \text{otherwise.} \end{cases}$$

Reference:

- [1] F. Smarandache, "Collected Papers", Vol. II, 200 p., <Functii Prime and Coprime>, p. 137, Kishinev University Press, Kishinev, 1997.

18) The Smallest Power Function:

SP(n) is the smallest number m such that m^k is divisible by n, where $k \geq 2$ is given.

The following sequence SP(n) is generated:

1, 2, 3, 2, 5, 6, 7, 4, 3, 10, 11, 6, 13, 14, 15, 4, 17, 6, 19, 10, 21, 22, 23, 6, 5, 26, 3, 14, 29, 30, 31, 4, 33, 34, 35, 6, 37, 38, 39, 20, 41, 42, ...

Remarks:

If p is prime, then $SP(p) = p$.
 If r is square free, then $SP(r) = r$.

If $n = (p_1^{s_1}) \times \dots \times (p_k^{s_k})$ and all $s_i \leq p_i$, then $SP(n) = n$.

If $n = p^s$, where p is prime, then:

$$SP(n) = \begin{cases} p, & \text{if } 1 \leq s \leq p; \\ p^2, & \text{if } p+1 \leq s \leq 2p^2; \\ p^3, & \text{if } 2p^2+1 \leq s \leq 3p^3; \\ \dots & \dots \\ p^t, & \text{if } (t-1)p^{(t-1)+1} \leq s \leq tp^t. \end{cases}$$

Generally, if $n = (p_1^{s_1}) \times \dots \times (p_k^{s_k})$, with all p_i prime, then:

$$SP(n) = (p_1^{t_1}) \times \dots \times (p_k^{t_k}), \text{ where}$$

$$t_i = u_i \text{ if } (u_i - 1)p_i^{(u_i - 1)+1} \leq s_i \leq u_i p_i^{u_i}$$

for $1 \leq i \leq k$.

Particular cases:

a) A second function ($k=2$):

1, 2, 3, 2, 5, 6, 7, 4, 3, 10, 11, 6, 13, 14, 15, 4, 17, 6, 19, 10,
21, 22, 23, 12, 5, 26, 9, 14, 29, 30, 31, 8, 33, ...

2

($S_2(n)$ is the smallest integer m such that m is divisible by n)

b) A third function ($k=3$):

1, 2, 3, 2, 5, 6, 7, 8, 3, 10, 11, 6, 13, 14, 15, 4, 17, 6, 19, 10,
21, 22, 23, 6, 5, 26, 3, 14, 29, 30, 31, 4, 33, ...

3

($S_3(n)$ is the smallest integer m such that m is divisible by n)

19) A $3n$ -digital subsequence:

13, 26, 39, 412, 515, 618, 721, 824, 927, 1030, 1133, 1236, ...
(numbers that can be partitioned into two groups such that the
second is three times bigger than the first)

20) A $4n$ -digital subsequence:

14, 28, 312, 416, 520, 624, 728, 832, 936, 1040, 1144, 1248, ...
(numbers that can be partitioned into two groups such that the
second is four times bigger than the first)

21) A $5n$ -digital subsequence:

15, 210, 315, 420, 525, 630, 735, 840, 945, 1050, 1155, 1260, ...
(numbers that can be partitioned into two groups such that the
second is five times bigger than the first)

22) Sequences of Sub-sequences

For all of the following sequences:

a) *Crescendo Sub-sequences:*

1, 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, 2, 3, 4, 5, 1, 2, 3, 4, 5, 6,
1, 2, 3, 4, 5, 6, 7, 1, 2, 3, 4, 5, 6, 7, 8, . . .

b) *Decrescendo Sub-sequences:*

1, 2, 1, 3, 2, 1, 4, 3, 2, 1, 5, 4, 3, 2, 1, 6, 5, 4, 3, 2, 1,
7, 6, 5, 4, 3, 2, 1, 8, 7, 6, 5, 4, 3, 2, 1, . . .

c) *Crescendo Pyramidal Sub-sequences:*

1, 1, 2, 1, 1, 2, 3, 2, 1, 1, 2, 3, 4, 3, 2, 1,
 1, 2, 3, 4, 5, 4, 3, 2, 1, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, . . .

d) *Decrescendo Pyramidal Sub-sequences:*

1, 2, 1, 2, 3, 2, 1, 2, 3, 4, 3, 2, 1, 2, 3, 4,
 5, 4, 3, 2, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, 2, 3, 4, 5, 6, . . .

