Soft Computing

Similarity measures between interval neutrosophic sets and their multicriteria decision-making method

--Manuscript Draft--

Manuscript Number:	SOCO-D-11-00309
Full Title:	Similarity measures between interval neutrosophic sets and their multicriteria decision-making method
Article Type:	Original Research
Keywords:	Neutrosophic set; Interval neutrosophic set; Hamming distance; Euclidean distance; Similarity measure; Decision-making
Abstract:	An interval neutrosophic set is an instance of a neutrosophic set, which can be used in real scientific and engineering applications. In the paper, the Hamming and Euclidean distances between interval neutrosophic sets (INSs) are defined and the similarity measures between INSs are proposed based on the relationship between similarity measures and distances. Then a multicriteria decision-making method is established in interval neutrosophic setting, in which criterion values for alternatives are INSs and the criterion weights are known information. We utilize the similarity measures between each alternative and the ideal alternative to rank the alternatives and to determine the best one. Finally, an illustrative example demonstrates the application of the proposed decision-making method.

Similarity measures between interval neutrosophic sets and their multicriteria decision-making method

Jun Ye*

Faculty of Engineering, Shaoxing University, Shaoxing, Zhejiang Province 312000, P.R. China

Abstract

An interval neutrosophic set is an instance of a neutrosophic set, which can be used in real scientific and engineering applications. In the paper, the Hamming and Euclidean distances between interval neutrosophic sets (INSs) are defined and the similarity measures between INSs are proposed based on the relationship between similarity measures and distances. Then a multicriteria decision-making method is established in interval neutrosophic setting, in which criterion values for alternatives are INSs and the criterion weights are known information. We utilize the similarity measures between each alternative and the ideal alternative to rank the alternatives and to determine the best one. Finally, an illustrative example demonstrates the application of the proposed decision-making method.

Keywords: Neutrosophic set; Interval neutrosophic set; Hamming distance; Euclidean distance; Similarity measure; Decision-making

---

* Corresponding author.
Tel.: +86-575-88327323
E-mail address: yejnn@yahoo.com.cn (Jun Ye)

1. Introduction

Neutrosophy is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra (Smarandache 1999). Neutrosophic set is a powerful general formal framework which generalizes the concept of the classic set, fuzzy set (Zadeh 1965), interval valued fuzzy set (Turksen 1986), intuitionistic fuzzy set (Atanassov 1986), interval valued intuitionistic fuzzy set (Atanassov and Gargov 1989), paraconsistent set (Smarandache 1999), dialetheist set (Smarandache 1999), paradoxist set (Smarandache 1999), tautological set (Smarandache 1999). In neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership, and false-membership are independent. This assumption is very important in many applications such as information fusion in which the data are combined from different sensors. Recently, neutrosophic sets had mainly been applied to image processing (Cheng and Guo, 2008; Guo and Cheng 2009).

Intuitionistic fuzzy sets and interval valued intuitionistic fuzzy sets can only handle incomplete information but not the indeterminate information and inconsistent information which exist commonly in real situations. For example, when we ask the opinion of an expert about certain statement, he or she may that the possibility that the statement is true is between 0.5 and 0.7, and the statement is false is between 0.2 and 0.4, and the degree that he or she is not sure is between 0.1 and 0.3. Here is another example, suppose there are 10 voters during a voting process. In time t_1, four vote "yes", three vote "no" and three are undecided. For neutrosophic notation, it can be expressed as x(0.4,0.3,0.3); in time t_2, two vote "yes", three vote "no", two give up, and three are undecided, then it can be expressed as x(0.2,0.3,0.3). That is beyond the scope of the intuitionistic fuzzy set. So the notion of neutrosophic set is more general and overcomes the aforementioned issues.

The neutrosophic set generalizes the above mentioned sets from philosophical point of view. From scientific or engineering point of view, the neutrosophic set and set-theoretic operators need to be specified. Otherwise, it will be difficult to apply in the real applications (Wang et al 2005). Therefore, Wang et al (2005) proposed the set-theoretic operators on an instance of neutrosophic set called interval neutrosophic set (INS). The interval neutrosophic set can represent uncertain, imprecise, incomplete and inconsistent information which exist in real world. However, to the best of our knowledge, the existing literature does not deal with similarity measures between INSs and the decision-making problems in interval neutrosophic setting. Therefore, the Hamming and Euclidean distances between INSs are defined and the distances-based similarity measures for INSs are proposed in this paper, which can be used in real scientific and engineering applications. Thus, a multicriteria decision-making method is established based on the proposed similarity measures. Through the similarity measures between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best one can be easily identified as well. An illustrative example demonstrates the application of the proposed decision-making method.

The rest of paper is organized as follows. Section 2 introduces the some concepts of neutrosophic sets (Smarandache 1999) and INSs (Wang et al 2005). The Hamming and Euclidean distances between INSs are defined and a similarity measure based on the Hamming distance and a similarity measure based on the Euclidean distance are proposed according to the relationship of similarity measures and distances in Section 3. A decision-making method is established in interval neutrosophic setting by means of the similarity measure between each alternative and the ideal alternative in Section 4. In Section 5, an illustrative example is presented to illustrate the developed approach. Finally, some final remarks of the similarity measures between INSs and the proposed decision-making method are given in Section 6.

