Single Valued Neutrosophic Graphs: Degree, Order and Size

Said Broumi [1] Mohamed Talea[2] Florentin Smarandache[3]

[1,2] Laboratory of Information processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco.

[3]Department of Mathematics, University of New Mexico,705 Gurley Avenue, Gallup, NM 87301, USA.

broumisaid78@gmail.com, taleamohamed@yahoo.fr, fsmarandache@gmail.com

Abstract *:* The single valued neutrosophic graph is a new version of graph theory presented recently as a generalization of fuzzy graph and intuitionistic fuzzy graph. The single valued neutrosophic graph (SVN-graph) is used when the relation between nodes (or vertices) in problems are indeterminate. In this paper, we examine the properties of various types of degrees, order and size of single valued neutrosophic graphs and a new definition for complete single valued neutrosophic graph and regular single valued neutrosophic graph is given.

Keywords: Single valued neutrosophic graph, effective degree, neighbourhood degree, Order, Size

1. Introduction

Neutrosophic set proposed by Smarandache [12, 13] is a powerful tool to deal with incomplete, indeterminate and inconsistent information in real world. It is a generalization of the theory of fuzzy set [17], intuitionistic fuzzy sets [22], interval-valued fuzzy sets [19] and interval-valued intuitionistic fuzzy sets [23], then the neutrosophic set is characterized by a truth-membership degree (t), an indeterminacy-membership degree (i) and a falsity-membership degree (f) independently, which are within the real standard or nonstandard unit interval $]^-0, 1^+[$. Therefore, if their range is restrained within the real standard unit interval [0, 1], the neutrosophic set is easily applied to engineering problems. For this purpose, Wang et al. [15] introduced the concept of a single valued neutrosophic set (SVNS) as a subclass of the neutrosophic set. The same authors introduced the notion of interval valued neutrosophic sets [16] as subclass of neutrosophic sets in which the value of truth-membership, indeterminacy-membership and falsity-membership degrees are intervals of numbers instead of the real numbers. neutrosophic sets and its extensions such as single valued neutrosophic sets, interval neutrosophic sets, simplified neutrosophic sets and so on have been applied in a wide variety of fields including computer science, enginnering, mathematics, medicine and economic [1, 2, 3, 7, 8, 11, 12, 13, 14, 18, 20, 28, 29, 30].

Lots of works on fuzzy graphs and intuitionistic fuzzy graphs [4, 5, 6, 25, 26] have been carried out and all of them have considered the vertex sets and edge sets as fuzzy and /or intuitionistic fuzzy sets. But, when the relations between nodes(or vertices) in problems are indeterminate, the fuzzy graphs and intuitionistic fuzzy graphs are failed. For this purpose, Smarandache [10] have defined four main categories of neutrosophic graphs, two based on literal indeterminacy (I), which called them; I-edge neutrosophic graph and I-vertex neutrosophic graph, these concepts are studied deeply and has gained popularity among the researchers due to its applications via real world problems [31, 32]. The two others graphs are based on (t, i, f) components and called them; The (t, i, f)-edge neutrosophic graph and the (t, i, f)-vertex neutrosophic graph, these concepts are not developed at all. Later on, Broumi et al. [27] introduced a third neutrosophic graph model combined the (t, i, f)-edge and and the (t, i, f)-vertex neutrosophic graph and investigated some of their properties. The third neutrosophic graph model is called 'single valued neutrosophic graph' (SVNG for short). The single valued neutrosophic graph is the generalization of fuzzy graph and intuitionistic fuzzy graph. This

paper introduces a degree of a vertex, an effective degree, a neighborhood degree of a vertex and a closed neighborhood degree of a vertex in single valued neutrosophic graph. In addition, this paper introduces a regular single valued neutrosophic graph. Finally, this paper investigates some of their results.

2. Preliminaires

We present some known definitions and results for ready reference to go through the work presented in this paper.

