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METHODOLOGIES AND APPLICATION

Single-valued neutrosophic similarity measures based on cotangent function and their application in the fault diagnosis of steam turbine

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Abstract Similarity measure is an important tool in pattern recognition and fault diagnosis. This paper proposes two cotangent similarity measures for single-valued neutrosophic sets (SVNSs) based on cotangent function. Then, the weighted cotangent similarity measures are introduced by considering the importance of each element. Moreover, by the comparison between the cotangent similarity measures of SVNSs and existing cosine similarity measure of SVNSs, the developed cotangent similarity measures demonstrate their advantages and rationality and in some cases can overcome some disadvantages of the cosine similarity measure defined in vector space. Finally, the cotangent similarity measures are applied to the fault diagnosis of steam turbine. The proposed fault diagnosis method demonstrates its effectiveness and rationality by the comparative analysis with the cosine similarity measure in the fault diagnosis of steam turbine.

Keywords Single-valued neutrosophic set · Cotangent similarity measure · Fault diagnosis · Steam turbine

1 Introduction

Various new concepts of high-order fuzzy sets have been proposed since Zadeh (1965) firstly presented fuzzy set theory. Among them, intuitionistic fuzzy sets (IFSs) introduced by Atanassov (1986) are the generalization of fuzzy sets. In IFS, two characteristic functions are expressed by the membership degree and non-membership degree of elements in the universe to the set, and the hesitation degree called hesitation margin is defined as 1 minus the sum of membership and non-membership degrees. Therefore, it provides a flexible mathematical framework to incomplete and uncertainty information. However, it can only handle incomplete and uncertainty information but not the indeterminate and inconsistent information which exists commonly in real situations. Therefore, Smarandache (1999) originally proposed the concept of a neutrosophic set from philosophical point of view. According to the definition of a neutrosophic set given by Smarandache (1999), a neutrosophic set A in a universal set X is characterized independently by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x)$: $X \to]^-0, 1^+[$, $I_A(x)$: $X \to]^-0, 1^+[$, and $F_A(x)$: $X \to]^-0, 1^+[$. However, the defined range of the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ in a neutrosophic set A is within the nonstandard unit interval $]^-0, 1^+[$, which is indicated in the definition of the neutrosophic set A (Smarandache 1999), it is only used for philosophical applications, especially when distinction is required between absolute and relative truth/falsity/indeterminacy. To easily use in technical applications of the neutrosophic set, the defined range of $T_A(x)$, $I_A(x)$ and $F_A(x)$ can be restrained to the normal standard real unit interval [0, 1] (Wang et al. 2005, 2010). Thus, Wang et al. (2005, 2010) introduced the concepts of a single-valued neutrosophic set (SVNS) and an interval neutrosophic set (INS), which are the subclasses of the neutrosophic set. Then, SVNSs and INSs are the generalization of the concepts of the classic sets, fuzzy sets, IFSs and interval-valued intuitionistic fuzzy sets (IVIFSs) and can deal with incom-

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plete, indeterminate and inconsistent information. Therefore, SVNSs and INSs have been applied to some engineering fields, such as decision making (Ye 2014c; Liu et al. 2014; Liu and Wang 2014; Peng et al. 2014), clustering analysis (Ye 2014d) and image processing (Guo et al. 2014). However, till now, SVNSs and INSs have not been applied to fault diagnosis field.

Similarity measure is an important topic in the neutrosophic theory. Many similarity measures have been proposed by some researchers. Broumi and Smarandache (2013) defined the Hausdorff distance between neutrosophic sets and some similarity measures based on the distance, set theoretic approach, and matching function to calculate the similarity degree between neutrosophic sets. Majumdar and Samanta (2014) introduced several similarity measures of SVNSs based on distances, a matching function, membership grades, and then proposed an entropy measure for a SVNS. Then, Ye (2014a) introduced the Hamming and Euclidean distances between INSs and the distance-based similarity measures and applied them to multiple attribute decision-making problems with interval neutrosophic information. Ye (2014b) further put forward the distance-based similarity measure of SVNSs and applied it to group decision-making problems with single-valued neutrosophic information. Furthermore, Ye (2014c) presented three vector similarity measures for simplified neutrosophic sets (SNSs), including the Jaccard, Dice and cosine similarity measures for SVNSs and INSs, based on the extension of the Jaccard, Dice and cosine similarity measures between vectors and applied them to multicriteria decision-making problems under a simplified neutrosophic environment.

Among various similarity measures, a type of similarity measures based on trigonometric functions has been introduced and applied to science and engineering areas. For example, (Ye 2011, 2014c) put forward some cosine similarity measures for IFSs, SVNSs and INSs and applied them to pattern recognition, medical diagnosis and decision making. Tian (2013) developed the cotangent similarity measure of IFSs and applied it to medical diagnosis. However, the cosine similarity measures defined in vector space (Ye 2011, 2014c) have some drawbacks in some situations. For instance, in some cases, they produce undefined (unmeaningful) phenomena or some results calculated by the cosine similarity measures are not reasonable in some real cases (details given in Sects. 2 and 4). As the cotangent measure can overcome the aforementioned drawbacks of the cosine measure, we should extend the cotangent measure to the similarity measure of SVNSs. Motivated by the cotangent similarity measure of IFSs, this paper proposes two cotangent similarity measures of SVNSs and apply them to the fault diagnosis of steam turbine. To do so, the rest of the article is organized as follows. Section 2 introduces some basic concepts and similarity measures of IFSs and SVNSs. Section 3 put forward two similarity measures of SVNSs based on cotangent function and investigates their properties. In Sect. 4, we give the comparative analysis between the developed cotangent measures and existing cosine measure for SVNSs. Section 5 applies the two cotangent similarity measures to the fault diagnosis of steam turbine and gives the comparative analysis with the cosine similarity measure in the fault diagnosis of steam turbine. Conclusions and further research are contained in Sect. 6.