e) *Crescendo Symmetric Sub-sequences:*

1, 1, 1, 2, 2, 1, 1, 2, 3, 3, 2, 1, 1, 2, 3, 4, 4, 3, 2, 1,
 1, 2, 3, 4, 5, 5, 4, 3, 2, 1, 1, 2, 3, 4, 5, 6, 6, 5, 4, 3, 2, 1, . . .

f) *Decrescendo Symmetric Sub-sequences:*

1, 1, 2, 1, 1, 2, 3, 2, 1, 1, 2, 3, 4, 3, 2, 1, 1, 2, 3, 4,
 5, 4, 3, 2, 1, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, 1, 2, 3, 4, 5, 6, . . .

g) *Permutation Sub-sequences:*

1, 2, 1, 3, 4, 2, 1, 3, 5, 6, 4, 2, 1, 3, 5, 7, 8, 6, 4, 2,
 1, 3, 5, 7, 9, 10, 8, 6, 4, 2, 1, 3, 5, 7, 9, 10, 8, 6, 4, 2, . . .

Find a formula for the general term of the sequence.

Solutions:

For purposes of notation in all problems, let

$$a(n)$$

denote the n-th term in the complete sequence and

$$b(n)$$

the n-th subsequence. Therefore, a(n) will be a number and b(n) a sub-sequence.

a) Clearly, b(n) contains n terms. Using a well-known summation formula, at the end of b(n) there would be a total of

$$\frac{n(n + 1)}{2}$$

terms.

Therefore, since the last number of b(n) is n,

$$a((n(n+1))/2) = n.$$

Finally, since this would be the terminal number in the sub-sequence

$$b(n) = 1, 2, 3, . . . , n$$

the general formula is

$$a\left(\frac{n(n+1)}{2} - i\right) = n - i$$

for $n \geq 1$ and $0 \leq i \leq n - 1$.

b) With modifications for decreasing rather than increasing, the proof is essentially the same. The final formula is

$$a\left(\frac{n(n+1)}{2} - i\right) = 1 + i$$

for $n \geq 1$ and $0 \leq i \leq n - 1$.

c) Clearly, $b(n)$ has $2n - 1$ terms. Using the well-known formula of summation

$$1 + 3 + 5 + \dots + (2n - 1) = n.$$

the last term of $b(n)$ is in position n^2 and $a(n^2) = 1$. The largest number in $b(n)$ is n , so counting back $n - 1$ positions, they increase in value by one each step until n is reached.

$$a(n^2 - i) = 1 + i, \quad \text{for } 0 \leq i \leq n-1.$$

After the maximum value at $n-1$ positions back from n^2 , the values decrease by one. So at the n th position back, the value is $n-1$, at the $(n-1)$ st position back the value is $n-2$ and so forth.

$$a(n^2 - n - i) = n - i - 1$$

for $0 \leq i \leq n - 2$.

d) Using similar reasoning

$$a(n^2) = n \quad \text{for } n \geq 1$$

and

$$a(n^2 - i) = n - i, \quad \text{for } 0 \leq i \leq n-1$$

$$a(n^2 - n - i) = 2 + i, \quad \text{for } 0 \leq i \leq n-2.$$

e) Clearly, $b(n)$ contains $2n$ terms. Applying another well-known summation formula

$$2 + 4 + 6 + \dots + 2n = n(n+1), \quad \text{for } n \geq 1.$$

Therefore, $a(n(n+1)) = 1$. Counting backwards $n-1$ positions, each term

decreases by 1 up to a maximum of n.

$$a((n(n+1))-i) = 1 + i, \text{ for } 0 \leq i \leq n-1$$

The value n positions down is also n and then the terms decrease by one back down to one.

$$a((n(n+1))-n-i) = n - i, \text{ for } 0 \leq i \leq n - 1.$$

f) The number of terms in b(n) is the same as that for (e). The only difference is that now the direction of increase/decrease is reversed.

$$a((n(n+1))-i) = n - i, \text{ for } 0 \leq i \leq n-1.$$

$$a((n(n+1))-n-i) = 1 + i, \text{ for } 0 \leq i \leq n - 1.$$

g) Given the following circular permutation on the first n integers.

