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Single Valued Neutrosophic Sets

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Abstract

Neutrosophic set is a part of neutrosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophic set is a powerful general formal framework that has been recently proposed. However, neutrosophic set needs to be specified from a technical point of view. To this effect, we define the set-theoretic operators on an instance of neutrosophic set, we call it single valued neutrosophic set (SVNS). We provide various properties of SVNS, which are connected to the operations and relations over SVNS.

Keywords: Neutrosophic set, single valued neutrosophic set, set-theoretic operator

1. Introduction

The concept of fuzzy sets was introduced by Zadeh in 1965[5]. Since then fuzzy sets and fuzzy logic have been applied in many real applications to handle uncertainty. The traditional fuzzy set uses one real value $\mu_A(x) \in [0,1]$ to represent the grade of membership of fuzzy set A defined on universe X. Sometimes $\mu_A(x)$ itself is uncertain and hard to be defined by a crisp value. So the concept of interval valued fuzzy sets was proposed [3] to capture the uncertainty of grade of membership. Interval valued fuzzy set uses an interval value $[\mu_A^L(x), \mu_A^U(x)]$ with $0 \le \mu_A^L(x) \le \mu_A^U(x) \le 1$ to represent the grade of membership of fuzzy set A. In some applications such as expert system, belief system and information fusion, we should consider not only the truth-membership supported by the evident but also the falsity-membership against by the evident. That is beyond the scope of fuzzy sets and interval valued fuzzy sets. In 1986, Atanassov introduced the intuitionistic fuzzy sets [1] which is a generalization of fuzzy sets and provably equivalent to interval valued fuzzy sets. The intuitionistic fuzzy sets consider both truth-membership $t_A(x)$ and falsity-membership $f_A(x)$, with $t_A(x)$, $f_A(x) \in [0,1]$ and $0 \le t_A(x)+f_A(x) \le 1$. Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information which exists commonly in belief system. In intuitionistic fuzzy sets, indeterminacy is $1-t_A(x)-f_A(x)$ by default. For example, when we ask the opinion of an expert about certain statement, he or she may that the possibility that the statement is true is 0.5 and the statement is false is 0.6 and the degree that he or she is not sure is 0.2.

In neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership and falsity-membership are independent. This assumption is very important in a lot of situations such as information fusion when we try to combine the data from different sensors. Neutrosophy was introduced by Smarandache in 1995. "It is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra" [2]. Neutrosophic set is a power general formal framework which generalizes the concept of the classic set, fuzzy set [5], interval valued fuzzy set [3], intuitionistic fuzzy set [1], etc. A neutrosophic set A defined on universe U. x = x(T,I,F) $\in$ A with T, I and F being the real standard or non-standard subsets of $]0^-,1^+[$. T is the degree of truth-membership function in the set A, I is the indeterminacy-membership function in the set A and F is the falsity-membership function in the set A.

The neutrosophic set generalizes the above mentioned sets from philosophical point of view. From scientific or engineering point of view, the neutrosophic set and set-theoretic operators need to be specified. Otherwise, it will be difficult to apply in the real applications. In this paper, we define the set-theoretic operators on an instance of neutrosophic set called single valued neutrosophic set (svns).

2. Neutrosophic Set

This section gives a brief overview of concepts of neutrosophic set defined in [2]. Here, we use different notations to express the same meaning. Let S_1 and S_2 be two real standard or non-standard subsets, then $S_1 + S_2 = \{x|x = s_1+s_2, s_1 \in S_1 \text{ and } s_2 \in S_2\}$, $\{1^+\} + S_2 = \{x|x = 1^+ + s_2, s_2 \in S_2\}$. $S_1 - S_2 = \{x|x=s_1-s_2, s_1 \in S_1 \text{ and } s_2 \in S_2\}$, $\{1^+\} - S_2 = \{x|x = 1^+ - s_2, s_2 \in S_2\}$. $S_1 \times S_2 = \{x|x = s_1 \times s_2, s_1 \in S_1 \text{ and } s_2 \in S_2\}$.