2 Some concepts and similarity measures of IFSs and SVNSs

2.1 IFS and its cotangent similarity measure

Atanassov (1986) presented an IFS concept as an extension of the concept of a fuzzy set (Zadeh 1965) and gave the following definition.

Definition 1 (Atanassov 1986) An IFS A in the universe of discourse X is defined as $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$, where $\mu_A(x)$: $X \rightarrow [0, 1]$ and $\nu_A(x)$: $X \rightarrow [0, 1]$ are the membership degree and non-membership degree, respectively, of the element x to the set A with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for $x \in X$.

Then $m_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called Atanassov's intuitionistic index or a hesitancy degree of the element x in the set A. Obviously, there is $0 \leq m_A(x) \leq 1$ for $x \in X$.

Similarity measures have the following definition (Tian 2013).

Definition 2 (Tian 2013) A real-valued function S: IFS$(X) \times$ IFS$(X) \rightarrow [0, 1]$ is called a similarity measure on IFS(X), if it satisfies the following axiomatic requirements:

(S1) $0 \leq S(A, B) \leq 1$;
(S2) $S(A, B) = 1$ if and only if $A = B$;
(S3) $S(A, B) = S(B, A)$;
(S4) If $A \subseteq B \subseteq C$, then $S(A, C) \leq S(A, B)$ and $S(A, C) \leq S(B, C)$.

Assume that there are two IFSs $A = \{\langle x_j, \mu_A(x_j), \nu_A(x_j)\rangle | x_j \in X\}$ and $B = \{\langle x_j, \mu_B(x_j), \nu_B(x_j)\rangle | x_j \in X\}$ in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$. Tian (2013) proposed the following cotangent similarity measure of the IFSs A and B:

$$CS(A, B) = \frac{1}{n}\sum_{j=1}^{n} \cot\left[\frac{\pi}{4} + \frac{\pi}{4}\max\left(\begin{array}{l}\left|\mu_A(x_j) - \mu_B(x_j)\right|,\\ \left|\nu_A(x_j) - \nu_B(x_j)\right|\end{array}\right)\right]. \quad (1)$$

2.2 Some concepts and cosine measure of SVNSs

Smarandache (1999) firstly gave the definition of a neutrosophic set from philosophical point of view.

Definition 3 (Smarandache 1999) Let X be a space of points (objects) with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, respectively. The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^{-}0, 1^{+}[$, such that $T_A(x)$: $X \to]^{-}0, 1^{+}[$, $I_A(x)$: $X \to]^{-}0, 1^{+}[$ and $F_A(x)$: $X \to]^{-}0, 1^{+}[$. Then, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ has no restriction. Hence, there is the condition $^{-}0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^{+}$.

To easily apply the neutrosophic set to science and engineering areas, Wang et al. (2010) introduced the concept of SVNS, which is a subclass of the neutrosophic set and gave the following definition.

Definition 4 (Wang et al. 2010) Let X be a universal set. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. Then, a SVNS A can be denoted by the following symbol:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \,|x \in X\},$$

where $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ for each x in X. Therefore, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

For two SVNSs $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \,|x \in X\}$ and $B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle \,|x \in X\}$, there are the following relations (Wang et al. 2010):

(1) *Complement*

$$A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle \,|x \in X\};$$

(2) *Inclusion*
$A \subseteq B$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$ for any x in X;
(3) *Equality*
$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;
(4) *Union*

$$A \cup B = \left\{\left\langle \begin{matrix} x, T_A(x) \vee T_B(x), \\ I_A(x) \wedge I_B(x), \\ F_A(x) \wedge F_B(x) \end{matrix} \right\rangle |x \in X\right\};$$

(5) *Intersection* $A \cap B = \left\{\left\langle \begin{matrix} x, T_A(x) \wedge T_B(x), \\ I_A(x) \vee I_B(x), \\ F_A(x) \vee F_B(x) \end{matrix} \right\rangle |x \in X\right\}$.