Definition 2.1 [5]: An intuitionistic fuzzy graph (IFG) is of the form G= (V, E) where

1.V= $\{v_1, v_2 ,..., v_n\}$ such that μ_1:V→[0, 1] and γ_1:V→[0, 1] denotes the degree of membership and non-membership of the element v_i ∈ V, respectively, and

$$0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1 \text{ for every } v_i \in V \text{ (i=1, 2, …,n)} \tag{1}$$

2. E ⊆ V x V where μ_2:V x V →[0, 1] and γ_2:V x V →[0, 1] are such that

$$\mu_2(v_i, v_j) \le \min [\mu_1(v_i), \mu_1(v_j)] \tag{2}$$
$$\gamma_2(v_i, v_j) \ge \max[\gamma_1(v_i), \gamma_1(v_j)] \tag{3}$$
$$0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1 \quad \text{for every } (v_i, v_j) \in E \text{ (i, j= 1, 2, …,n)} \tag{4}$$

Definition 2.2 [27] A single valued neutrosophic graph (SVN-graph) is of the form G= (V, E) where

1.V= $\{v_1 , v_2 ,... , v_n\}$ such that T_1:V→[0, 1], I_1:V→[0, 1] and F_1:V→[0, 1] denotes the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element v_i ∈ V, respectively, and

$$0 \le T_1(v_i) + I_1(v_i) + F_1(v_i) \le 3 \qquad \text{for every } v_i \in V \text{ (i=1, 2, …,n)} \tag{5}$$

2. E ⊆ V x V where T_2:V x V →[0, 1], I_2:V x V →[0, 1] and F_2:V x V →[0, 1] are such that $T_2(v_i, v_j) \le \min [T_1(v_i), T_1(v_j)]$, $I_2(v_i, v_j) \ge \max[I_1(v_i), I_1(v_j)]$ and $F_2(v_i, v_j) \ge \max[F_1(v_i), F_1(v_j)]$ and

$$0 \le T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \le 3 \quad \text{for every } (v_i, v_j) \in E \text{ (i, j= 1, 2, …,n)} \tag{6}$$

Definition 2.3 [27]: A SVN-graph H = (V', E') is said to be a single valued neutrosophic subgraph (SVNSG) of the SVNG G= (V, E) if $V' \subseteq V$ and $E' \subseteq E$. In other words, if $T'_{1i} \le T_{1i}, I'_{1i} \ge I_{1i},   F'_{1i} \ge F_{1i}$ for every i, j = 1, 2.........n. and $T'_{2ij} \le T_{2ij}, I'_{2ij} \ge I_{2ij},   F'_{2ij} \ge F_{2ij}$ for every i, j = 1, 2.........n $\tag{7}$

Definition 2.4 [27]: A single valued neutrosophic graph is complete if $T_{2ij}= \min (T_{1i}, T_{1j})$ and $I_{2ij}= \max (I_{1i}, I_{1j})$ and $F_{2ij}= \max (F_{1i}, F_{1j})$ for all $(v_i , v_j) \in E$ $\tag{8}$

The example of a single valued neutrosophic and complete single valued neutrosophic graph with four vertices is given in the Figure 1 and Figure 2.
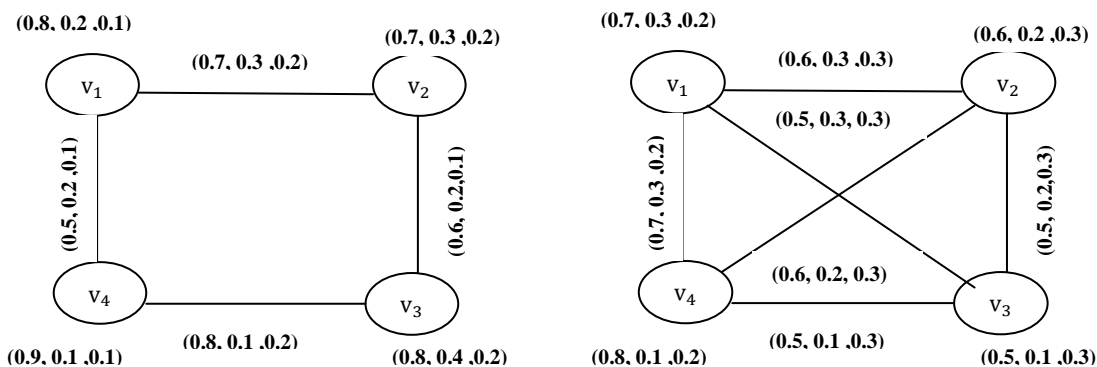
3. Vertex Degree