2. Some concepts of neutrosophic sets

This section gives a brief overview of concepts of neutrosophic sets (Smarandache 1999) and interval neutrosophic sets (Wang et al 2005).

2.1. Neutrosophic sets

Neutrosophic set is a part of neutrosophy, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra (Smarandache 1999), and is a powerful general formal framework, which generalizes the above mentioned sets from philosophical point of view. The relationship of neutrosophic set and other sets is illustrated in Fig. 1 (Wang et al 2005).

Smarandache (1999) gave the following definition of a neutrosophic set.

Definition 1 (Smarandache 1999) Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$. That is $T_A(x)$: $X \rightarrow]0^-, 1^+[$, $I_A(x)$: $X \rightarrow]0^-, 1^+[$, and $F_A(x)$: $X \rightarrow]0^-, 1^+[$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2 (Smarandache 1999) The complement of a neutrosophic set A is denoted by A^c and is defined as $T_A{}^c(x) = \{1^+\} \ominus T_A(x)$, $I_A{}^c(x) = \{1+\} \ominus I_A(x)$, and $F_A{}^c(x) = \{1+\} \ominus F_A(x)$ for every x in X.

Definition 3 (Smarandache 1999) A neutrosophic set A is contained in the other neutrosophic set B, $A \subseteq B$ if and only if $\inf T_A(x) \le \inf T_B(x)$, $\sup T_A(x) \le \sup T_B(x)$, $\inf I_A(x) \ge \inf I_B(x)$, $\sup I_A(x) \ge \sup I_B(x)$, $\inf F_A(x) \ge \inf F_B(x)$, and $\sup F_A(x) \ge \sup F_B(x)$ for every x in X.

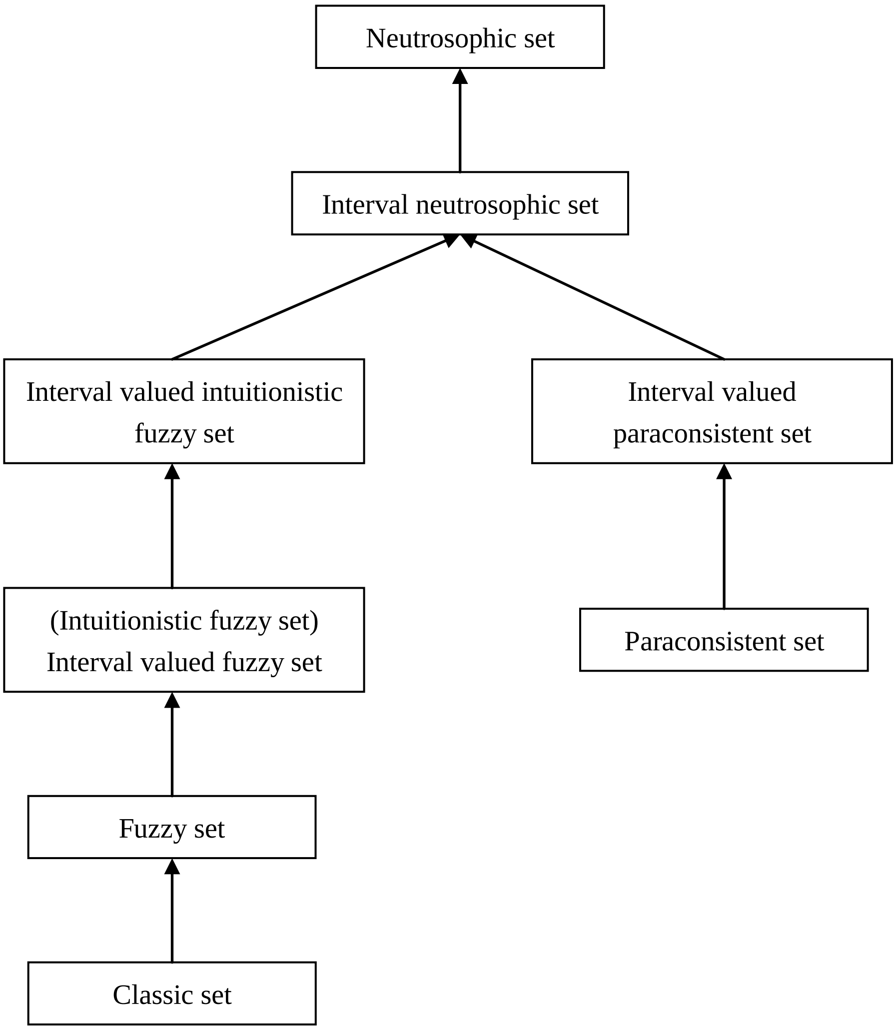

Fig. 1. Relationship of neutrosophic set and other sets

2.2. Interval neutrosophic sets

An INS is an instance of a neutrosophic set, which can be used in real scientific and engineering applications. In the following, we introduce the definition of an INS (Wang et al. 2005).