Assume that there are two SVNSs $A = \{\langle x_j, T_A(x_j), I_A(x_j), F_A(x_j)\rangle | x_j \in X\}$ and $B = \{\langle x_j, T_B(x_j), I_B(x_j), F_B(x_j)\rangle | x_j \in X\}$ in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$. Ye (2014c) presented the cosine similarity measure of SVNSs in vector space:

$$\mathrm{Cos}(A, B) = \frac{1}{n}\sum_{j=1}^{n} \frac{\begin{pmatrix} T_A(x_j)T_B(x_j)+ \\ I_A(x_j)I_B(x_j)+ \\ F_A(x_j)F_B(x_j) \end{pmatrix}}{\begin{pmatrix} \sqrt{T_A^2(x_j) + I_A^2(x_j) + F_A^2(x_j)} \\ \sqrt{T_B^2(x_j) + I_B^2(x_j) + F_B^2(x_j)} \end{pmatrix}}. \tag{2}$$

However, one can find some drawbacks of Eq. (2) as follows:

(1) For two SVNSs A and B, if $T_A(x_j) = I_A(x_j) = F_A(x_j) = 0$ and/or $T_B(x_j) = I_B(x_j) = F_B(x_j) = 0$ for any x_j in $X (j = 1, 2, \ldots, n)$, Eq. (2) is undefined or unmeaningful. In this case, one cannot utilize it to calculate the cosine similarity measure between A and B.
(2) If $T_A(x_j) = 2T_B(x_j)$, $I_A(x_j) = 2I_B(x_j)$ and $F_A(x_j) = 2F_B(x_j)$ or $2T_A(x_j) = T_B(x_j)$, $2I_A(x_j) = I_B(x_j)$ and $2F_A(x_j) = F_B(x_j)$ for any x_j in $X (j = 1, 2, \ldots, n)$, the measure value of Eq. (2) is as follows:

$$\begin{aligned} &\mathrm{Cos}(A, B) \\ &= \frac{1}{n}\sum_{j=1}^{n} \frac{\begin{pmatrix} 2T_A(x_j)T_B(x_j)+ \\ 2I_A(x_j)I_B(x_j)+ \\ 2F_A(x_j)F_B(x_j) \end{pmatrix}}{\begin{pmatrix} 2\sqrt{T_A^2(x_j) + I_A^2(x_j) + F_A^2(x_j)} \\ \sqrt{T_B^2(x_j) + I_B^2(x_j) + F_B^2(x_j)} \end{pmatrix}}. \\ &= \frac{1}{n}\sum_{j=1}^{n} \frac{T_A^2(x_j)+I_A^2(x_j)+F_A^2(x_j)}{T_A^2(x_j)+I_A^2(x_j)+F_A^2(x_j)} = 1 \end{aligned}$$

Since $A \ne B$, the measure value of $\mathrm{Cos}(A, B)$ is equal to 1. This means that it only satisfies the necessary condition of the property (S2) in Definition 2, but not the sufficient condition. Therefore, in this case, it is unreasonable to apply to pattern recognition and fault diagnosis.

To overcome these disadvantages, we shall propose two cotangent similarity measures of SVNSs in the following section.

3 Cotangent similarity measures of SVNSs

Based on cotangent function, the section proposes two cotangent similarity measures between SVNSs and investigates their properties.

Definition 5 Let $A = \{< x_j, T_A(x_j), I_A(x_j), F_A(x_j) > |x_j \in X\}$ and $B = \{< x_j, T_B(x_j), I_B(x_j), F_B(x_j) > |x_j \in X\}$ be any two SVNSs in $X = \{x_1, x_2, \ldots, x_n\}$. Then, we define two cotangent similarity measures between A and B, respectively, as follows:

$$\mathrm{Cot}_1(A, B) = \frac{1}{n}\sum_{j=1}^{n} \cot\left[\frac{\pi}{4} + \frac{\pi}{4}\max\begin{pmatrix}\left|T_A(x_j) - T_B(x_j)\right|, \\ \left|I_A(x_j) - I_B(x_j)\right|, \\ \left|F_A(x_j) - F_A(x_j)\right|\end{pmatrix}\right], \quad (3)$$

$$\mathrm{Cot}_2(A, B) = \frac{1}{n}\sum_{j=1}^{n} \cot\left[\frac{\pi}{4} + \frac{\pi}{12}\begin{pmatrix}\left|T_A(x_j) - T_B(x_j)\right| + \\ \left|I_A(x_j) - I_B(x_j)\right| + \\ \left|F_A(x_j) - F_B(x_j)\right|\end{pmatrix}\right]. \quad (4)$$

Proposition 1 *For two SVNSs A and B in X= \{x_1, x_2, …, x_n\}, the cotangent similarity measure $Cot_k(A, B)$ (k = 1, 2) should satisfy the axiomatic requirements (S1–S4) in Definition 2.*

Proof (S1) Since the truth-membership, indeterminacy-membership, and falsity-membership functions in SVNS and the cotangent function within the interval $[\pi/4, \pi/2]$ lie between 0 and 1, the similarity measures based on cotangent function also lie between 0 and 1. Hence, there is $0 \le \mathrm{Cot}_k(A, B) \le 1$ for $k = 1, 2$.