Degree of a vertex of a single valued neutrosophic is defined below

Definition 3.1: Let G = (V, E) be a SVN-graph. The ordinary degree(simply *degree*) of a vertex v in G, denoted by $d(v)$ is defined as $d(v) = (d_T(v), d_I(v), d_F(v))$, where

$d_T(v) = \sum_{u \neq v} T_2(u, v)$ denotes the T- degree of a vertex v.

$d_I(v) = \sum_{u \neq v} I_2(u, v)$ denotes the I- degree of a vertex v.

$d_F(v) = \sum_{u \neq v} F_2(u, v)$ denotes the F- degree of a vertex v.

Definition 3.2: The minimum degree of G is $\delta(G) = (\delta_T(G),\ \delta_I(G), \delta_F(G))$, where

$\delta_T = \wedge \{d_T(v) \mid v \in V\}$ denotes the minimum T- degree.

$\delta_I = \wedge \{d_I(v) \mid v \in V\}$ denotes the minimum I- degree.

$\delta_F = \wedge \{d_F(v) \mid v \in V\}$ denotes the minimum F- degree.

Definition 3.3: The maximum degree of G is $\Delta(G) = (\Delta_T(G),\ \Delta_I(G), \Delta_F(G))$,where

$\Delta_T = \vee \{d_T(v) \mid v \in V\}$ denotes the maximum T- degree.

$\Delta_I = \vee \{d_I(v) \mid v \in V\}$ denotes the maximum I- degree.

$\Delta_F = \vee \{d_F(v) \mid v \in V\}$ denotes the maximum F- degree.

Example 3.4: Consider a SVN-graph G= (V, E), such that V={v_1, v_2, v_3, v_4}and E={(v_1, v_2), (v_2, v_3), (v_3, v_4), (v_4, v_1)}

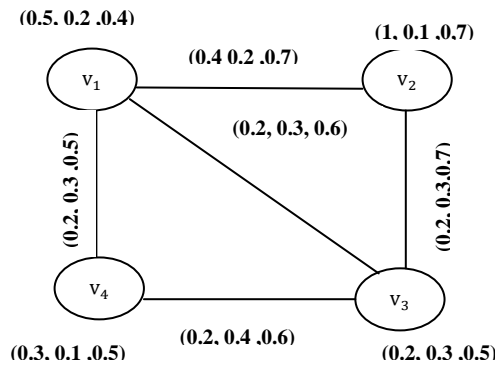**Fig. 3.degree vertex of single valued neutrosophic graph**

By usual computation, we have the degrees for all vertices:

$d(v_1)$=(0.8, 0.8, 1.8) $d(v_2)$=(0.6, 0.5, 1.4)

$d(v_3)$=(0.6, 1, 1.9) $d(v_4)$=(0.4, 0.7, 1.1)

And the minimum degree, the maximum degree of G are

$\delta(G) = (0.4 , 0.5, 1.1)$ $\Delta(G) = (0.8, 1, 1.9)$

Proposition 3.5: In any single valued neutrosophic graph G=(V, E), the sum of the degree of   truth-membership value of all vertices is equal to twice the sum of the truth-membership value of all edges, the sum of the degree of indeterminacy-membership value of all vertices is equal to twice the sum of the indeterminacy-membership value of all edges and the sum of the degree of falsity-membership value of all vertices is equal to twice the sum of the falsity-membership value of all edges.

