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# SINGLE VALUED NEUTROSOPHIC TRAPEZOID LINGUISTIC AGGREGATION OPERATORS BASED MULTI-ATTRIBUTE DECISION MAKING

**Said Broumi[1] and Florentin Smarandache[2]**
[1] Faculty of Lettres and Humanities, Hay El Baraka Ben M'sik Casablanca B.P. 7951, University of Hassan II Casablanca , Morocco
Email: **broumisaid78@gmail.com**
[2]Department of Mathematics, University of New Mexico,705 Gurley Avenue, Gallup, NM 87301, USA
Email :**fsmarandache@gmail.com**

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**ABSTRACT**

Multi-attribute decision making (MADM). Play an important role in many applications, due to the efficiency to handle indeterminate and inconsistent information, single valued neutrosophic sets is widely used to model indeterminate information. In this paper, a new MADM method based on neutrosophic trapezoid linguistic weighted arithmetic averaging aggregation SVNTrLWAA operator and neutrosophic trapezoid linguistic weighted geometric aggregation SVNTrLWGA operator is presented. A numerical example is presented to demonstrate the application and efficiency of the proposed method.

**Keywords:** Single valued neutrosophic trapezoid linguistic weighted arithmetic averaging aggregation (SVNTrLWAA) operator, neutrosophic trapezoid linguistic weighted weighted geometric aggregation (SVNTrLWGA) operator, single valued neutrosophic sets.

## 1.INTRODUCTION

F. Smarandache [6] proposed the neutrosophic set (NS) by adding an independent indeterminacy-membership function. The concept of neutrosophic set is generalization of classic set, fuzzy set [26], intuitionistic fuzzy set [22], interval intuitionistic fuzzy set [24,25] and so on. In NS, the indeterminacy is quantified explicitly and truth-membership, indeterminacy membership, and false-membership are completely independent. From scientific or engineering point of view, the neutrosophic set and set-theoretic view, operators need to be specified. Otherwise, it will be difficult to apply in the real applications. Therefore, H. Wang et al [7] defined a single valued neutrosophic set (SVNS) and then provided the set theoretic operations and various properties of single valued neutrosophic sets. Furthermore, H. Wang et al.[8] proposed the set theoretic operations on an instance of neutrosophic set called interval valued neutrosophic set (IVNS) which is more flexible and practical than NS. The works on neutrosophic set (NS) and interval valued neutrosophic set (IVNS), in theories and application have been progressing rapidly (e.g, [1,2,3,4,5,7,9,10,11,12,13,14,15,16,17, 21,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49].

Multiple attribute decision making (MADM) problem are of importance in most kinds of fields such as engineering, economics, and management. In many situations decision makers have incomplete , indeterminate and inconsistent information about alternatives with respect to attributes. It is well known that the conventional and fuzzy or intuitionistic fuzzy decision making analysis [27, 50, 51, 52] using different techniques tools have been found to be inadequate to handle indeterminate an inconsistent data. So ,Recently, neutrosophic multicriteria decision making problems have been proposed to deal with such situation.

In addition, because the aggregation operators are the important tools to process the neutrosophic decision making problems. Lately, research on aggregation methods and multiple attribute decision making theories under neutrosophic environment is very active and lot of results have been obtained from neutrosophic information. Based on the aggregation operators, J. Ye [20] developed some new weighted arithmetic averaging and weighted geometric averaging operators for simplified neutrosophic sets. P. Liu [28] present the generalized neutrosophic Hamacher aggregation operators such as Generalized neutrosophic number Hamacher weighted averaging (GNNHWA) operator, Generalized neutrosophic number Hamacher ordered weighted averaging (GNNHOWA) operator, and Generalized neutrosophic number Hamacher hybrid averaging (GNNHA) operator and studied some properties of these operators and analyzed some special cases and gave a decision-making method based on these operators for multiple attribute group decision making with neutrosophic numbers. Based on the idea of Bonferroni mean, P. Liu [32] proposed some Bonferroni mean operators such a s the single-valued neutrosophic normalized weighted Bonferroni mean. J. J. Peng et al [22 ] defined the novel operations and aggregation operators, which were based on the operations in J. Ye [20].