(S2) For the two SVNSs A and B, if $A = B$, this implies $T_A(x_j) = T_B(x_j)$, $I_A(x_j) = I_B(x_j)$, $F_A(x_j) = F_B(x_j)$ for j= 1, 2, …, n and $x_j \in X$. Hence $\left|T_A(x_j) - T_B(x_j)\right| = 0$, $\left|I_A(x_j) - I_B(x_j)\right| = 0$, and $\left|F_A(x_j) - F_B(x_j)\right| = 0$. Thus $\mathrm{Cot}_k(A, B) = 1$ for $k = 1, 2$.
If $Cot_k(A, B) = 1$ for $k = 1, 2$, this implies $\left|T_A(x_j) - T_B(x_j)\right| = 0$, $\left|I_A(x_j) - I_B(x_j)\right| = 0$, and $\left|F_A(x_j) - F_B(x_j)\right| = 0$ since $\cot(\pi/4)$ is equal to 1. Then, these equalities indicate that $T_A(x_j) = T_B(x_j)$, $I_A(x_j) = I_B(x_j)$, and $F_A(x_j) = F_B(x_j)$ for j= 1, 2, …, n and $x_j \in X$. Hence $A = B$.

(S3) The proof is straightforward.

(S4) If $A \subseteq B \subseteq C$,then this implies $T_A(x_j) \le T_B(x_j) \le T_C(x_j)$, $I_A(x_j) \ge I_B(x_j) \ge I_C(x_j)$, and $F_A(x_j) \ge F_B(x_j) \ge F_C(x_j)$ for j= 1, 2, …, n and $x_j \in X$. Then, we have

$$\left|T_A(x_j) - T_B(x_j)\right| \le \left|T_A(x_j) - T_C(x_j)\right|,$$
$$\left|T_B(x_j) - T_C(x_j)\right| \le \left|T_A(x_j) - T_C(x_j)\right|,$$
$$\left|I_A(x_j) - I_B(x_j)\right| \le \left|I_A(x_j) - I_C(x_j)\right|,$$
$$\left|I_B(x_j) - I_C(x_j)\right| \le \left|I_A(x_j) - I_C(x_j)\right|,$$
$$\left|F_A(x_j) - F_B(x_j)\right| \le \left|F_A(x_j) - F_C(x_j)\right|,$$
$$\text{and} \quad \left|F_B(x_j) - F_C(x_j)\right| \le \left|F_A(x_j) - F_C(x_j)\right|.$$

Hence, $\mathrm{Cot}_k(A, C) \le \mathrm{Cot}_k(A, B)$ and $\mathrm{Cot}_k(A, C) \le \mathrm{Cot}_k(B, C)$ for $k = 1, 2$ since the cotangent function is a decreasing function within the interval $[\pi/4, \pi/2]$.

Therefore, the proofs of these axiomatic requirements are completed. □

Usually, one takes the weight of each element x_j for $x_j \in X$ into account. Assume that the weight of an element x_j is w_j ($j = 1, 2, \ldots, n$) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Thus, we can introduce the following weighted cotangent similarity measures of SVNSs:

$$W\mathrm{Cot}_1(A, B) = \sum_{j=1}^{n} w_j \cot\left[\tfrac{\pi}{4} + \tfrac{\pi}{4}\max\begin{pmatrix}\left|T_A(x_j) - T_B(x_j)\right|, \\ \left|I_A(x_j) - I_B(x_j)\right|, \\ \left|F_A(x_j) - F_A(x_j)\right|\end{pmatrix}\right], \quad (5)$$

$$W\mathrm{Cot}_2(A, B) = \sum_{j=1}^{n} w_j \cot\left[\tfrac{\pi}{4} + \tfrac{\pi}{12}\begin{pmatrix}\left|T_A(x_j) - T_B(x_j)\right| + \\ \left|I_A(x_j) - I_B(x_j)\right| + \\ \left|F_A(x_j) - F_A(x_j)\right|\end{pmatrix}\right]. \quad (6)$$

Especially when $w_j = 1/n$ for $j = 1, 2, \ldots, n$, Eqs. (5) and (6) reduce to Eqs. (3) and (4).

4 Comparative analysis

To compare the proposed cotangent measures with the cosine measure in Ye (2014c) in single-valued neutrosophic setting, we provide an example to demonstrate the effectiveness and rationality of the proposed two cotangent similarity measures of SVNSs.

Example 1 We consider any two SVNSs A_1 and A_2 and compare the proposed cotangent similarity measures with existing cosine similarity measure in Ye (2014c) by the following cases:

Case 1 Assume that there are two SVNSs in $X = \{x\}$: $A_1 = \{< x, 0.1, 0.2, 0.2> |x \in X\}$ and $A_2 = \{< x, 0.2, 0.4, 0.4> |x \in X\}$.

Then, using Eqs. (2)–(4), we can obtain $\mathrm{Cos}(A_1, A_2) = 1$, $\mathrm{Cot}_1(A_1, A_2) = 0.7265$ and $\mathrm{Cot}_2(A_1, A_2) = 0.7673$. The measure value of $\mathrm{Cos}(A_1, A_2)$ is equal to 1, which is not reasonable since $A_1 \ne A_2$. In this case, it is also unreasonable in the applications of pattern recognition and fault diagnosis. Furthermore, this also means that it only satisfies the necessary condition of the axiomatic requirement (S2) in Definition 2, but not the sufficient condition. Then, the results of the cotangent similarity measures $\mathrm{Cot}_1(A_1, A_2)$ and $\mathrm{Cot}_2(A_1, A_2)$ are reasonable.