$$\begin{array}{l} \text{phi} \\ n \end{array} = \begin{array}{c} | \ 1 \ 2 \ 3 \ 4 \ \dots \ n-2 \ n-1 \ n \ | \\ | \ 1 \ 3 \ 5 \ 7 \ \dots \ 6 \ 4 \ 2 \ | \end{array}$$

Once again, b(n) has 2n terms. Therefore,

$$a(n(n+1)) = 2.$$

Counting backwards n-1 positions, each term is two larger than the successor

$$a((n(n+1))-i) = 2 + 2i, \text{ for } 0 \leq i \leq n-1.$$

The next position down is one less than the previous and after that, each term is again two less the successor.

$$a((n(n+1))-n-i) = 2n - 1 - 2i, \text{ for } 0 \leq i \leq n-1.$$

As a single formula using the permutation

$$a((n(n+1))-i) = \text{phi}_n(2n-i), \text{ for } 0 \leq i \leq 2n-1.$$

Reference:

- F. Smarandache, "Numerical Sequences", University of Craiova, 1975;
- [See Arizona State University, Special Collection, Tempe, AZ, USA].

GENERAL REFERENCES

BOOKS:

- Muller, R. (editor), "Unsolved Problems Related to the Smarandache Function", Number Theory Publishing co., Phoenix, AZ, USA, 1993.
- Dumitrescu C., Seleacu, V. (editors), "Some Notions and Questions in Number Theory", Erhus Univ. Press, Glendale, USA, 1994, 66p.
- Ashbacher, Charles, "An Introduction to the Smarandache Function", Erhus University Press, Vail, USA, 1995, 61p.

Ashbacher, Charles, "Collection of Problems on Smarandache Notions", Erhus University Press, Vail, 1996, 73p.

Kashihara, Kenichiro, (Japon), "Comments and Topics on Smarandache Notions and Problems", Erhus University Press, Vail, 1996, 50p.

Dumitrescu C., Seleacu, V., "The Smarandache Function", Erhus University Press, Vail, 1996, 135p.

Ibstedt, Henry, "Surfing on the Ocean of Numbers / A few Smarandache Notions and Similar Topics", Erhus University Press, Vail, 1997, 76p.

Radescu, Nicolae, "Elemente de Aritmetica si Teoria Numerelor", Editura Didactica Nova, Craiova, Romania, 1996, pp. 132-6.

Ibstedt, Henry, "Computer Analysis Systems", American Research Press, Lupton, two editions, 1998, 86p.

Ashbacher, Charles, "Plucking From the Tree of Smarandache Sequences and Functions", American Research Press, Lupton, 1998, 88 p.

ARTICLES:

Le, Charles T., "The Smarandache Class of Paradoxes", in <Journal of Indian Academy of Mathematics>, Indore, India, Vol. 18, No. 1, 1996, pp. 53-5;
and in <Bulletin of the Transilvania University of Brasov>, Mathematics, Vol. 1 (36) - New Series, Series B, Transilvania University Press, Romania, 1994, pp. 7-8.

Le, Charles T., "The Most Paradoxist Mathematician of the World: Florentin Smarandache", in <Bulletin of Pure and Applied Sciences>, India, Vol. 15E (Maths & Statistics), No. 1, January-June 1996, pp. 81-100.

Smith, Sylvester, "A Set of Conjectures on Smarandache Sequences", in <Bulletin of Pure and Applied Sciences>, India, Vol. 15E (Maths & Statistics), No. 1, January-June 1996, pp. 101-107.

Seagull, Larry, "Poem in Arithmetic Space", in <World Poetry 95>, The 13th Series, Editors: Kim Joung Woong, Kang Shin-Il, Ul Chi Publishing Co., Seoul, Korea, 1995, pp. 288-90;

and in <Henry C. Bunner / An Anthology in Memoriam>, Bristol Banner Books, Bristol, IN, USA, 1996, p. 149.

Le, Charles T., "The Smarandache Class of Paradoxes / (mathematical poem)", in <Henry C. Bunner / An Anthology in Memoriam>, Bristol Banner Books, Bristol, IN, USA, 1996, p. 94.

Seagull, L., "O Scurt Biografie", in <Tempus>, Bucharest, Anul V, Nr. 1, Iulie 1996, p. 7.

Tilton, Homer B., "Smarandache's Paradoxes", Tucson, AZ, USA, Vol. 2, No. 9, September 1996, pp. 1-2.