$\sum \mathbf{d(v_i)} = [\sum \mathbf{d_T(v_i)}, \sum \mathbf{d_I(v_i)}, \sum \mathbf{d}_F(\mathbf{v_i})] = [\ 2\sum_{u \neq v} T_2\ (u,v),\ 2\sum_{u \neq v} I_2\ (u,v),\ 2\sum_{u \neq v} F_2\ (u,v)]\ (9)$
proof:

Let G = (V, E) be a SVN-graph where V={$\mathbf{v_1, v_2,}$…….. $\mathbf{v_n}$}
$\sum \mathbf{d(v_i)} = [\sum \mathbf{d_T(v_i)}, \sum \mathbf{d_I(v_i)}, \sum \mathbf{d}_F(\mathbf{v_i})] = [(d_T(\mathbf{v_1}), d_I(\mathbf{v_1}), d_F(\mathbf{v_1})) + (d_T(\mathbf{v_2}), d_I(\mathbf{v_2}),$
$d_F(\mathbf{v_2})) + …. + (d_T(\mathbf{v_n}),\ d_I(\mathbf{v_n}),\ d_F(\mathbf{v_n}))]$

$= [(T_2(\mathbf{v_1, v_2}), I_2(\mathbf{v_1, v_2}), F_2(\mathbf{v_1, v_2})) + (T_2(\mathbf{v_1, v_3}), I_2(\mathbf{v_1, v_3}), F_2(\mathbf{v_1, v_3}))$
$+ …… + (T_2(\mathbf{v_1, v_n}), I_2(\mathbf{v_1, v_n}), F_2(\mathbf{v_1, v_n})) + (T_2(\mathbf{v_2, v_1}), I_2(\mathbf{v_2, v_1}), F_2(\mathbf{v_2, v_1})) +$
$(T_2(\mathbf{v_2, v_3}), I_2(\mathbf{v_2, v_3}), F_2(\mathbf{v_2, v_3})) + …… + (T_2(\mathbf{v_2, v_n}), I_2(\mathbf{v_2, v_n}), F_2(\mathbf{v_2, v_n})) +$
………………………………………………………………………………+
$(T_2(\mathbf{v_n, v_1}), I_2(\mathbf{v_n, v_1}), F_2(\mathbf{v_n, v_1})) + (T_2(\mathbf{v_n, v_2}), I_2(\mathbf{v_n, v_2}), F_2(\mathbf{v_n, v_2}))$
$+ …… + (T_2(\mathbf{v_{n-1}, v_n}), I_2(\mathbf{v_{n-1}, v_n}), F_2(\mathbf{v_{n-1}, v_n}))\ ]$
$= 2[(T_2(\mathbf{v_1, v_2}), I_2(\mathbf{v_1, v_2}), F_2(\mathbf{v_1, v_2})) + (T_2(\mathbf{v_1, v_3}), I_2(\mathbf{v_1, v_3}), F_2(\mathbf{v_1, v_3}))$
$+ …… + (T_2(\mathbf{v_1, v_n}), I_2(\mathbf{v_1, v_n}), F_2(\mathbf{v_1, v_n}))]$
$= [\ 2\sum_{u \neq v} T_2\ (u,v),\ 2\sum_{u \neq v} I_2\ (u,v),\ 2\sum_{u \neq v} F_2\ (u,v)]$. Hence the proof.