Case 2 Assume that there are two SVNSs in $X = \{x\}$: $A_1 = \{< x, 0.1, 0.2, 0.3> |x \in X\}$ and $A_2 = \{< x, 0.0, 0.0, 0.0> |x \in X\}$.

Then, by applying Eq. (2) the cosine similarity measure $\mathrm{Cos}(A_1, A_2)$ is undefined or unmeaningful. In this case, one cannot utilize it to calculate the cosine similarity measure between A_1 and A_2. However, by applying Eqs. (3)–(4) we get $\mathrm{Cot}_1(A_1, A_2) = 0.6128$ and $\mathrm{Cot}_2(A_1, A_2) = 0.7265$. The results show that the two cotangent similarity measures are effective and reasonable.

The example demonstrates that the proposed two cotangent similarity measures of SVNSs are effective and reasonable and in some cases can overcome the disadvantages of the cosine similarity measure defined in vector space (Ye 2014c).

5 Application in the fault diagnosis of steam turbine

The vibration of huge steam turbine generator sets is usually a typical fault type, which suffers the influence of a lot of factors like the mechanical structure, load, vacuum degree, hot inflation of cylinder body and rotor, fluctuation of network load, temperature of lubricant oil, ground, and so on. In steam turbine, interaction effects in these factors show the vibration of steam turbine. In the vibration fault diagnosis of steam turbine, the relation between the cause and the fault phenomena of steam turbine has been investigated in Ye (2009). For the vibration fault diagnosis problem of steam turbine, the fault diagnosis of the turbine realized by the frequency features, which are extracted from the vibration signals of steam turbine, is a simple and effective method. However, a volume of fault feature information obtained from modem measurement technologies usually contains a lot of incomplete, uncertain and inconsistent information. In some practical situations, the frequency features may include incomplete and indeterminate information, which is expressed suitably by SVNSs. Here, we apply the cotangent similarity measures of SVNSs to the vibration fault diagnosis of steam turbine.

In the fault diagnosis problem of the turbine, we consider a set of ten fault patterns $A = \{A_1$ (Unbalance), A_2 (Pneumatic force couple), A_3 (Offset center), A_4 (Oil-membrane oscillation), A_5 (Radial impact friction of rotor), A_6 (Symbiosis looseness), A_7 (Damage of antithrust bearing), A_8 (Surge), A_9 (Looseness of bearing block) and A_{10}(Non-uniform bearing stiffness)$\}$ as the fault knowledge and a set of nine frequency ranges for different frequency spectrum $C = \{C_1(0.01\text{–}0.39f), C_2(0.4\text{–}0.49f), C_3(0.5f), C_4(0.51\text{–}0.99f), C_5(f), C_6(2f), C_7(3\text{–}5f), C_8$ (Odd times of f) and C_9 (High frequency $> 5f$)$\}$ under operating frequency f as a characteristic set (attribute set). Then, the information of the fault knowledge A_i ($i = 1, 2, \ldots, 10$) with respect to the frequency range (attribute)C_j ($j = 1, 2, \ldots, 9$) can be introduced from Ye (2009), which is shown in Table 1 (Ye 2009). Here, assume that the weight of each characteristic

Table 1 Fault knowledge with vague values (Ye 2009)

A_i (Fault knowledge)	Frequency range (f: operating frequency)								
	C_1 (0.01–0.39f)	C_2 (0.4–0.49f)	C_3 (0.5f)	C_4 (0.51–0.99f)	C_5 (f)	C_6(2f)	C_7 (3–5f)	C_8 (Odd times of f)	C_9 (High frequency >5f)
A_1 (unbalance)	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.85, 1.00]	[0.04, 0.06]	[0.04, 0.07]	[0.00, 0.00]	[0.00, 0.00]
A_2 (pneumatic force couple)	[0.00, 0.00]	[0.28, 0.31]	[0.09, 0.12]	[0.55, 0.70]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.08, 0.13]
A_3 (offset center)	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.30, 0.58]	[0.40, 0.62]	[0.08, 0.13]	[0.00, 0.00]	[0.00, 0.00]
A_4 (oil-membrane oscillation)	[0.09, 0.11]	[0.78, 0.82]	[0.00, 0.00]	[0.08, 0.11]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]
A_5 (radial impact friction of rotor)	[0.09, 0.12]	[0.09, 0.11]	[0.08, 0.12]	[0.09, 0.12]	[0.18, 0.21]	[0.08, 0.13]	[0.08, 0.13]	[0.08, 0.12]	[0.08, 0.12]
A_6 (symbiosis looseness)	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.18, 0.22]	[0.12, 0.17]	[0.37, 0.45]	[0.00, 0.00]	[0.22, 0.28]
A_7 (damage of antithrust bearing)	[0.00, 0.00]	[0.00, 0.00]	[0.08, 0.12]	[0.86, 0.93]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]
A_8 (surge)	[0.00, 0.00]	[0.27, 0.32]	[0.08, 0.12]	[0.54, 0.62]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]
A_9 (looseness of bearing block)	[0.85, 0.93]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.08, 0.12]	[0.00, 0.00]
A_{10} (non-uniform bearing stiffness)	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.00, 0.00]	[0.77, 0.83]	[0.19, 0.23]	[0.00, 0.00]	[0.00, 0.00]

C_j is $w_j = 1/9$ for $j = 1, 2, \ldots, 9$. To show the effectiveness of the fault diagnosis method using the cotangent similarity measures of SVNSs, each vague value $[T_{ij}, 1-F_{ij}]$ (basic element) in a vague set A_i ($i = 1, 2, \ldots, 10$; $j = 1, 2, \ldots, 9$) in Table 1 can be transformed it into a single-valued neutrosophic value denoted by $\langle T_{ij}, I_{ij}, F_{ij}\rangle$, which is shown in Table 2.