Based on the linguistic variable and the concept of interval neutrosophic sets, J. Ye [18] defined interval neutrosophic linguistic variable, as well as its operation principles, and developed some new aggregation operators for the interval neutrosophic linguistic information, including interval neutrosophic linguistic arithmetic weighted average(INLAWA) operator, linguistic geometric weighted average(INLGWA) operator and discuss some properties. Furthermore, he proposed the decision making method for multiple attribute decision making (MAGDM) problems with an illustrated example to show the process of decision making and the effectiveness of the proposed method.

In order to deal with the more complex neutrosophic information. J. Ye [19] ,further proposed the interval neutrosophic uncertain linguistic variables by extending an uncertain linguistic variables with an interval neutrosophic set, and proposed the operational rules, score function, accuracy function and certainty function of interval neutrosophic uncertain linguistic variables. Then, the interval neutrosophic uncertain linguistic weighted arithmetic averaging operator and interval neutrosophic uncertain linguistic weighted geometric averaging operator are developed, and a multiple attribute decision making method with interval neutrosophic linguistic information is proposed.

To the our knowledge, The existing approaches under the neutrosophic linguistic environment are not suitable for dealing with MADM problems under single valued neutrosophic trapezoid linguistic environment. Indeed, human judgments including preference information may be stated by possible trapezoid linguistic variable which has a membership ,indeterminacy and non-membership degree. Therefore, it is necessary to pay enough attention on this issue and propose more appropriate methods for dealing with MADM, which is also our motivation. Based on Trapezoid linguistic terms and the single valued neutrosophic sets, in this paper, we define a new concept called single valued neutrosophic trapezoid linguistic variable, then propose score function and and some new aggregation operators, and an approach for dealing with single valued neutrosophic trapezoid linguistic environment in the MADM process. The main advantage of the SVNTrLS is that is composed of trapezoid linguistic term, which is generalization case of SVINLS, a special case of INLS, proposed by J. Ye [18].

In order to process incomplete, indeterminate and inconsistent information more efficiency and precisely, it is necessary to make a further study on the extended form of the single valued neutrosophic uncertain linguistic variables by combining trapezoid fuzzy linguistic variables and single valued neutrosophic set. For example, we can evaluate the investment alternatives problem by the linguistic set: S={$s_0$(extremely low); $s_1$(very low); $s_2$(low); $s_3$(medium); $s_4$(high); $s_5$(very high); $s_6$(extermley high)}.Perhaps, we can use the trapezoid fuzzy linguistic [$s_\theta$, $s_\rho$,$s_\mu$, $s_\nu$], ($0 \le \theta \le \rho \le \mu \le \nu \le l-1$) to describe the evaluation result, but this is not accurate, because it merely provides a linguistic range. In this paper , we can use single valued neutrosophic trapezoid linguistic (SVNNTrL), $[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}],(T_A(x), I_A(x), F_A(x))$ to describe the investment problem giving the membership degree, indeterminacy degree, and non-membership degree to $[s_\theta, s_\rho, s_\mu, s_\nu]$. This is the motivation of our study .As a fact, SVNTrL avoids the information distortions and losing in decision making process, and overcomes the shortcomings of the single valued neutrosophic linguistic variables [18] and single valued neutrosophic uncertain linguistic variables [19 ].

To achieve the above purposes, The remainder of this paper is organized as follows: some basic definitions of trapezoid linguistic term set, neutrosophic set, single valued neutrosophic set and single valued neutrosophic uncertain linguistic set are briefly reviewed in section 2. In section3, the concept, operational laws, score function, accuracy function and certainty function of including single valued neutrosophic trapezoid linguistic elements are defined. In Section 4, some single valued neutrosophic trapezoid linguistic aggregation operators are proposed, such single valued neutrosophic trapezoid linguistic weighted average (SVNTrLWAA) operator, single valued neutrosophic trapezoid linguistic weighted average (SVNTrLWGA) operators , then some desirable properties of the proposed operators are investigated. In section 5, we develop an approach for multiple attribute decision making problems with single valued neutrosophic trapezoid linguistic information based on the proposed operators. In section 6, a numerical example is given to illustrate the application of the proposed method. The paper is concluded in section 7.