Proposition 3.6: The maximum degree of any vertex in a SVN-graph with n vertices is n-1.
Proof:
Let G = (V, E) be a SVN-graph. The maximum truth-membership value given to an edge is 1 and the number of edges incident on a vertex can be at most n-1. Hence the maximum truth- membership degree $\mathbf{d_T(v_i)}$ of any vertex $\mathbf{v_i}$ in a SVN-graph with n vertices is n-1.
Similarly, the maximum indeterminacy –membership value given to an edge is 1 and the number of edges incident on a vertex can be at most n-1. Hence the maximum indeterminacy- membership degree $\mathbf{d_I(v_i)}$
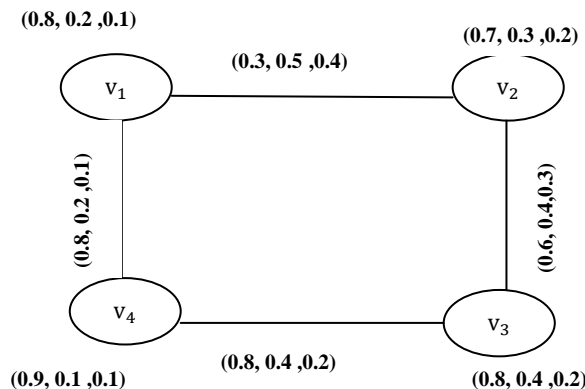Also, the maximum falsity-membership value given to an edge is 1 and the number of edges incident on a vertex can be at most n-1. Hence the maximum falsity-membership degree $\mathbf{d}_F(\mathbf{v_i})$ of any vertex $\mathbf{v_i}$ in a SVN-graph with n vertices is n-1. Hence the result.
4.Effective Degree
Definition 4.1: An edge e = (v, w) of a SVN-graph G = (V, E) is called an effective edge if T_2(v, w)= T_1(v) $\wedge$ T_1(w), I_2(v, w)= I_1(v) $\vee$ I_1(w) and F_2(v, w)= F_1(v) $\vee F_1$(w) for all (v, w) $\in$ E. In this case, the vertex v is called a neighbor of w and conversely.
N(v) ={ w $\in$ V : w is a neighbor of v} is called the neighborhood of v.

Example 4.2. Consider a SVN-graph G= (V, E), such that V={$\mathbf{v_1, v_2, v_3, v_4}$}and E={($\mathbf{v_1, v_2}$), ($\mathbf{v_2, v_3}$), ($\mathbf{v_3, v_4}$), ($\mathbf{v_4, v_1}$)}

(0.8, 0.2 ,0.1)   **(0.7, 0.3 ,0.2)**
(0.3, 0.5 ,0.4)
v_1 v_2
(0.8, 0.2 ,0.1)   **(0.6, 0.4,0.3)**
v_4 v_3
(0.8, 0.4 ,0.2)
(0.9, 0.1 ,0.1)   **(0.8, 0.4 ,0.2)**

Fig. 4. Single valued neutrosophic graph

In this example, v_4v_1 and v_4v_3 are effective edges. Also $N(v_4) =\{ v_1, v_3\}$, $N(v_3)=\{ v_4\}$, $N(v_1) =\{ v_4\}$, $N(v_2) = \emptyset$ (the empty set).

Definition 4.3: The effective degree of a vertex 'v' in G is defined by $d_E(v) = (d_{ET}(v), d_{EI}(v), d_{EF}(v))$ where $d_{ET}(v)$ is the sum of the truth-membership values of the effective edges incident with v, $d_{EI}(v)$ is the sum of the indeterminacy-membership values of the effective edges incident with v and $d_{EF}(v)$ is the sum of the falsity-membership values of effective edges incident with v.

Definition 4.4: The minimum effective degree of G is $\delta_E[G]= (\delta_{ET}[G], \delta_{EI}[G], \delta_{EF}[G])$ where
$\delta_{ET}[G]= \wedge\{d_{ET}[v] \,|v \in V\}$ denotes the minimum effective T- degree.
$\delta_{EI}[G]= \wedge\{d_{EI}[v] \,| \, v \in V\}$ denotes the minimum effective I- degree.
$\delta_{EF}[G]= \wedge\{d_{EF}[v] \,| \, v \in V\}$ denotes the minimum effective F- degree.

Definition 4.5: The maximum effective degree of G is $\Delta_E[G]= (\Delta_{ET}[G], \Delta_{EI}[G], \Delta_{EF}[G])$ where
$\Delta_{ET}[G]= \vee\{d_{ET}[v] \,|v \in V\}$ denotes the maximum effective T- degree.
$\Delta_{EI}[G]= \vee\{d_{EI}[v] \,| \, v \in V \}$ denotes the maximum effective I- degree.
$\Delta_{EF}[G]= \vee\{d_{EF}[v] \,| \, v \in V\}$ denotes the maximum effective F- degree.