In the vibration fault diagnosis of steam turbine, two real testing samples are introduced from Ye (2009), which are represented by the form of two single-valued neutrosophic sets: $B_1 = \{\langle 0, 0, 1\rangle, \langle 0, 0, 1\rangle, \langle 0.1, 0, 0.9\rangle, \langle 0.9, 0, 0.1\rangle, \langle 0, 0, 1\rangle, \langle 0, 0, 1\rangle, \langle 0, 0, 1\rangle, \langle 0, 0, 1\rangle, \langle 0, 0, 1\rangle\}$ and $B_2 = \{\langle 0.39, 0, 0.61\rangle, \langle 0.07, 0, 0.93\rangle, \langle 0, 0, 1\rangle, \langle 0.06, 0, 0.94\rangle, \langle 0, 0, 1\rangle, \langle 0.13, 0, 0.87\rangle, \langle 0, 0, 1\rangle, \langle 0, 0, 1\rangle, \langle 0.35, 0, 0.65\rangle\}$. Then, ranking all faults according to the cotangent similarity measure values between A_i ($i = 1, 2, \ldots, 10$) and B_s ($s = 1, 2$) in a decreasing order, we can give the diagnosis results, which are described as follows.

5.1 Fault diagnosis of the first testing sample B_1

The cotangent similarity measures between A_i ($i = 1, 2, \ldots, 10$) and B_1 are calculated by Eq. (5) as follows:

$W\mathrm{Cot}_1(A_1, B_1) = 0.7487$, $W\mathrm{Cot}_1(A_2, B_1) = 0.8811$, $W\mathrm{Cot}_1(A_3, B_1)$ 0.7108, $W\mathrm{Cot}_1(A_4, B_1) = 0.7755$, $W\mathrm{Cot}_1(A_5, B_1) = 0.7505$, $W\mathrm{Cot}_1(A_6, B_1) = 0.7216$, $W\mathrm{Cot}_1(A_7, B_1) = 0.9816$, $W\mathrm{Cot}_1(A_8, B_1) = 0.8978$, $W\mathrm{Cot}_1(A_9, B_1) = 0.7572$, and $W\mathrm{Cot}_1(A_{10}, B_1) = 0.7509$.

Therefore, the ranking order of all faults is $A_7 \to A_8 \to A_2 \to A_4 \to A_9 \to A_{10} \to A_5 \to A_1 \to A_6 \to A_3$.

Or using Eq. (6), we can also calculate the cotangent similarity measures between A_i ($i = 1, 2, \ldots, 10$) and B_1 as follows:

$W\mathrm{Cot}_2(A_1, B_1) = 0.8180$, $W\mathrm{Cot}_2(A_2, B_1) = 0.9164$, $W\mathrm{Cot}_2(A_3, B_1) = 0.7908$, $W\mathrm{Cot}_2(A_4, B_1) = 0.8372$, $W\mathrm{Cot}_2(A_5, B_1) = 0.8243$, $W\mathrm{Cot}_2(A_6, B_1) = 0.8011$, $W\mathrm{Cot}_2(A_7, B_1) = 0.9876$, $W\mathrm{Cot}_2(A_8, B_1) = 0.9277$, $W\mathrm{Cot}_2(A_9, B_1) = 0.8238$, and $W\mathrm{Cot}_2(A_{10}, B_1) = 0.8195$.

Therefore, the ranking order of all faults is $A_7 \to A_8 \to A_2 \to A_4 \to A_5 \to A_9 \to A_{10} \to A_1 \to A_6 \to A_3$.

According to above two kinds of fault ranking orders, the vibration faults of the turbine are firstly resulted from damage of antithrust bearing (A_7), next surge (A_8), and then pneumatic force couple (A_2), and so on. By actual checking, we discover that one of antithrust bearings is damage, which is in accordance with the actual fault. Thus, it causes the violent vibration of the turbine.

5.2 Fault diagnosis of the second testing sample B_2

The cotangent similarity measures between A_i ($i = 1, 2, \ldots, 10$) and B_2 are calculated by Eq. (5) as follows:

$W\mathrm{Cot}_1(A_1, B_2) = 0.7387$, $W\mathrm{Cot}_1(A_2, B_2) = 0.7530$, $W\mathrm{Cot}_1(A_3, B_2) = 0.7183$, $W\mathrm{Cot}_1(A_4, B_2) = 0.7900$, $W\mathrm{Cot}_1(A_5, B_2) = 0.8017$, $W\mathrm{Cot}_1(A_6, B_2) = 0.8030$, $W\mathrm{Cot}_1(A_7, B_2)$ 0.7466, $W\mathrm{Cot}_1(A_8, B_2) = 0.7499$, $W\mathrm{Cot}_1(A_9, B_2) = 0.8205$, and $W\mathrm{Cot}_1(A_{10}, B_2) = 0.7574$.