Definition 4 (Wang et al 2005) Let X be a space of points (objects) with generic elements in X denoted by x. An INS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x)$, $I_A(x)$, $F_A(x) \subseteq [0, 1]$.

We call it "interval" because it is the subclass of a neutrosophic set, that is, we only consider the subunitary interval of [0, 1]. Therefore, All INSs are clearly neutrosophic sets.

An INS in R^1 is illustrated in Fig. 2 (Wang et al 2005).

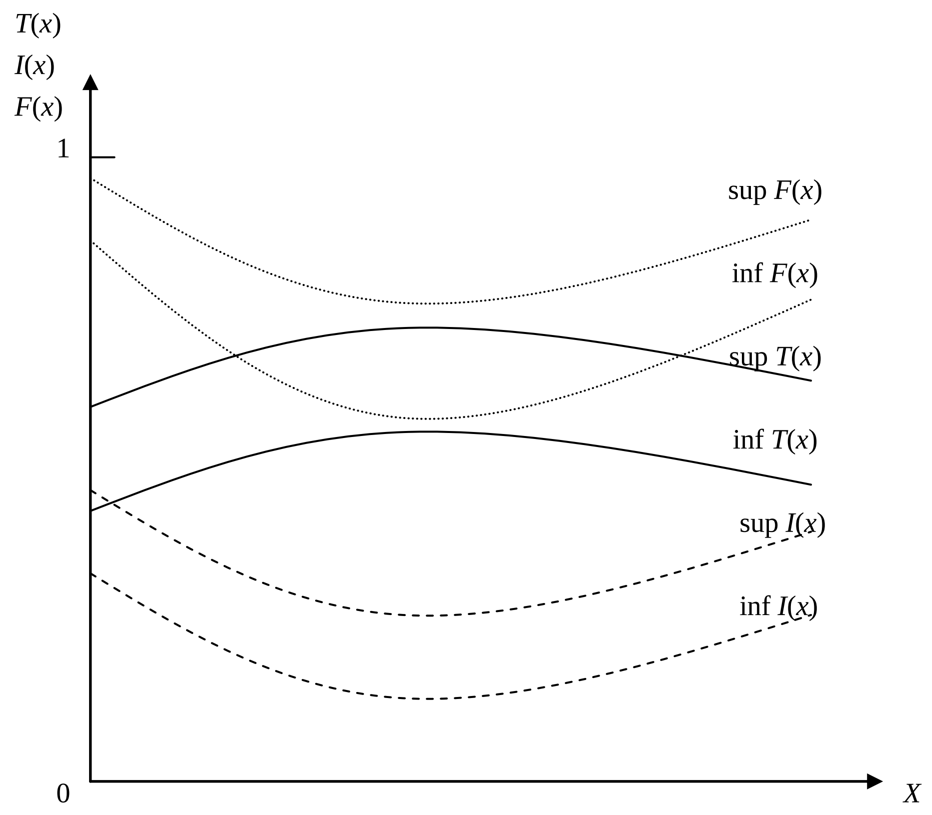

Fig. 2. Illustration of an INS in R^1

Definition 5 (Wang et al 2005) An INS A is empty if and only if its inf $T_A(x)$ = sup $T_A(x)$ = 0, inf $I_A(x)$

= sup $I_A(x)$ = 1, and inf $F_A(x)$ = sup $F_A(x)$ = 0 for any x in X.

Definition 6 (Wang et al 2005) The complement of an INS A is denoted by A^c and is defined as $T_A{}^c(x)$ = $F_A(x)$, inf $I_A{}^c(x) = 1 - \sup I_A(x)$, sup $I_A{}^c(x) = 1 - \inf I_A(x)$, $F_A{}^c(x) = T_A(x)$ for any x in X.

Let $\underline{0}$ = <0, 1, 1> and $\underline{1}$ = <1, 0, 0>. Then, $\underline{0}^c$ = <1, 0, 0> and $\underline{1}^c$ = <0, 1, 1>.

Definition 7 (Wang et al 2005) An interval neutrosophic set A is contained in the other INS B, $A \subseteq B$, if and only if inf $T_A(x) \le$ inf $T_B(x)$, sup $T_A(x) \le$ sup $T_B(x)$, inf $I_A(x) \ge$ inf $I_B(x)$, sup $I_A(x) \ge$ sup $I_B(x)$, inf $F_A(x) \ge$ inf $F_B(x)$, and sup $F_A(x) \ge$ sup $F_B(x)$ for any x in X.

Definition 8 (Wang et al 2005) Two INSs A and B are equal, written as $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.

3. Similarity measures between INSs

In this section, we present the definitions of the Hamming and Euclidean distances between INSs and the similarity measures between INSs based on the distances, which can be used in real scientific and engineering applications.

For convenience, two INSs A and B in $X = \{x_1, x_2, \ldots, x_n\}$ are denoted by $A = \{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\rangle \mid x_i \in X\}$ and $B = \{\langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\rangle \mid x_i \in X\}$, where $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i) \subseteq [0, 1]$ for every $x_i \in X$. Then we define the following distances for A and B.