Definition 1 (Neutrosophic Set) Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-,1^+[$. That is

$$T_A: X \rightarrow]0^-,1^+[\tag{1}$$

$$I_A: X \rightarrow]0^-,1^+[\tag{2}$$

$$F_A: X \rightarrow]0^-,1^+[\tag{3}$$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x)+\sup I_A(x)+\sup F_A(x) \leq 3^+$.

Definition 2 The complement of a neutrosophic set A is denoted by c(A) and is defined by

$$T_{c(A)}(x) = \{1^+\} - T_A(x), \tag{4}$$

$$I_{c(A)}(x) = \{1^+\} - I_A(x), \tag{5}$$

$$F_{c(A)}(x) = \{1^+\} - F_A(x), \tag{6}$$

for all x in X.

Definition 3 (Containment) A neutrosophic set A is contained in the other neutrosophic set B, $A \subseteq B$, if and only if

$$\inf T_A(x) \leq \inf T_B(x),\ \sup T_A(x) \leq \sup T_B(x) \tag{7}$$

$$\inf F_A(x) \geq \inf F_B(x),\ \sup F_B(x) \geq \sup F_B(x) \tag{8}$$

Definition 4 (Union) The union of two neutrosophic sets A and B is a neutrosophic set C, written as $C = A \cup B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x) = T_A(x)+T_B(x)-T_A(x)\times T_B(x), \tag{9}$$

$$I_C(x) = I_A(x)+I_B(x)-I_A(x)\times I_B(x), \tag{10}$$

$$F_C(x) = F_A(x)+F_B(x)-F_A(x)\times F_B(x), \tag{11}$$

for all x in X.

Definition 5 (Intersection) The intersection of two neutrosophic sets A and B is a neutrosophic set C, written as $C = A \cap B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x) = T_A(x) \times T_B(x), \tag{12}$$

$$I_C(x) = I_A(x) \times I_B(x), \tag{13}$$

$$F_C(x) = F_A(x) \times F_B(x), \tag{14}$$

for all x in X.

3. Single Valued Neutrosophic Set

In this section, we present the notion of single valued neutrosophic set (SVNS). SVNS is an instance of neutrosophic set which can be used in real scientific and engineering applications.

Definition 6 (Single Valued Neutrosophic Set) Let X be a space of points (objects), with a generic element in X denoted by x. A single valued neutrosophic set (SVNS) A in X is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A. For each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0,1]$.

When X is continuous, a SVNS A can be written as

$$A=\int_X \langle T(x), I(x), F(x)\rangle / x, x \in X \tag{15}$$

When X is discrete, a SVNS A can be written as

$$A=\sum_{i=1}^{n} \langle T(xi), I(xi), F(xi)\rangle / xi, xi \in X \tag{16}$$

Consider parameters such as capability, trustworthiness and price of semantic Web services. These parameters are commonly used to define quality of service of semantic Web services. In this section, we will use the evaluation of quality of service of semantic Web services [4] as running example to illustrate every set-theoretic operation on single valued neutrosophic sets.

Example 1 Assume that $X = [x_1,x_2,x_3]$. x_1 is capability, x_2 is trustworthiness and x_3 is price. The values of x_1,x_2 and x_3 are in [0,1]. They are obtained from the questionnaire of some domain experts, their option could be a degree of "good service", a degree of indeterminacy and a degree of "poor service". A is a single valued neutrosophic set of X defined by $A = \langle 0.3,0.4,0.5\rangle/x_1 + \langle 0.5,0.2,0.3\rangle/x_2 + \langle 0.7,0.2,0.2\rangle/x_3$. B is a single valued neutrosophic set of X defined by $B = \langle 0.6,0.1,0.2\rangle/x_1 + \langle 0.3,0.2,0.6\rangle/x_2 + \langle 0.4,0.1,0.5\rangle/x_3$.