Therefore, the ranking order of all faults is $A_9 \to A_6 \to A_5 \to A_4 \to A_{10} \to A_2 \to A_8 \to A_7 \to A_1 \to A_3$.

Or using Eq. (6), the cotangent similarity measures between A_i ($i = 1, 2, \ldots, 10$) and B_2 are also calculated as follows:

$W\mathrm{Cot}_2(A_1, B_2) = 0.8133$, $W\mathrm{Cot}_2(A_2, B_2) = 0.8245$, $W\mathrm{Cot}_2(A_3, B_2) = 0.7991$, $W\mathrm{Cot}_2(A_4, B_2) = 0.8499$, $W\mathrm{Cot}_2(A_5, B_2) = 0.8623$, $W\mathrm{Cot}_2(A_6, B_2) = 0.8613$, $W\mathrm{Cot}_2(A_7, B_2) = 0.8189$, $W\mathrm{Cot}_2(A_8, B_2) = 0.8224$, $W\mathrm{Cot}_2(A_9, B_2) = 0.8733$, and $W\mathrm{Cot}_2(A_{10}, B_2) = 0.8269$.

Hence, the ranking order of all faults is $A_9 \to A_5 \to A_6 \to A_4 \to A_{10} \to A_2 \to A_8 \to A_7 \to A_1 \to A_3$.

According to above two kinds of fault ranking orders, we can see that the vibration fault of the turbine is firstly resulted from the looseness of bearing block (A_9), then either symbiosis looseness (A_6) and radial impact friction of rotor (A_5) or radial impact friction of rotor (A_5) and symbiosis looseness (A_6), and so on. By actual checking, we discover the friction between the rotor and cylinder body in the turbine, and then the vibration values of four ground bolts of the bearing between the turbine and the gearbox are very difference. We also discover that the gap between the nuts and the bearing block is oversize. Thus, the looseness of the bearing block causes the violent vibration of the turbine, which is in accordance with the actual fault.

Obviously, the results of all fault diagnoses based on the two cotangent similarity measures are the same as the actual faults of the turbine. The fault diagnosis results of the turbine show that the fault diagnosis method not only indicates the main fault types of the turbine, but also provides useful information for multi-fault analyses and fault trend.

5.3 Comparative analysis with the cosine similarity measure

To show the advantages and rationality of the fault diagnosis method of steam turbine proposed in this paper, we compare the proposed cotangent similarity measures with the cosine similarity measure proposed by Ye (2014c) in the fault diagnosis of steam turbine.

For the fault diagnosis of the testing sample B_1, the cosine similarity measures between A_i ($i = 1, 2, \ldots, 10$) and B_1 are calculated by Eq. (2) as follows:

$\mathrm{Cos}(A_1, B_1) = 0.7891$, $\mathrm{Cos}(A_2, B_1) = 0.9799$, $\mathrm{Cos}(A_3, B_1) = 0.8282$, $\mathrm{Cos}(A_4, B_1) = 0.8236$, $\mathrm{Cos}(A_5, B_1) = 0.9057$, $\mathrm{Cos}(A_6, B_1) = 0.8714$, $\mathrm{Cos}(A_7, B_1) = 0.9995$, $\mathrm{Cos}(A_8, B_1) = 0.9773$, $\mathrm{Cos}(A_9, B_1) = 0.7979$, and $\mathrm{Cos}(A_{10}, B_1) = 0.8099$.

Table 2 Fault knowledge with single-valued neutrosophic values

A_i (Fault knowledge)	Frequency range (f: operating frequency)				
	C_1 (0.01–0.39f)	C_2 (0.4–0.49f)	C_3 (0.5f)	C_4 (0.51–0.99f)	C_5 (f)
A_1 (unbalance)	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0.85, 0.15, 0>
A_2 (pneumatic force couple)	<0, 0, 1>	<0.28, 0.03, 0.69>	<0.09, 0.03, 0.88>	<0.55, 0.15, 0.3>	<0, 0, 1>
A_3 (offset center)	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0.30, 0.28, 0.42>
A_4 (oil-membrane oscillation)	<0.09, 0.02, 0.89>	<0.78, 0.04, 0.18>	<0, 0, 1>	<0.08, 0.03, 0.89>	<0, 0, 1>
A_5 (radial impact friction of rotor)	<0.09, 0.03, 0.88>	<0.09, 0.02, 0.89>	<0.08, 0.04, 0.88>	<0.09, 0.03, 0.88>	<0.18, 0.03, 0.79>
A_6 (symbiosis looseness)	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0.18, 0.04, 0.78>
A_7 (damage of antithrust bearing)	<0, 0, 1>	<0, 0, 1>	<0.08, 0.04, 0.88>	<0.86, 0.07, 0.07>	<0, 0, 1>
A_8 (surge)	<0, 0, 1>	<0.27, 0.05, 0.68>	<0.08, 0.04, 0.88>	<0.54, 0.08, 0.38>	<0, 0, 1>
A_9 (looseness of bearing block)	<0.85, 0.08, 0.07>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>
A_{10} (non-uniform bearing stiffness)	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>
	C_6(2f)	C_7 (3–5f)	C_8 (odd times of f)	C_9 (high frequency >5f)	
A_1 (unbalance)	<0.04, 0.02, 0.94>	<0.04, 0.03, 0.93>	<0, 0, 1>	<0, 0, 1>	
A_2 (pneumatic force couple)	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0.08, 0.05, 0.87>	
A_3 (offset center)	<0.40, 0.22, 0.38>	<0.08, 0.05, 0.87>	<0, 0, 1>	<0, 0, 1>	
A_4 (oil-membrane oscillation)	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	
A_5 (radial impact friction of rotor)	<0.08, 0.05, 0.87>	<0.08, 0.05, 0.87>	<0.08, 0.04, 0.88>	<0.08, 0.04, 0.88>	
A_6 (symbiosis looseness)	<0.12, 0.05, 0.83>	<0.37, 0.08, 0.55>	<0, 0, 1>	<0.22, 0.06, 0.72>	
A_7 (damage of antithrust bearing)	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	
A_8 (surge)	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	
A_9 (looseness of bearing block)	<0, 0, 1>	<0, 0, 1>	<0.08, 0.04, 0.88>	<0, 0, 1>	
A_{10} (non-uniform bearing stiffness)	<0.77, 0.06, 0.17>	<0.19, 0.04, 0.77>	<0, 0, 1>	<0, 0, 1>	