(i) The Hamming distance:

$$d_1(A,B) = \frac{1}{6}\sum_{i=1}^{n}\Big[\left|\inf T_A(x_i) - \inf T_B(x_i)\right| + \left|\sup T_A(x_i) - \sup T_B(x_i)\right| + \left|\inf I_A(x_i) - \inf I_B(x_i)\right| \\ + \left|\sup I_A(x_i) - \sup I_B(x_i)\right| + \left|\inf F_A(x_i) - \inf F_B(x_i)\right| + \left|\sup F_A(x_i) - \sup F_B(x_i)\right|\Big], \quad (1)$$

(ii) The normalized Hamming distance:

$$d_2(A,B)=\frac{1}{6n}\sum_{i=1}^{n}\Big[\left|\inf T_A(x_i)-\inf T_B(x_i)\right|+\left|\sup T_A(x_i)-\sup T_B(x_i)\right|+\left|\inf I_A(x_i)-\inf I_B(x_i)\right| \\ +\left|\sup I_A(x_i)-\sup I_B(x_i)\right|+\left|\inf F_A(x_i)-\inf F_B(x_i)\right|+\left|\sup F_A(x_i)-\sup F_B(x_i)\right|\Big]. \quad (2)$$

(iii) The Euclidean distance:

$$d_3(A,B)=\left\{\frac{1}{6}\sum_{i=1}^{n}\Big[\left(\inf T_A(x_i)-\inf T_B(x_i)\right)^2+\left(\sup T_A(x_i)-\sup T_B(x_i)\right)^2+\left(\inf I_A(x_i)-\inf I_B(x_i)\right)^2 \\ +\left(\sup I_A(x_i)-\sup I_B(x_i)\right)^2+\left(\inf F_A(x_i)-\inf F_B(x_i)\right)^2+\left(\sup F_A(x_i)-\sup F_B(x_i)\right)^2\Big]\right\}^{1/2}, \quad (3)$$

(IV) The normalized Euclidean distance:

$$d_4(A,B)=\left\{\frac{1}{6n}\sum_{i=1}^{n}\Big[\left(\inf T_A(x_i)-\inf T_B(x_i)\right)^2+\left(\sup T_A(x_i)-\sup T_B(x_i)\right)^2+\left(\inf I_A(x_i)-\inf I_B(x_i)\right)^2 \\ +\left(\sup I_A(x_i)-\sup I_B(x_i)\right)^2+\left(\inf F_A(x_i)-\inf F_B(x_i)\right)^2+\left(\sup F_A(x_i)-\sup F_B(x_i)\right)^2\Big]\right\}^{1/2}. \quad (4)$$

Proposition 1 The above defined distance $d_k(A, B)$ (k = 1, 2, 3,4) between INSs A and B satisfies the following properties (D1-D4):

(D1) $d_k(A, B) \geq 0$;

(D2) $d_k(A, B) = 0$ if and only if $A = B$;

(D3) $d_k(A, B) = d_k(B, A)$;

(D4) If $A \subseteq B \subseteq C$, C is an INS in X, then $d_k(A, C) \geq d_k(A, B)$ and $d_k(A, C) \geq d_k(B, C)$.

Proof It is easy to see that $d_k(A, B)$ (k = 1, 2, 3,4) satisfies the properties (D1)–(D3). Therefore, we only prove (D4). Let $A \subseteq B \subseteq C$, then, $\inf T_A(x_i) \leq \inf T_B(x_i) \leq \inf T_C(x_i)$, $\sup T_A(x_i) \leq \sup T_B(x_i) \leq \sup T_C(x_i)$, $\inf I_A(x_i) \geq \inf I_B(x_i) \geq \inf I_C(x_i)$, $\sup I_A(x_i) \geq \sup I_B(x_i) \geq \sup I_C(x_i)$, $\inf F_A(x_i) \geq \inf F_B(x_i) \geq \inf F_C(x_i)$, and $\sup F_A(x_i) \geq \sup F_B(x_i) \geq \sup F_C(x_i)$ for every $x_i \in X$. For p = 1, 2, we have

$\left|\inf T_A(x_i)-\inf T_B(x_i)\right|^p \leq \left|\inf T_A(x_i)-\inf T_C(x_i)\right|^p$, $\left|\sup T_A(x_i)-\sup T_B(x_i)\right|^p \leq \left|\sup T_A(x_i)-\sup T_C(x_i)\right|^p$,

$\left|\inf T_B(x_i)-\inf T_C(x_i)\right|^p \leq \left|\inf T_A(x_i)-\inf T_C(x_i)\right|^p$, $\left|\sup T_B(x_i)-\sup T_C(x_i)\right|^p \leq \left|\sup T_A(x_i)-\sup T_C(x_i)\right|^p$,