Example 4.6: Consider a SVN-graph as in Figure 3. By usual computation, we have the effective degrees for all vertices
$d_E(v_1)=(0, 0, 0)$ $\qquad$ $d_E(v_2)=(0.2, 0.3, 0.7)$
$d_E(v_3)=(0.2, 0.3, 0.7)$ $\quad$ $d_E(v_4)=(0, 0, 0)$
$\delta_E(G) = (0, 0, 0)$ $\qquad$ $\Delta_E(G) = (0.2, 0.3, 0.7)$
Here v_2v_3 is only effective degree.

Note: $d_E(v_1)=(0, 0, 0)$ means that there is no effective edge incident on v_1 .

Now, we can defined the neighborhood concept in SVN-graph as analogue of intuitionistic fuzzy graph.

5. Neighbourhood Degree

Definition 5.1: Let G $=(V, E)$ be a SVN-graph. The neighbourhood of any vertex v is defined as $N(v)= (N_T(v), N_I(v), N_F(v))$ where
$N_T(v)=\{T_2(v, w)= T_1(v) \wedge T_1(w) \,; w \in V \}$ denotes the neighbourhood T- vertex.
$N_I(v)= \{I_2(v, w)= I_1(v) \vee I_1(w); w \in V\}$ denotes the neighbourhood I- vertex.
$N_F(v)= \{F_2(v, w)= F_1(v) \vee F_1(w); w \in V \}$ denotes the neighbourhood F- vertex.
and $N[v]= N(v) \cup \{v\}$ is called the closed neighbourhood of v.

Definition 5.2: Let G=(V, E) be a single valued neutrosophic graph (SVN-graph). The neighbourhood degree of a vertex 'v' is defined as the sum of truth-membership, indeterminacy- membership and falsity-membership value of the neighbourhood vertices of v and is denoted by
$d_N(v) = (d_{NT}(v), d_{NI}(v), d_{NF}(v))$ where
$d_{NT}(v) = \sum_{w\in N(v)} T_1(w)$ denotes the neighbourhood T- degree.
$d_{NI}(v) = \sum_{w\in N(v)} I_1(w)$ denotes the neighbourhood I- degree.
$d_{NF}(v) = \sum_{w\in N(v)} F_1(w)$ denotes neighbourhood F- degree.

Definition 5.3: The minimum neighbourhood degree is defined as
$\delta_N(G) = (\delta_{NT}(G), \delta_{NI}(G), \delta_{NF}(G))$ where
$\delta_{NT}(G) = \wedge\{d_{NT}(v) \,|v \in V\}$ denotes the minimum neighbourhood T- degree.
$\delta_{NI}(G) = \wedge\{d_{NI}(v) \,| v \in V \}$ denotes the minimum neighbourhood I- degree.
$\delta_{NF}(G) = \wedge\{d_{NF}(v) \,| v \in V\}$ denotes the minimum neighbourhood F- degree.

Definition 5.4: The maximum neighbourhood degree is defined as
$\Delta_N(G) = (\Delta_{NT}(G), \Delta_{NI}(G), \Delta_{NF}(G))$ where
$\Delta_{NT}(G) = \vee\{d_{NT}(v) \,|v \in V\}$ denotes the maximum neighbourhood T- degree.

$\Delta_{NI}(G) = V\{d_{NI}(v) \mid v \in V \}$ denotes the maximum neighbourhood I- degree.
$\Delta_{NF}(G) = V\{d_{NF}(v) \mid v \in V\}$ denotes the maximum neighbourhood F- degree.