## 2-PRELIMINARIES

In the following, we shall introduce some basic concepts related to trapezoidal fuzzy linguistic variables, single valued neutrosophic set , single valued neutrosophic linguistic sets and single valued neutrosophic uncertain linguistic sets.

### *2.1 Trapezoid fuzzy linguistic variables*

A linguistic set is defined as a finite and completely ordered discreet term set,

$S$=($s_0$, $s_1$,…, $s_{l-1}$), where l is the odd value. For example, when l=7,the linguistic term set S can be defined as follows: S={$s_0$(extremely low); $s_1$(very low); $s_2$(low); $s_3$(medium); $s_4$(high); $s_5$(very high); $s_6$(extermley high)}

**Definition 2.1 :[49]**

**Let** $\bar{S}$ = {$s_\theta$| $s_0 \leq s_\theta \leq s_{l-1}$ $\theta$ ∈[0, $l$ -1]}, which is the continuous form of linguistic set S. $s_\theta, s_\rho, s_\mu, s_v$ are four linguistic terms in , and $s_0 \leq s_\theta \leq s_\rho \leq s_\mu \leq s_v \leq s_{l-1}$ if $0 \leq \theta \leq \rho \leq \mu \leq v \leq l-1$, then the trapezoid linguistic variable is defined as $\hat{s}$= [$s_\theta, s_\rho, s_\mu, s_v$], and $\hat{S}$ denotes a set of the trapezoid linguistic variables.

In particular , if any two of $s_\theta, s_\rho, s_\mu, s_\nu$ are equal, then $\hat{s}$ is reduced to triangular fuzzy linguistic variable; if any three of $s_\theta, s_\rho, s_\mu, s_\nu$ are equal, then $\hat{s}$ is reduced to uncertain linguistic variable

### *2.2 The expected value of trapezoid fuzzy linguistic variable*

Let $\hat{s}$= ([ $s_\theta$ , $s_\rho$ , $s_\mu$ , $s_\nu$ ]) be a trapezoid fuzzy linguistic variable, then the expected value E($\hat{s}$) of $\hat{s}$ is defined as:

$$E(\hat{s})=\frac{\theta+\rho+\mu+\nu}{4}$$

### *2.3 Neutrosophic sets*

**Definition 2.2 [7]**
Let U be a universe of discourse then the neutrosophic set A is an object having the form
A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$ >, x ∈ X },

Where the functions $T_A(x)$, $I_A(x)$, $F_A(x)$ : U→]$^-$0,1+[define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$$^-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+.$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of ]$^-$0,1$^+$[. So instead of ]$^-$0,1$^+$[ we need to take the interval [0,1] for technical applications, because ]$^-$0,1$^+$[will be difficult to apply in the real applications such as in scientific and engineering problems.

### *2.4 Single valued Neutrosophic Sets*

**Definition 2.3 [7]**
Let X be an universe of discourse then the neutrosophic set A is an object having the form
A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$>, x ∈ X },
where the functions $T_A(x)$, $I_A(x)$, $F_A(x)$ :U→[0,1] define respectively the degree of membership , the degree of indeterminacy, and the degree of non-membership of the element x ∈ X to the set A with the condition.

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

**Definition 2.4 [7]**
A single valued neutrosophic set A is contained in another single valued neutrosophic set B i.e. A ⊆ B if ∀x ∈ U, $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$

Based on interval neutrosophic set and linguistic variables, J. Ye [18] presented the extension form of the linguistic set, i.e, interval neutrosophic linguistic set. The interval neutrosophic linguistic set is reduced to single valued neutrosophic linguistic sets if the components $T_A^L(x) = T_A^U(x) = T_A(x)$, $I_A^L(x) = I_A^U(x) = I_A(x)$ and $F_A^L(x) = F_A^U(x) = F_A(x)$ and is defined as follows as follows:

***2.5 Single valued neutrosophic linguistic set***

Based on single valued neutrosophic set and linguistic variables, Ye [18] presented the extension form of the linguistic set, i.e., single valued neutrosphic linguistic set, which is shown as follows:

**Definition 2.5: [18]** A single valued neutrosophic linguistic set A in X can be defined as

A ={<x, $s_{\theta(x)}$, $(T_A(x), I_A(x), F_A(x))$>| x $\in$ X}

Where $s_{\theta(x)} \in \hat{S}$, $T_A(x) \subseteq [0.1]$, $I_A(x) \subseteq [0.1]$, and $F_A(x) \subseteq [0.1]$ with the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for any x $\in$ X. The function $T_A(x), I_A(x)$ and $F_A(x)$ express, respectively, the truth-membership degree, the indeterminacy –membership degree, and the falsity-membership degree with values of the element x in X to the linguistic variable $s_{\theta(x)}$.