$\left|\inf I_A(x_i)-\inf I_B(x_i)\right|^p \leq \left|\inf I_A(x_i)-\inf I_C(x_i)\right|^p$, $\left|\sup I_A(x_i)-\sup I_B(x_i)\right|^p \leq \left|\sup I_A(x_i)-\sup I_C(x_i)\right|^p$,

$\left|\inf I_B(x_i)-\inf I_C(x_i)\right|^p \leq \left|\inf I_A(x_i)-\inf I_C(x_i)\right|^p$, $\left|\sup I_B(x_i)-\sup I_C(x_i)\right|^p \leq \left|\sup I_A(x_i)-\sup I_C(x_i)\right|^p$,

$\left|\inf F_A(x_i)-\inf F_B(x_i)\right|^p \leq \left|\inf F_A(x_i)-\inf F_C(x_i)\right|^p$, $\left|\sup F_A(x_i)-\sup F_B(x_i)\right|^p \leq \left|\sup F_A(x_i)-\sup F_C(x_i)\right|^p$,

$|\inf F_B(x_i) - \inf F_C(x_i)|^p \le |\inf F_A(x_i) - \inf F_C(x_i)|^p$, $|\sup F_B(x_i) - \sup F_C(x_i)|^p \le |\sup F_A(x_i) - \sup F_C(x_i)|^p$.

Hence

$$\begin{aligned}&|\inf T_A(x_i) - \inf T_B(x_i)|^p + |\sup T_A(x_i) - \sup T_B(x_i)|^p + |\inf I_A(x_i) - \inf I_B(x_i)|^p + |\sup I_A(x_i) - \sup I_B(x_i)|^p \\ &+ |\inf F_A(x_i) - \inf F_B(x_i)|^p + |\sup F_A(x_i) - \sup F_B(x_i)|^p \le |\inf T_A(x_i) - \inf T_C(x_i)|^p + |\sup T_A(x_i) - \sup T_C(x_i)|^p \\ &+ |\inf I_A(x_i) - \inf I_C(x_i)|^p + |\sup I_A(x_i) - \sup I_C(x_i)|^p + |\inf F_A(x_i) - \inf F_C(x_i)|^p + |\sup F_A(x_i) - \sup F_C(x_i)|^p\end{aligned},$$

$$\begin{aligned}&|\inf T_B(x_i) - \inf T_C(x_i)|^p + |\sup T_B(x_i) - \sup T_C(x_i)|^p + |\inf I_B(x_i) - \inf I_C(x_i)|^p + |\sup I_B(x_i) - \sup I_C(x_i)|^p \\ &+ |\inf F_B(x_i) - \inf F_C(x_i)|^p + |\sup F_B(x_i) - \sup F_C(x_i)|^p \le |\inf T_A(x_i) - \inf T_C(x_i)|^p + |\sup T_A(x_i) - \sup T_C(x_i)|^p \\ &+ |\inf I_A(x_i) - \inf I_C(x_i)|^p + |\sup I_A(x_i) - \sup I_C(x_i)|^p + |\inf F_A(x_i) - \inf F_C(x_i)|^p + |\sup F_A(x_i) - \sup F_C(x_i)|^p\end{aligned}.$$

Combining the above inequalities with the above defined distance formulas (1)-(4), we can obtain that

$d_k(A,B) \le d_k(A,C)$ and $d_k(B,C) \le d_k(A,C)$ for k = 1, 2, 3, 4.

Thus the property (D4) is obtained. □

However, the differences of importance are considered in the elements in the universe. Therefore, we need to take the weights of the elements x_i (i = 1, 2,…, n) into account. In the following, we develop some weighted distance measures between INSs.

Let $w = \{w_1, w_2,\ldots, w_n\}$ is the weight vector of the elements x_i (i = 1, 2,…, n), then we have the following the weighted Hamming distance and the weighted Euclidean distance:

$$\begin{aligned}d_5(A,B) = \frac{1}{6}\sum_{i=1}^{n} w_i\big[&|\inf T_A(x_i) - \inf T_B(x_i)| + |\sup T_A(x_i) - \sup T_B(x_i)| + |\inf I_A(x_i) - \inf I_B(x_i)| \\ &+ |\sup I_A(x_i) - \sup I_B(x_i)| + |\inf F_A(x_i) - \inf F_B(x_i)| + |\sup F_A(x_i) - \sup F_B(x_i)|\big]\end{aligned}, \quad (5)$$

$$\begin{aligned}d_6(A,B) = \Big\{\frac{1}{6}\sum_{i=1}^{n} w_i\big[&(\inf T_A(x_i) - \inf T_B(x_i))^2 + (\sup T_A(x_i) - \sup T_B(x_i))^2 + (\inf I_A(x_i) - \inf I_B(x_i))^2 \\ &+ (\sup I_A(x_i) - \sup I_B(x_i))^2 + (\inf F_A(x_i) - \inf F_B(x_i))^2 + (\sup F_A(x_i) - \sup F_B(x_i))^2\big]\Big\}^{1/2}\end{aligned}. \quad (6)$$

If $w = \{1/n, 1/n,\ldots, 1/n\}$, then Eqs. (5) and (6) are reduced to the normalized Hamming distance Eq. (2) and the normalized Euclidean distance Eq. (4), respectively.