Mudge, Mike, "Not Numerology, but Numerology!", in <Personal Computer Worls>, London, February 1997, pp. 279-280.

Mudge, Mike, "The Smarandache Function", in <Personal Computer Worls>, London, July 1992, p. 420.

Dumitrescu, Constantin, "La Fonction de Smarandache -- une Nouvelle Fonction dans la Theorie des Nombres", Congres International <<Henry Pointcare>>, Universite Nancy 2, France, 14-18 Mai, 1994.

NOTES:

Kashihara, Kenichiro, "Introducing the Pseudo-Smarandache Function", in <Personal Computer World>, London, September 1997, p. 249.

Traian, Ilie, "Scrisul în Diaspora / Florentin Smarandache", in <Curierul Românesc>, Bucharest, Anul IX, Nr. 5(124), mai 1997, p. 17.

Coanda, George, "'Emigrant la Infinit'", in <Dâmbovita>, Târgoviste, Romania, Nr. 1284, 28 Ianuarie 1997, p. 4.

Vlad, Mihail I., "Universul Florentin Smarandache", in the volume: <Emigrant la Infinit>, by F. Smarandache, Ed. Macarie, Târgoviste, 1996, p. 5.

Barbulescu, Radu, "Florentin Smarandache: Exist Împotriva Mea", in <Observator-Munchen>, Germany, Anul VIII, No. 2-4 (27-29), Martie-Decembrie 1995, p. 72.

CC, "Un Român Lanseaza Noi Teorii în Matematica", in <Jurnalul National>, Bucharest, No. 806, 22 Ianuarie 1996.

Policarp, Gane, "Anunt Important", in <Caiet de Informare Matematica>, Câmpina, Romania, Anul XVIII, No. 35, Mai 1996, coperta III; and Anul XVIII, No. 36, Decembrie 1996, coperta III.

Castillo, Jose, "Dear Editor", in <Mathematical Spectrum>, University of Sheffield, U.K., Editor D.W. Sharpe, Vol. 29, No. 1, 1996-7, p. 21.

Castini, Peter, "Dear Editor", in <Mathematical Spectrum>, University of Sheffield, U.K., Editor D.W. Sharpe, Vol. 28, No. 3, 1995-6, p. 68.

Seagull, L., "The Smarandache Function and the Number of Primes up to x ", in <Mathematical Spectrum>, Vol. 28, No. 3, 1995-6, p. 53.

ABSTRACTS:

Ibstedt, Henry, "Smarandache Iterations of First and Second Kind", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 4, Issue 106, 1996, 96T-11-194, p. 680.

Smarandoiu, Stefan, "Convergence of Smarandache Continued Fractions", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 4, Issue 106, 1996, 96T-11-195, p. 680.

Smarandoiu, Stefan, "Roots of Smarandache type Polynomials", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 4, Issue 106, 1996, 96T-12-196, p. 680.

Grenier, Marie, "The Smarandache Multi-Structure and Multi-Space", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-03-101, p. 579.

Haufmann, Hans, "The Smarandache Inconsistent Systems of Axioms", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-03-143, p. 579.

Dinca, C., "The Smarandache General-Recurrence Type Sets", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-03-154, pp. 579-80.

Dinca, C., "Examples of Smarandache Recurrence Sets (I)", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-03-155, p. 580.

Brewster, Joan, "The Smarandache Multiplication and Division", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-102, pp. 582-3.

Mudge, Micael R., "Introducing the Smarandache-Kurepa and Smarandache-Wagstaff Functions", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-108, p. 583.

Mudge, Micael R., "Examples of Smarandache Magic Squares of Order n ", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-110, p. 583.

Tutescu, Lucian, "On a Conjecture Concerning the Smarandache Function", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-113, p. 583.

Sandor, Jozsef, "On Certain Inequalities involving the Smarandache Function", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-114, p. 583.

Balacenoiu, Ion, "The Monotony of Smarandache Functions of First Kind", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-115, p. 584.

Andrei, M., "A Linear Combination with Smarandache Function to Obtain an Identity", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-116, p. 584.

Bencze, Mihaly, "Proposed Problems Related to Smarandache Function", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-118, p. 584.

Burton, Emil, "An Isomorphism used to study the Smarandache Function", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-119, p. 584.