Also. Based on interval neutrosophic set and linguistic variables, J. Ye [19] presented the extension form of the uncertain linguistic set, i.e., interval neutrosophic uncertain linguistic set. The interval neutrosophic uncertain linguistic set is reduced to single valued neutrosophic uncertain linguistic sets if the components $T_A^L(x) = T_A^U(x) = T_A(x)$, $I_A^L(x) = I_A^U(x) = I_A(x)$ and $F_A^L(x) = F_A^U(x) = F_A(x)$ and is defined as follows:

**2.6 Single valued neutrosophic uncertain linguistic set.**

**Definition2.6:[19]** A single valued neutrosophic uncertain linguistic set A in X can be defined as

A ={<x, $[s_{\theta(x)}, s_{\rho(x)}]$,$(T_A(x), I_A(x), F_A(x))$>: x $\in$ X}

Where $s_{\theta(x)}, s_{\rho(x)} \in \hat{S}$, $T_A(x) \in [0.1]$, $I_A(x) \in [0.1]$, and $F_A(x) \in [0.1]$ with the condition $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for any x $\in$ X. $[s_{\theta(x)}, s_{\rho(x)}]$ is an uncertain linguistic term, The function $T_A(x), I_A(x)$ and $F_A(x)$ express, respectively, the truth-membership degree , the indeterminacy –membership degree, and the falsity-membership degree of the element x in X belonging to the linguistic term $[s_{\theta(x)}, s_{\rho(x)}]$.

**Definition 2.7** Let A ={<x, $[s_{\theta(x)}, s_{\rho(x)}]$, $(T_A(x), I_A(x), F_A(x))$ >: x $\in$ X} be a SVNULN. Then the eight tuple < $[s_{\theta(x)}, s_{\rho(x)}]$,$(T_A(x), I_A(x), F_A(x))$) > is called an NULV and A can be viewed as a collection of NULVs. Thus, the SVNULs can also be expressed as

A ={<x, $[s_{\theta(x)}, s_{\rho(x)}]$ , $(T_A(x), I_A(x), F_A(x))$) >: x $\in$ X}

For any two SVNULVNs $\tilde{a}_1$=< $[s_{\theta(\tilde{a}_1)}, s_{\rho(\tilde{a}_1)}]$, $(T(\tilde{a}_1), I(\tilde{a}_1), F(\tilde{a}_1))$> and $\tilde{a}_2$ = < $[s_{\theta(\tilde{a}_2)}, s_{\rho(\tilde{a}_2)}]$, $(T(\tilde{a}_2), I(\tilde{a}_2), F(\tilde{a}_2))$> and $\lambda \geq 0$, defined the following operational rules:

$\tilde{a}_1 \oplus \tilde{a}_2$ =< $[s_{\theta(\tilde{a}_1)+\theta(\tilde{a}_2)}, s_{\rho(\tilde{a}_1)+\rho(\tilde{a}_2)}]$,$((T(\tilde{a}_1)+T(\tilde{a}_2)-T(\tilde{a}_1)\,T(\tilde{a}_2)),\ I(\tilde{a}_1)\,I(\tilde{a}_2),\ F(\tilde{a}_1)\,F(\tilde{a}_2))$>

$\tilde{a}_1 \otimes \tilde{a}_2$=< $[s_{\theta(\tilde{a}_1)\times\theta(\tilde{a}_2)}, s_{\rho(\tilde{a}_1)\times\rho(\tilde{a}_2)}]$, $(T(\tilde{a}_1)\,T(\tilde{a}_2), (I(\tilde{a}_1)+I(\tilde{a}_2)-I(\tilde{a}_1)\,I(\tilde{a}_2)), (F(\tilde{a}_1)+F(\tilde{a}_2)$ - $F(\tilde{a}_1)\,F(\tilde{a}_2))$>