It is easy to check that the weighted distance $d_k(A, B)$ (k = 5, 6) between INSs A and B also satisfy the above properties (D1-D4).

It is well known that similarity measures can be generated from distance measures. Therefore, we may use the proposed distance measures to define similarity measures. Based on the relationship of similarity measures and distance measures, we can define some similarity measures between INSs *A* and *B* as follows:

$$S_1(A,B)=1-\frac{1}{6}\sum_{i=1}^{n}w_i\Big[\left|\inf T_A(x_i)-\inf T_B(x_i)\right|+\left|\sup T_A(x_i)-\sup T_B(x_i)\right|+\left|\inf I_A(x_i)-\inf I_B(x_i)\right| \\ +\left|\sup I_A(x_i)-\sup I_B(x_i)\right|+\left|\inf F_A(x_i)-\inf F_B(x_i)\right|+\left|\sup F_A(x_i)-\sup F_B(x_i)\right|\Big] \quad (7)$$

$$S_2(A,B)=1-\left\{\frac{1}{6}\sum_{i=1}^{n}w_i\Big[\left(\inf T_A(x_i)-\inf T_B(x_i)\right)^2+\left(\sup T_A(x_i)-\sup T_B(x_i)\right)^2+\left(\inf I_A(x_i)-\inf I_B(x_i)\right)^2 \\ +\left(\sup I_A(x_i)-\sup I_B(x_i)\right)^2+\left(\inf F_A(x_i)-\inf F_B(x_i)\right)^2+\left(\sup F_A(x_i)-\sup F_B(x_i)\right)^2\Big]\right\}^{1/2} \quad (8)$$

According to the above distance properties (D1-D4), it is easy to check that the similarity measure $S_k(A, B)$ (k = 1, 2) has the following properties (P1-P4):

(P1) $0 \le S_k(A, B) \le 1$;

(P2) $S_k(A, B) = 1$ if and only if $A = B$;

(P3) $S_k(A, B) = S_k(B, A)$;

(P4) If $A \subseteq B \subseteq C$, C is an INS in X, then $S_k(A, C) \le S_k(A, B)$ and $S_k(A, C) \le S_k(B, C)$.

It is clear that the larger the value of $S_k(A, B)$ (k = 1, 2), the more the similarity between INSs *A* and *B*.

4. Decision-making method based on the similarity measures

In this section, we present a handling method for the multicriteria decision-making problem in interval neutrosophic setting by means of the similarity measures between INSs.

Let $A = \{A_1, A_2,\ldots, A_m\}$ be a set of alternatives and let $C = \{C_1, C_2,\ldots, C_n\}$ be a set of criteria. Assume that the weight of the criterion C_j (j = 1, 2,…, n), entered by the decision-maker, is w_j, $w_j \in$

[0, 1] and $\sum_{j=1}^{n} x_j = 1$. In this case, the characteristic of the alternative A_i (i = 1, 2,…, m) is represented by the following INS:

$$\begin{aligned} A_i &= \{\langle C_j, T_{A_i}(C_j), I_{A_i}(C_j), F_{A_i}(C_j)\rangle \mid C_j \in C\} \\ &= \{\langle C_j, [\inf T_{A_i}(C_j), \sup T_{A_i}(C_j)], [\inf I_{A_i}(C_j), \sup I_{A_i}(C_j)], [\inf F_{A_i}(C_j), \sup F_{A_i}(C_j)]\rangle \mid C_j \in C\}, \end{aligned}$$

where

$T_{A_i}(C_j) = [\inf T_{A_i}(C_j), \sup T_{A_i}(C_j)]$, $I_{A_i}(C_j) = [\inf I_{A_i}(C_j), \sup I_{A_i}(C_j)]$, $F_{A_i}(C_j) = [\inf F_{A_i}(C_j), \sup F_{A_i}(C_j)] \subseteq$ [0, 1], $0 \le \sup T_{A_i}(C_j) + \sup I_{A_i}(C_j) + \sup F_{A_i}(C_j) \le 3$, j = 1, 2, …, n, and i = 1, 2, …, m. An INS, which is the pair of intervals $T_{A_i}(C_j) = [\inf T_{A_i}(C_j), \sup T_{A_i}(C_j)]$, $I_{A_i}(C_j) = [\inf I_{A_i}(C_j), \sup I_{A_i}(C_j)]$, $F_{A_i}(C_j) = [\inf F_{A_i}(C_j), \sup F_{A_i}(C_j)]$ for $C_j \in C$, is denoted by α_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}], [e_{ij}, f_{ij}]) for convenience. Here, an INS is usually derived from the evaluation of an alternative A_i with respect to a criterion C_j by means of a score law and data processing in practice. Therefore, we can elicit a interval neutrosophic decision matrix $D = (\alpha_{ij})_{m\times n}$.

In multicriteria decision making environments, the concept of ideal point has been used to help identify the best alternative in the decision set. Although the ideal alternative does not exist in real world, it does provide a useful theoretical construct against which to evaluate alternatives.