Example 5.5: Consider a SVN-graph as in Figure 2. By usual computation, we have the neighbourhood degrees for all vertices, minimum and maximum neighbourhood degrees

$$d_N(v_1) = (1.9,\ 0.4\ ,0.8) \quad d_N(v_2) = (2,\ 0.5\ ,0.7)$$
$$d_N(v_3) = (2.1, 0.6\ ,0.7) \quad d_N(v_4) = (1.8,\ 0.6\ ,0.8)$$
$$\delta_N(G) = (1.8,\ 0.4, 0.7) \quad \Delta_N(G) = (2.1, 0.6\ ,0.8)$$

Definition 5.6: A vertex v $\in$ V of SVN-graph G= (V, E) is said to be an isolated vertex if $T_2(v_i, v_j) = I_2(v_i, v_j) = F_2(v_i, v_j) = 0$ For all v $\in$ V, $v_i \neq v_j$ that is N(v) = $\emptyset$ (the empty set).

Definition 5.7: Let G= (V, E) be a single valued neutrosophic graph (SVN-graph). The closed neighbourhood degree of a vertex 'v' is defined as the sum of truth-membership, indeterminacy- membership and falsity-membership value of the neighbourhood vertices of v and including truth-membership, indeterminacy- membership and falsity-membership value of v, and is denoted by $d_N[v] = (d_{NT}[v], d_{NI}[v], d_{NF}[v])$ where

$d_{NT}[v] = \sum_{w \in N(v)} T_1(w) + T_1(v)$ denotes the closed neighbourhood T- degree.

$d_{NI}[v] = \sum_{w \in N(v)} I_1(w) + I_1(v)$ denotes the closed neighbourhood I- degree.

$d_{NF}[v] = \sum_{w \in N(v)} F_1(w) + F_1(v)$ denotes the closed neighbourhood F- degree.

Definition 5.8: The minimum closed neighbourhood degree is defined as

$\delta_N[G] = (\delta_{NT}[G], \delta_{NI}[G], \delta_{NF}[G])$ where

$\delta_{NT}[G] = \Lambda\{d_{NT}[v] \mid v \in V\}$ denotes the minimum closed neighbourhood T- degree

$\delta_{NI}[G] = \Lambda\{d_{NI}[v] \mid v \in V \}$ denotes the minimum closed neighbourhood I- degree

$\delta_{NF}[G] = \Lambda\{d_{NF}[v] \mid v \in V\}$ denotes the minimum closed neighbourhood F- degree

Definition 5.9: The maximum closed neighbourhood degree is defined as

$\Delta_N[G] = (\Delta_{NT}[G], \Delta_{NI}[G], \Delta_{NF}[G])$ where

$\Delta_{NT}[G] = V\{d_{NT}[v] \mid v \in V\}$ denotes the maximum closed neighbourhood T- degree

$\Delta_{NI}[G] = V\{d_{NI}[v] \mid v \in V \}$ denotes the maximum closed neighbourhood I- degree

$\Delta_{NF}[G] = V\{d_{NF}[v] \mid v \in V\}$ denotes the maximum closed neighbourhood F- degree

6. Regular single valued neutrosophic graph.

Definition 6.1: A single valued neutrosophic graph G = (V, E) is said to be regular single valued neutrosophic graph (RSVN-graph), if all the vertices have the same closed neighbourhood degree. (i.e) if $\delta_{NT}[G] = \Delta_{NT}[G]$, $\delta_{NI}[G] = \Delta_{NI}[G]$ and $\delta_{NF}[G] = \Delta_{NF}[G]$

Example 6.2: Consider a SVN-graph as in Figure 2. By usual computation, we have the closed neighbourhood degrees for all vertices, minimum and maximum neighbourhood degrees

$d_N[v_1] = d_N[v_2] = d_N[v_3] = d_N[v_4] = (2.6, 0.7, 1)$

$\delta_N[G] = \Delta_N[G] = (2.6, 0.7, 1)$

It is clear from calculation that G is regular single valued neutrosophic graph (RSVN-graph).