$\lambda\tilde{a}_1$ =<$[s_{\lambda\theta(\tilde{a}_1)}, s_{\lambda\rho(\tilde{a}_1)}]$, $(1-(1-T(\tilde{a}_1))^{\lambda}, (I(\tilde{a}_1))^{\lambda}, (F(\tilde{a}_1))^{\lambda})$>

$\tilde{a}_1^{\lambda}$ =<$[s_{\theta^{\lambda}(\tilde{a}_1)}, s_{\rho^{\lambda}(\tilde{a}_1)}]$, $((T(\tilde{a}_1))^{\lambda}, (1-(1-I(\tilde{a}_1))^{\lambda}), (1-(1-F(\tilde{a}_1))^{\lambda})$>

**Definition 2.8** Let $\tilde{a}_i$ =< $[s_{\theta(\tilde{a}_i)}, s_{\rho(\tilde{a}_i)}]$, $(T(\tilde{a}_i), I(\tilde{a}_i), F(\tilde{a}_i))$> be a SVNULN, the expected function E($\tilde{a}_i$), the accuracy H($\tilde{a}_i$) and the certainty C($\tilde{a}_i$) are define as follows:

$$E(\tilde{a}) = \frac{1}{3}(2+T(\tilde{a})-I(\tilde{a})-F(\tilde{a}))\times S_{\frac{(\theta(\tilde{a})+\rho(\tilde{a}))}{2}}$$

$$= S_{\frac{1}{6}(2+T(\tilde{a})-I(\tilde{a})-F(\tilde{a}))\times(\theta(\tilde{a})+\rho(\tilde{a}))}$$

$$H(\tilde{a}) = (T(\tilde{a})-F(\tilde{a}))\times S_{\frac{(\theta(\tilde{a})+\rho(\tilde{a}))}{2}}$$

$$= S_{\frac{1}{2}(T(\tilde{a})-F(\tilde{a}))\times(\theta(\tilde{a})+\rho(\tilde{a}))}$$

$$C(\tilde{a}) = (T(\tilde{a}))\times S_{\frac{(\theta(\tilde{a})+\rho(\tilde{a}))}{2}}$$

$$= S_{\frac{1}{2}(T(\tilde{a}))\times(\theta(\tilde{a})+\rho(\tilde{a}))}$$

Assume that $\tilde{a}_i$ and $\tilde{a}_j$ are two SVNULNs, they can be compared by the following rules:

1.If E ($\tilde{a}_i$) > E ($\tilde{a}_j$), then $\tilde{a}_i > \tilde{a}_j$;

2.If E ($\tilde{a}_i$) = E ($\tilde{a}_j$), then

If H ($\tilde{a}_i$) > H ($\tilde{a}_j$), then $\tilde{a}_i > \tilde{a}_j$,

If H ($\tilde{a}_i$) = H ($\tilde{a}_j$), then $\tilde{a}_i = \tilde{a}_j$,

H ($\tilde{a}_i$) < H ($\tilde{a}_j$), then $\tilde{a}_i < \tilde{a}_j$,

## 3- SINGLE VALUED NEUTROSOPHIC TRAPEZOID LINGUISTIC SETS.

Based on the concept of SVNS and trapezoid linguistic variable, we extend the SVNLS to define the SVNTrLS and SVNTrLNs. The operations and ranking method of SVNTrLNs are also given in this section

**Definition 3.1** Let X be a finite universal set and $[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}] \in \hat{s}$ be trapezoid linguistic variable. A SVNTrLs in X is defined as

A ={<x, $[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}]$, $(T_A(x),\ I_A(x),\ F_A(x))$>| x $\in$ X}

Where $s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)} \in \hat{s}$, $T_A(x) \in [0.1]$, $I_A(x) \in [0.1]$, and $F_A(x) \in [0.1]$ with the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for any x $\in$X. $[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}]$ is a trapezoid linguistic term, The function $T_A(x)$, $I_A(x)$ and $F_A(x)$ express, respectively, the truth-membership degree , the indeterminacy –membership degree, and the falsity-membership degree of the element x in X belonging to the linguistic term $[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}]$.