Generally, the evaluation criteria can be categorized into two kinds, benefit criteria and cost criteria. Let H be a collection of benefit criteria and L be a collection of cost criteria. Then we define an ideal INS for a benefit criterion in the ideal alternative A^* as $\alpha_j^* = ([a_j^*, b_j^*], [c_j^*, d_j^*], [e_j^*, f_j^*]) = ([1,1], [0,0], [0,0])$ for $j \in H$; while for a cost criterion, we define an ideal INS in the ideal alternative A^* as $\alpha_j^* = ([a_j^*, b_j^*], [c_j^*, d_j^*], [e_j^*, f_j^*]) = ([0,0], [1,1], [1,1])$ for $j \in L$.

Thus, by applying Eqs. (7) and (8) two similarity measures between an alternative A_i and the ideal alternative A^* represented by the INSs are defined as follows:

$$S_1(A^*, A_i) = 1 - \frac{1}{6}\sum_{j=1}^{n} w_j \left[\left|a_j^* - a_{ij}\right| + \left|b_j^* - b_{ij}\right| + \left|c_j^* - c_{ij}\right| + \left|d_j^* - d_{ij}\right| + \left|e_j^* - e_{ij}\right| + \left|f_j^* - f_{ij}\right| \right], \tag{9}$$

$$S_2(A^*, A_i) = 1 - \left\{ \frac{1}{6}\sum_{j=1}^{n} w_j \left[\left(a_j^* - a_{ij}\right)^2 + \left(b_j^* - b_{ij}\right)^2 + \left(c_j^* - c_{ij}\right)^2 + \left(d_j^* - d_{ij}\right)^2 + \left(e_j^* - e_{ij}\right)^2 + \left(f_j^* - f_{ij}\right)^2 \right] \right\}^{1/2}. \tag{10}$$

Through the similarity measure $S_1(A^*, A_i)$ or $S_2(A^*, A_i)$ (i = 1, 2,…, m) between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best one can be easily identified as well.

5. Illustrative example

In this section, an example for the multicriteria decision-making problem of alternatives is used as the demonstration of the application of the proposed decision-making method, as well as the effectiveness of the proposed method.

Let us consider the decision-making problem adapted from (Ye 2009). There is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money: (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; (4) A_4 is an arms company. The investment company must take a decision according to the following three criteria: (1) C_1 is the risk analysis; (2) C_2 is the growth analysis; (3) C_3 is the environmental impact analysis, where C_1 and C_2 are benefit criteria, and C_3 is a cost criterion. The weight vector of the criteria is given by w = (0.35, 0.25, 0.40). The four possible alternatives are to be evaluated under the above three criteria by corresponding to the INSs, as shown in the following interval neutrosophic decision matrix D:

$$D=\begin{bmatrix} ([0.4,0.5],[0.2,0.3],[0.3,0.4]) & ([0.4,0.6],[0.1,0.3],[0.2,0.4]) & ([0.7,0.9],[0.2,0.3],[0.4,0.5]) \\ ([0.6,0.7],[0.1,0.2],[0.2,0.3]) & ([0.6,0.7],[0.1,0.2],[0.2,0.3]) & ([0.3,0.6],[0.3,0.5],[0.8,0.9]) \\ ([0.3,0.6],[0.2,0.3][0.3,0.4]) & ([0.5,0.6],[0.2,0.3],[0.3,0.4]) & ([0.4,0.5],[0.2,0.4],[0.7,0.9]) \\ ([0.7,0.8],[0.0,0.1],[0.1,0.2]) & ([0.6,0.7],[0.1,0.2],[0.1,0.3]) & ([0.6,0.7],[0.3,0.4],[0.8,0.9]) \end{bmatrix}.$$

Then, we utilize the developed approach to obtain the most desirable alternative(s).

By using Eq. (9) we can obtain the following similarity measures of $S_1^*(A^*, A_i)$ (i =1, 2, 3, 4):

$S_1^*(A^*, A_1) = 0.5025$, $S_1^*(A^*, A_2) = 0.6900$, $S_1^*(A^*, A_3) = 0.5983$, and $S_1^*(A^*, A_4) = 0.6958$.

Therefore, the ranking order of the four alternatives is A_4, A_2, A_3, and A_1. Obviously, amongst them A_4 is the best alternative.

Or by applying Eq. (10) we can give the similarity measures of $S_2^*(A^*, A_i)$ (i =1, 2, 3, 4) as follows:

$S_2^*(A^*, A_1) = 0.4572$, $S_2^*(A^*, A_2) = 0.6455$, $S_2^*(A^*, A_3) = 0.5599$, and $S_2^*(A^*, A_4) = 0.6200$.

Thus, the ranking order of the four alternatives is A_2, A_4, A_3, and A_1, obviously, amongst them A_2 is the best alternative.