Seleacu, V., "On the Θ_s and Θ_{s^2} Functions", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-120, p. 584.

Ibstedt, Henry, "Base Solution (The Smarandache Function)", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-123, p. 584.

Mudge, Michael R., "The Smarandache Near-To-Primorial Function", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-124, p. 585.

Dumitrescu, C., "On the Calculation of the Smarandache Function", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-126, p. 585.

Ashbacher, Charles, "A Note on the Smarandache Near-To-Primordial Function", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-128, p. 585.

Burton, E., "Some Convergence Problems Involving the Smarandache Function", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-129, p. 585.

Cojocaru, Ion, "The Constant of Smarandache", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-130, p. 585.

Cojocaru, Ion, "The Irrationality of the Sum of a Remarkable Series", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-131, p. 585.

Cojocaru, Ion, "Solving some Diophantine Equations Concerning the Smarandache Function", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-132, p. 586.

Cojocaru, Ion, "On some Series Involving the Smarandache Function", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-133, p. 586.

Tutescu, Lucian, "Four Solved Equations the Smarandache Function", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-134, p. 586.

Raymond, George, "The Smarandache Wrong Numbers", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-136, p. 586.

Raymond, George, "Some Special Sequences", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-137, pp. 586-7.

Raymond, George, "The Smarandache Random Sieve", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-138, p. 587.

Raymond, George, "The Smarandache Cubic Base", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-139, p. 587.

Tolu, Ali, "On a Concatenate Sequence", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-145, p. 587.

Grebenikova, Irina, "Some Bases of Numeration", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-146, p. 588.

Tolu, Amir, "On the Smarandache U-Sequence(s)", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-147, p. 588.

Ibstedt, Henry, "Radu's problem", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-160, p. 588.

Ibstedt, Henry, "Smarandache-Fibonacci Triplets", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-11-161, p. 588.

Mullin, Albert A., "On the Smarandache Function and the Fixed-Point Theorem", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-33-156, p. 592.

Tolu, Ali, "Series of the Reciprocal of Smarandache Non-U-Sequences", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-40-148, p. 594.

Dinca, Constantin, "Series Related to Smarandache Type S-Sequences", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-40-149, p. 594.

Tolu, Ali, "Series of the Reciprocal of Smarandache Non-Multiplicative Sequences", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-40-150, p. 594.

Tolu, Ali, "Series of the Reciprocal of Smarandache Anti-Symmetric Sequence", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-40-157, p. 594.

Popov, M. R., "The Smarandache Non-Geometry", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-51-98, p. 595.

Brown, Jerry L., "The Smarandache Counter-Projective Geometry", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 3, Issue 105, 1996, 96T-51-112, pp. 595-6.

Kashihara, Kenichiro, "Comments and Topics on Smarandache Notions and Problems", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 4, Issue 106, 1996, 96T-11-186, p. 678.

Kashihara, Kenichiro, "Pseudo-Smarandache Function", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 4, Issue 106, 1996, 96T-11-187, pp. 678-9.

Popescu, Marian, "Implementation of the Smarandache Integer Algorithms", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 1, Issue 103, 1996, 96T-99-14, p. 264; and in <Abstracts of Papers Presented to the American Mathematical Society>, Vol. 17, No. 2, Issue 104, 1996, 96T-11-52, p. 447.

Popescu, Marian, "A Model of the Smarandache Paradoxist Geometry", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 1, Issue 103, 1996, 96T-99-15, p. 265.

Popescu, Florentin, "Remarks on some Smarandache Type Sequences", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 2, Issue 104, 1996, 96T-11-51, p. 447.

Popescu, Florentin, "Deformability of some Smarandache Wild Knots", in <Abstracts of Papers Presented to the American Mathematical Society>, New Providence, RI, USA, Vol. 17, No. 2, Issue 104, 1996, 96T-54-53, p. 452.

PROPOSED PROBLEMS AND SOLUTIONS:

Ashbacher, Charles, "A Smarandache Problem", in <Journal of Recreational Mathematics>, USA, Vol. 28(2), 1996-7, Proposed Problem # 2335, p. 144.

Ashbacher, Charles, "A Pseudo-Smarandache Function Problem", in <Journal of Recreational Mathematics>, USA, Vol. 28(2), 1996-7, Proposed Problem # 2336, pp. 145-6.

Yau, T., "Aufgabe 1118", in <Elemente der Mathematik>, Basel, Switzerland, Vol. 52 (1997), p. 37.