Definition 7 (Complement) The complement of a single valued neutrosophic set A is denoted by c(A) and is defined by

$$T_{c(A)}(x) = F_A(x), \quad (17)$$
$$I_{c(A)}(x) = 1 - I_A(x), \quad (18)$$
$$F_{c(A)}(x) = T_A(x), \quad (19)$$

for all x in X.

Example 2 Let A be the single valued neutrosophic set defined in Example 1. Then, $c(A) = \langle 0.5,0.6,0.3\rangle/x_1 + \langle 0.3,0.8,0.5\rangle/x_2 + \langle 0.2,0.8,0.7\rangle/x_3$.

Definition 8 (Containment) A single valued neutrosophic set A is contained in the other single valued neutrosophic set B, $A \subseteq B$, if and only if

$$T_A(x) \le T_B(x), \quad (20)$$
$$I_A(x) \le I_B(x), \quad (21)$$
$$F_A(x) \ge F_B(x), \quad (22)$$

for all x in X.

Note that by the definition of containment, X is partial order not linear order. For example, let A and B be the single valued neutrosophic sets defined in Example 1. Then, A is not contained in B and B is not contained in A.

Definition 9 Two single valued neutrosophic sets A and B are equal, written as A = B, if and only if $A \subseteq B$ and $B \subseteq A$.

Theorem 3 $A \subseteq B \leftrightarrow c(B) \subseteq c(A)$
Proof: $A \subseteq B \Leftrightarrow T_A \le T_B$, $I_A \le I_B$, $F_A \ge F_B \Leftrightarrow F_B \le F_A$, $1 - I_B \le 1 - I_A$, $T_B \ge T_A \Leftrightarrow c(B) \subseteq c(A)$.

Definition 10 (Union) The union of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as $C = A \cup B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x) = \max(T_A(x),T_B(x)), \quad (23)$$
$$I_C(x) = \max(I_A(x),I_B(x)), \quad (24)$$
$$F_C(x) = \min(F_A(x),F_B(x)), \quad (25)$$

for all x in X.

Example 3 Let A and B be the single valued neutrosophic sets defined in Example 1. Then, $A \cup B = \langle 0.6,0.4,0.2\rangle/x_1 + \langle 0.5,0.2,0.3\rangle/x_2 + \langle 0.7,0.2,0.2\rangle/x_3$.

Theorem 2 $A \cup B$ is the smallest single valued neutrosophic set containing both A and B.
Proof: It is straightforward from the definition of the union operator.

Definition 11 (Intersection) The intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as $C = A \cap B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x) = \min(T_A(x),T_B(x)), \quad (26)$$
$$I_C(x) = \min(I_A(x),I_B(x)), \quad (27)$$
$$F_C(x) = \max(F_A(x),F_B(x)), \quad (28)$$

for all x in X.

Example 4 Let A and B be the single valued neutrosophic sets defined in Example 1. Then, $A \cap B = \langle 0.3,0.1,0.5\rangle/x_1 + \langle 0.3,0.2,0.6\rangle/x_2 + \langle 0.4,0.1,0.5\rangle/x_3$.

Theorem 3 $A \cap B$ is the largest single valued neutrosophic set contained in both A and B.
Proof: It is direct from the definition of intersection operator.

Definition 12 (Difference) The difference of two single valued neutrosophic set C, written as $C = A \setminus B$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of A and B by

$$T_C(x) = \min(T_A(x),F_B(x)), \quad (29)$$
$$I_C(x) = \min(I_A(x),1-I_B(x)), \quad (30)$$
$$F_C(x) = \max(F_A(x),T_B(x)), \quad (31)$$

for all x in X.

Example 5 Let A and B be the single valued neutrosophic sets defined in Example 1. Then $A \setminus B = \langle 0.2,0.4,0.6\rangle/x_1 + \langle 0.5,0.2,0.3\rangle/x_2 + \langle 0.5,0.2,0.4\rangle/x_3$.