6. Conclusion

In this paper, we defined the Hamming and Euclidean distances and proposed the similarity measures between INSs based on the relationship between similarity measures and distances. Then a multicriteria decision-making method has been established in interval neutrosophic setting by means of the similarity measure between each alternative and the ideal alternative. Through the similarity measures, the ranking order of all alternatives can be determined and the best alternative can be easily identified as well. Finally, an illustrative example illustrated the application of the developed approach. The proposed similarity measures between INSs are more suitable for real scientific and engineering applications. Then the techniques proposed in this paper extend existing decision-making methods and can provide a useful way for decision-makers. In the future, we shall continue working in the application of the similarity measures between INSs to other domains.

References

Atanassov K, Gargov G (1989) Interval valued intuitionistic fuzzy sets. Fuzzy Sets and Systems 31: 343-349

Atanassov K (1986) Intuitionistic fuzzy sets. Fuzzy Sets and Systems 20: 87–96

Cheng HD, Guo Y (2008) A new neutrosophic approach to image thresholding. New Mathematics and Natural Computation 4 (3): 291–308

Guo Y, Cheng HD (2009) new neutrosophic approach to image segmentation. Pattern Recognition 42: 587-595

Smarandache F (1999) A unifying field in logics. neutrosophy*:* Neutrosophic probability, set and logic, American Research Press, Rehoboth

Smarandache F (2005) Neutrosophic set. A generalization of the intuitionistic fuzzy set. International Journal of Pure and Applied Mathematics 24: 287-297

Turksen I (1986) Interval valued fuzzy sets based on normal forms. Fuzzy Sets and Systems 20: 191–210

Wang H, Smarandache F, Zhang YQ, Sunderraman R (2005) Interval neutrosophic sets and Logic: Theory and Applications in Computing, Hexis, Phoenix, AZ

Ye J (2009) Multicriteria fuzzy decision-making method based on a novel accuracy function under interval-valued intuitionistic fuzzy environment. Expert Systems With Applications 36: 6899-6902

Zadeh LA (1965) Fuzzy Sets. Information and Control 8: 338-353

Figure(1)

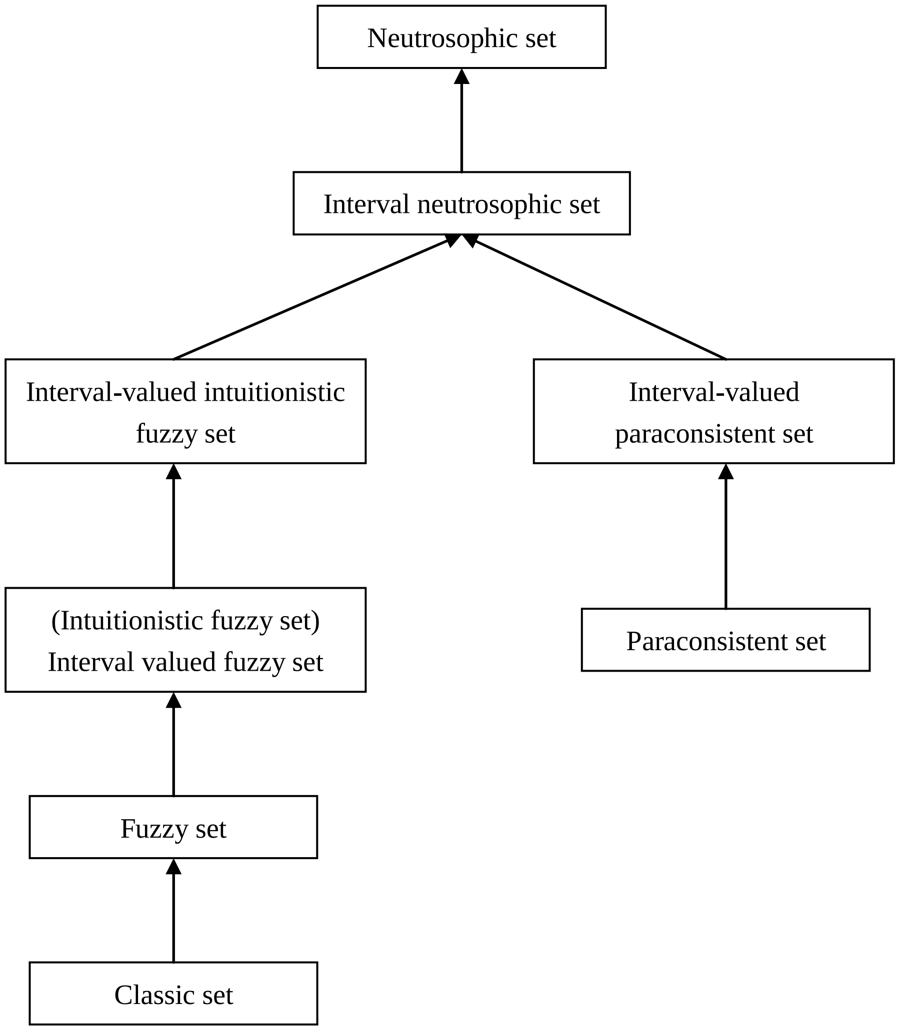

Fig. 1. Relationship of neutrosophic set and other sets

Figure(2)