Ashbacher, Charles, "Problem / Solution", in <Mathematics and Informatics Quarterly>, Bulgaria, Vol. 7, No. 2, June 1997, p. 80.

Ashbacher, Charles, "Problem AF-7", in <The AMATYC Review>, USA, Vol. 18, No. 2, Spring 1997, p. 70.

Ashbacher, Charles, "Problem 4616", in <School Science and Mathematics>, USA, Vol. 97(4), April 1997, p. 221.

Brown, Jerry, & Castillo, Jose, "Problem 4619", in <School Science and Mathematics>, USA, Vol. 97(4), April 1997, pp. 221-2.

Ashbacher, Charles, "Smarandache Lucky-Digital Subsequence", in <Journal of Recreational Mathematics>, USA, Vol. 28(1), 1996-7, Proposed Problem # 2301, p. 61.

Castillo, Jose, "Problem MH", in <Personal Computer World>, February 1996, p. 337.

Mudge, Mike, "Proposed Funy Problem", in <Math Power>, Pima Community College, Tucson, AZ, USA, Editor Homer B. Tilton, Vol. 2, No. 6, 1996 (WK12), p. 26.

Ashbacher, Charles, "Problem 864", in <PI MU Epsilon Journal>, USA, Vol. 10 (3), Fall 1995, p. 226.

Rodriguez, J., "Problem 866", in <PI MU Epsilon Journal>, USA, Vol. 10 (3), Fall 1995, p. 226.

Seagull, L., Problem 4541, in <School Science and Mathematics>, USA, Vol. 97, No. 7, November 1996, p. 392.

Kuenzi, N.J., and Prielipp, Bob, Solution to Problem 4541, in <School Science and Mathematics>, USA, Vol. 97, No. 7, November 1996, p. 392.

Ashbacher, Charles, Problem 4604, in <School Science and Mathematics>, USA, Vol. 97, No. 2, February 1996, p. 109.

Ashbacher, Charles, "Problem 501", in <The Pentagon>, Fall 1996, p. 51.

Rodriguez, J., Problem H-484, in <The Fibonacci Quarterly>, USA, Vol. 32, No. 1, February 1994, pp. 91-2.

Rose, W. A., and Economides, Gregory, Solution to Problem 26.5, in <Mathematical Spectrum>, Vol. 26, No. 4, 1993/4, p. 125.

Martin, Thomas, "Aufgabe 1075", in <Elemente der Mathematik>, Switzerland, Vol. 49, No. 3, 1994, p. 127.

REVIEWS:

Ashbacher, Charles, "'The Most Paradoxist Mathematician of the World', by Charles T. Le", in <Journal of Recreational Mathematics>, USA, Vol. 28(2), 1996-7, p. 130.

Mudge, Mike, "Loose ends / February 1997", in <Personal Computer World>, London, October 1997, p. 277.

Mercier, Armel, "Kashihara, Kenichiro / '*Comments and topics on Smarandache notions and problems'", in <Mathematical Reviews>, Ann Arbor, USA, 97k:11008.

Mercier, Armel, "Dumitrescu, C.; Seleacu, V. / '*The Smarandache function'", in <Mathematical Reviews>, Ann Arbor, USA, 97i:11004.

Wolfson, Paul, "Dumitrescu, C.: Seleacu, V. 'Some Notions and Questions in Number Theory'", in <Historia Mathematica>, USA, Vol. 24, No. 2, May 1997, p. 219 (#24.2.48).

Le, Charles T., "Kashihara, Kenichiro / 'Comments and topics on Smarandache notions and problems'", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 861-55, 1997, 11004.

Le, Charles T., "Ashbacher, Charles / 'Collection of problems on Smarandache notions'", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 846-52, 1992, 11002.

LCL, "Recreational Mathematics", in <American Mathematical Monthly>, USA, May 1997.

Minh, Ngan (Vietnam), "Poetical Numbers", in <Paul Laurence Dunbar / An Anthology in Memoriam>, Bristol Banner Books, Bristol, IN, USA, Editor M. Myers, 1997, p. 105.

Castillo, Jose, "Dear Editor", in <Mathematical Spectrum>, University of Sheffield, U.K., Editor D.W. Sharpe, Vol. 29, No. 1, 1996-7, p. 21.