Now we will define two operators: truth-favorite (Δ) and falsity-favorite(∇) to remove the indeterminacy in the single valued neutrosophic sets and transform it into intuitionistic fuzzy sets or paraconsistent sets. These two operators are unique on single valued neutrosophic sets.

Definition 13 (Truth-favorite) The truth-favorite of a single valued neutrosophic set A is a single valued neutrosophic set B, written as $B = \Delta A$, whose truth-membership and falsity-membership functions are related to those of A by

$$T_B(x) = \min(T_A(x)+I_A(x),1), \quad (32)$$
$$I_B(x) = 0, \quad (33)$$
$$F_B(x) = F_A(x), \quad (34)$$

for all x in X.

Example 6 Let A be the single valued neutrosophic set defined in Example 1. Then $\Delta A = \langle 0.7,0,0.5\rangle/x_1 + \langle 0.7,0,0.3\rangle/x_2 + \langle 0.9,0,0.2\rangle/x_3$.

Definition 14 (Falsity-favorite) The falsity-favorite of a single valued neutrosophic set B, written as $B = \nabla A$, whose truth-membership and falsity-membership functions are related to those of A by

$$T_B(x) = T_A(x), \quad (33)$$

$$I_B(x) = 0, \quad (34)$$

$$F_B(x) = \min(F_A(x)+I_A(x),1), \quad (35)$$

for all x in X.

Example 8 Let A be the single valued neutrosophic set defined in Example 1. Then $\nabla A = \langle 0.3,0,0.9\rangle/x_1 + \langle 0.5,0,0.5\rangle/x_2 + \langle 0.7,0,0.4\rangle/x_3$.

4. Properties of Set-theoretic Operators

In this section, we will give some properties of set-theoretic operators defined on single valued neutrosophic sets as in Section 3.

Property 1 (Commutativity) $A \cup B = B \cup A$, $A \cap B = B \cap A$, $A \times B = B \times A$.

Property 2 (Associativity) $A \cup (B \cup C) = (A \cup B) \cup C$, $A \cap (B \cap C) = (A \cap B) \cap C$, $A \times (B \times C) = (A \times B) \times C$.

Property 3 (Distributivity) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Property 4 (Idempotency) $A \cup A = A$, $A \cap A = A$, $\Delta\Delta A = \Delta A$, $\nabla\nabla A = \nabla A$.

Property 5 $A \cap \phi = \phi$, $A \cup X = X$, where $T\phi = I\phi = 0$, $F\phi = 1$ and $T_X = I_X = 1$, $F_X = 0$.

Property 6 $A \cup \phi = A$, $A \cap X = A$, where $T\phi = I\phi = 0$, $F\phi = 1$ and $T_X = I_X = 1$, $F_X = 0$.

Property 7 (Absorption) $A \cup (A \cap B) = A$, $A \cap (A \cup B) = A$.

Property 8 (De Morgan's Laws) $c(A \cup B) = c(A) \cap c(B)$, $c(A \cap B) = c(A) \cup c(B)$.

Property 9 (Involution) $c(c(A)) = A$.

Here, we notice that by the definition of complement, union and intersection of single valued neutrosophic sets, single valued neutrosophic sets satisfy the most properties of classic set, fuzzy set and intuitionistic fuzzy set. Same as fuzzy set and intuitionistic fuzzy set, it does not satisfy the principle of middle exclude.

5. Conclusions

In this paper, we have presented an instance of neutrosophic set called single valued neutrosophic set (SVNS). The single valued neutrosophic set is a generalization of classic set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set and paraconsistent set. The notion of inclusion, complement, union, intersection, have been defined on single valued neutrosophic sets. Various properties of set-theoretic operators have been provided. In the future, we will create the logic inference system based on single valued neutrosophic sets and apply the theory to solve practical applications in areas such as expert system, information fusion system, question-answering system, bioinformatics and medical informatics, etc.

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