Theorem 6.3: Every complete single valued neutrosophic is a regular single valued neutrosophic graph

Proof:

Let G = (V, E) be a complete SVN-graph then by definition of complete SVN-graph we have

$T_2(v, w) = T_1(v) \wedge T_1(w), \quad I_2(v, w) = I_1(v) \vee I_1(w)$ and $F_2(v, w) = F_1(v) \vee F_1(w)$ for every v, w $\in$ V.

By definition, the closed neighbourhood **T**-degree of each vertex is the sum of the truth-membership values of the vertices and itself, the closed neighbourhood **I**-degree of each vertex is the sum of the indeterminacy- membership values of the vertices and itself and the closed neighbourhood **T**-degree of each vertex is the sum of the falsity-membership values of the vertices and itself, Therefore all the vertices will have the same closed neighbourhood **T**- degree, closed neighbourhood **I**-degree and closed neighbourhood **F** -degree. This implies minimum closed neighbourhood degree is equal to maximum closed neighbourhood degree (i.e) $\delta_{NT}[G]=\Delta_{NT}[G]$, $\delta_{NI}[G]=\Delta_{NI}[G]$ and $\delta_{NF}[G]=\Delta_{NF}[G]$. This implies G is a regular single valued neutrosophic graph. Hence the theorem.

7.Order and size of single valued neutrosophic graph
In this section we introduce the definition of order and size of a single valued neutrosophic graph which are an important terms in single valued neutrosophic graph theory.

Definition 7.1: Let G =(V, E) be a SVN-graph. The order of G, denoted O(G) is defined as $O(G)=(O_T(G), O_I(G), O_F(G))$, where
$O_T(G)=\sum_{v\in V} T_1(v)$ denotes the T- order of G.
$O_I(G)=\sum_{v\in V} I_1(v)$ denotes the I- order of G.
$O_F(G)=\sum_{v\in V} F_1(v)$ denotes the F- order of G.
i.e. the order of G means also the number of vertices (or the cardinality of V).

Definition 7.2: Let G =(V, E) be a SVN-graph. The size of G, denoted S(G) is defined as $S(G) = (S_T(G), S_I(G), S_F(G))$, where
$S_T(G)=\sum_{u\neq v} T_2(u,v)$ denotes the T- size of G.
$S_I(G)=\sum_{u\neq v} I_2(u,v)$ denotes the I- size of G
$S_F(G)=\sum_{u\neq v} F_2(u,v)$ denotes the F- size of G
i.e. The size of G means also the number of edges (or the cardinality of E).

Example 7.3: Consider a SVN-graph as in Figure 3. O (G) = (2, 0.7, 2.1), S (G) = (1.2, 1.5, 3.1)

Proposition 7.4: In a complete single valued neutrosophic graph G=(V, E), the closed neighbourhood degree of any vertex is equal to the order of single valued neutrosophic graph (i.e) $O_T(G)=(d_{NT}[v] \mid v \in V)$, $O_I(G)=(d_{NI}[v] \mid v \in V)$ and $O_F(G)=(d_{NF}[v] \mid v \in V)$

Proof:
Let G = (V, E) be a complete single valued neutrosophic graph. The **T**-order of G, $O_T(G)$ is the sum of the truth-membership value of all the vertices, the **I**-order of G, $O_I(G)$ is the sum of the indeterminacy- membership value of all the vertices and the **F**-order of G, $O_F(G)$ is the sum of the falsity-membership value of all the vertices. Since G is a complete SVN-graph, the closed neighbourhood **T**-degree of each vertex is the sum of the truth-membership value of vertices, the closed neighbourhood **I**-degree of each vertex is the sum of the indeterminacy-membership value of vertices and the closed neighbourhood **F**-degree of each vertex is the sum of the falsity-membership value of vertices. Hence the result.

Conclusion

In this paper we have described degree of a vertex, order, size of single valued neutrosophic graphs. The necessary and sufficient conditions for a single valued neutrosophic graph to be the regular single valued neutrosophic graphs have been presented. Further, we are going to study some types of single valued neutrosophic graphs such irregular and totally irregular single valued neutrosophic graphs and bipolar single valued neutrosophic graphs.

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