Therefore, the ranking order of all faults is $A_7 \rightarrow A_2 \rightarrow A_8 \rightarrow A_5 \rightarrow A_6 \rightarrow A_3 \rightarrow A_4 \rightarrow A_{10} \rightarrow A_9 \rightarrow A_1$.

For the fault diagnosis of the testing sample B_2, the cosine similarity measures between A_i ($i = 1, 2, \ldots, 10$) and B_2 are calculated by Eq. (2) as follows:

$\mathrm{Cos}(A_1, B_2) = 0.8568$, $\mathrm{Cos}(A_2, B_2) = 0.9128$, $\mathrm{Cos}(A_3, B_2) = 0.9066$, $\mathrm{Cos}(A_4, B_2) = 0.8953$, $\mathrm{Cos}(A_5, B_2) = 0.9738$, $Cos(A_6, B_2) = 0.9567$, $\mathrm{Cos}(A_7, B_2) = 0.8720$, $\mathrm{Cos}(A_8, B_2) = 0.9201$, $\mathrm{Cos}(A_9, B_2) = 0.9403$, and $\mathrm{Cos}(A_{10}, B_2) = 0.8938$.

Thus, the ranking order of all faults is $A_5 \rightarrow A_6 \rightarrow A_9 \rightarrow A_8 \rightarrow A_2 \rightarrow A_3 \rightarrow A_4 \rightarrow A_{10} \rightarrow A_7 \rightarrow A_1$.

By the comparison between the cotangent similarity measures and the cosine similarity measure in the fault diagnosis of the turbine, the main fault of the testing sample B_1 obtained by the cotangent and cosine similarity measures is the damage of antithrust bearing (A_7), which is in accordance with the actual fault; while for the testing sample B_2, the main faults obtained by the cotangent and cosine similarity measures indicate different diagnosis results. The main fault obtained based on the cosine similarity measure is the radial impact friction of rotor (A_5), which is not in accordance with the actual fault of the looseness of the bearing block, and then the main fault obtained based on the two cotangent similarity measures is the looseness of bearing block (A_9), which is in accordance with the actual fault. Therefore, the fault diagnosis results show that the cotangent similarity measures outperform the cosine similarity measure in Ye (2014c) and demonstrate the effectiveness and rationality in the fault diagnosis of steam turbine. As mentioned before, the advantage of the cotangent similarity measures is that they can overcome the drawbacks such as undefined and unreasonable phenomena, which the cosine similarity measure implies in some cases.

However, the comparative analysis demonstrates that the proposed cotangent similarity measures for the fault diagnosis of steam turbine not only are effective and reasonable, but also can overcome the drawbacks of the cosine similarity measure defined in vector space. Therefore, the cotangent similarity measures provide a new method for the fault diagnosis of steam turbine under a single valued neutrosophic environment.

6 Conclusion

This paper proposed two new cotangent similarity measures for SVNSs based on the cotangent function. Then, the weighted cotangent similarity measures were introduced by considering the importance of each element. By the comparison between the cotangent similarity measures and existing cosine similarity measure under single-valued neutrosophic environment, the developed two cotangent measures demonstrated their advantages and can overcome the drawbacks of the cosine similarity measure in some cases. Finally, the proposed cotangent similarity measures of SVNSs were applied to the fault diagnosis of steam turbine. The comparative analysis demonstrated the effectiveness and rationality of the proposed fault diagnosis method. The fault diagnosis method based on the cotangent similarity measures not only provides a new way for the fault diagnosis of steam turbine under a single-valued neutrosophic environment but also extends existing fault diagnosis methods for steam turbine.

In further work, it is necessary and meaningful to extend the cotangent similarity measures of SVNSs to interval neutrosophic cotangent measures and their applications, such as decision making, pattern recognition, and medical diagnosis.

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