Rotaru, Ion, "Din nou despre Florentin Smarandache", in <Vatra>, Tg. Mure , Romania, Nr. 2 (299), 1996, pp. 93-4.

Zitarelli, David E., "Mudge, Michael R. / A Paradoxist Mathematician: His Function, Paradoxist Geometry, and Class of Paradoxes", in <Historia Mathematica>, USA, February 1997, Vol. 24, No. 1, # 24.1.119, p. 114.

Ashbacher, Charles, "Collected Papers of Florentin Smarandache, Volume I", in <Mathematics and Computer Education>, USA, Vol. 31, No.1, Winter 1997, pp. 104-5.

Zitarelli, David E., "Le, Charles T. / The Most Paradoxist Mathematician of the World", in <Historia Mathematica>, USA, November 1995, Vol. 22, No. 4, # 22.4.110, p. 460.

Le, Ch. T., "Vasiliu, Florin / Paradoxism's Main Roots", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 830 - 17, 03001, 1996/05.

Atanassov, K., "Popescu, Marcela; Popescu, Paul; Seleacu, Vasile / On Some Numerical Functions", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 831 - 44, 11003, 1996/06.

Atanassov, K., "B l cenoiu, I.; Seleacu, V.; Vîrlan, N. / Properties of the Numerical Function F_s ", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 831 - 44, 11004, 1996/06.

Atanassov, K., "Seleacu, Vasile; Vîrlan, Narcisa / On a Limit of a Sequence of the Numerical Function", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 831 - 45, 11005, 1996/06.

Atanassov, K., "Burton, Emil / On some Series Involving Smarandache Function", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 831 - 45, 11006, 1996/06.

Atanassov, K., "B l cenoiu, Ion; Seleacu, Vasile / Some Properties of Smarandache Functions of the Type I", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 831 - 45, 11007, 1996/06.

Atanassov, K., "Ashbacher, Charles / Some Problems on Smarandache Function", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 831 - 45, 11008, 1996/06.

Atanassov, K., "B l cenoiu, Ion; Popescu, Marcela; Seleacu, Vasile / About the Smarandache Square's Complementary Function", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 831 - 45, 11009, 1996/06.

Atanassov, K., "Tomi , Tiberiu Florin / Some Remarks Concerning the Distribution of the Smarandache Function", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 831 - 45, 11010, 1996/06.

Atanassov, K., "R descu, E.; R descu, N.; Dumitrescu, C. / Some Elementary Algebraic considerations Inspired by the Smarandache Function", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 831 - 46, 11011, 1996/06.

Atanassov, K., "B l cenoiu, Ion; Dumitrescu, Constantin / Smarandache Functions of the Second Kind", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 831 - 46, 11012, 1996/06.

Atanassov, K., "Popescu, Marcela; Popescu, Paul / The Problem of Lipschitz Condition", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 831 - 46, 11013, 1996/06.

Atanassov, K., "Dumitrescu, Constantin / A Brief History of the 'Smarandache Function'", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 831 - 46, 11014, 1996/06.

Atanassov, K., "Dumitrescu, C.; Seleacu, V. / Some Notions and Questions in Number Theory", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 840 - 70, 11001, 1996/15.

Atanassov, K., "Smarandache, Florentin / Only Problems, Not Solutions! 4th ed", in <Zentralblatt fuer Mathematik>, Berlin, Germany, 840 - 70, 11002, 1996/15.

CITATIONS:

Gardner, Martin, "Lucky Numbers and 2187", in <The Mathematical Inteligencer>, USA, Vol. 19, No. 2, 1997, p. 29.

----, Mathematics Calendar, "First International Conference on Smarandache Type Notions in Number Theory", in <Notices of the American Mathematical Society>, Providence, RI, USA, September 1996, p. 1057.

----, "Smarandache Notions", in <Ulrich's International Periodicals Directory 1997>, Mathematics, USA, pp. 4396, 8074, 9117.

----, "Smarandache function", in <Subject Headings>, Library of Congress, 17th Edition, Vol. III (K-P), Washington, D.C., 1994, p. 3530.

INTERNATIONAL CONGRESS:

The First International Conference on Smarandache Type Notions in Number Theory, August 21-24, Department of Mathematics, University of Craiova, Romania; This Conference has been organized by Dr. C. Dumitrescu & Dr. V. Seleacu, under the auspices of UNESCO.