**Definition 3.2** Let A ={<x, $[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}]$,$(T_A(x),\ I_A(x),\ F_A(x))$ >: x $\in$ X} be an SVNTrLN. Then the eight tuple < $[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}]$, $(T_A(x),\ I_A(x),\ F_A(x))$ > is called an SVNTrLV and A can be viewed as a collection of SVNTrLV s. Thus, the SVNTrLVs can also be expressed as

A ={<x, $[s_{\theta(x)}, s_{\rho(x)}, s_{\mu(x)}, s_{\nu(x)}]$, $(T_A(x), I_A(x), F_A(x))$ >: x $\in$ X}$[s_{\theta(\tilde{a}_1)}, s_{\rho(\tilde{a}_1)}, s_{\mu(\tilde{a}_1)}, s_{\nu(\tilde{a}_1)}]$

**Definition 3.3** Let $\tilde{a}_1$=< $[s_{\theta(\tilde{a}_1)}, s_{\rho(\tilde{a}_1)}, s_{\mu(\tilde{a}_1)}, s_{\nu(\tilde{a}_1)}]$, $(T(\tilde{a}_1), I(\tilde{a}_1), F(\tilde{a}_2))$> and $\tilde{a}_2$={<x, $[s_{\theta(\tilde{a}_2)}, s_{\rho(\tilde{a}_2)}, s_{\mu(\tilde{a}_2)}, s_{\nu(\tilde{a}_2)}]$, $(T(\tilde{a}_2), I(\tilde{a}_2), F(\tilde{a}_2))$> be two SVNTrLVs and $\lambda \geq 0$,then the operational laws of SVNTrLVs are defined as follows:

1. $\tilde{a}_1 \oplus \tilde{a}_2$ =< $[s_{\theta(\tilde{a}_1)+\theta(\tilde{a}_2)}, s_{\rho(\tilde{a}_1)+\rho(\tilde{a}_2)}, s_{\mu(\tilde{a}_1)+\mu(\tilde{a}_2)}, s_{\nu(\tilde{a}_1)+\nu(\tilde{a}_2)}]$, $((T(\tilde{a}_1)+T(\tilde{a}_2) -$ $T(\tilde{a}_1)\,T(\tilde{a}_2)), I(\tilde{a}_1)\ I(\tilde{a}_2),\ \ F(\tilde{a}_1)\ F(\tilde{a}_2))$>

2. $\tilde{a}_1 \otimes \tilde{a}_2$ =< [ $s_{\theta(\tilde{a}_1)\times\theta(\tilde{a}_2)}$ , $s_{\rho(\tilde{a}_1)\times\rho(\tilde{a}_2)}$ , $s_{\mu(\tilde{a}_1)\times\mu(\tilde{a}_2)}$ , $s_{\nu(\tilde{a}_1)\times\nu(\tilde{a}_2)}$ ], ( $T(\tilde{a}_1)\ T(\tilde{a}_2)$ , ( $I(\tilde{a}_1)+I(\tilde{a}_2)$ - $I(\tilde{a}_1)\ I(\tilde{a}_2)$ ), ( $F(\tilde{a}_1)+F(\tilde{a}_2)$ - $F(\tilde{a}_1)\ F(\tilde{a}_2)$ )>

3. $\lambda\tilde{a}_1$ =<[ $s_{\lambda\theta(\tilde{a}_1)}$ , $s_{\lambda\rho(\tilde{a}_1)}$ , $s_{\lambda\mu(\tilde{a}_1)}$ , $s_{\lambda\nu(\tilde{a}_1)}$ ],((1- $(1-T(\tilde{a}_1))^{\lambda}$ , $(I(\tilde{a}_1))^{\lambda}$ , $(F(\tilde{a}_1))^{\lambda}$ )>

4. $\tilde{a}_1^{\lambda}$ =<[ $s_{\theta^{\lambda}(\tilde{a}_1)}$ , $s_{\rho^{\lambda}(\tilde{a}_1)}$ , $s_{\mu^{\lambda}(\tilde{a}_1)}$ , $s_{\nu^{\lambda}(\tilde{a}_1)}$ ] , ( $(T(\tilde{a}_1))^{\lambda}$ , (1- $(1-I(\tilde{a}_1))^{\lambda}$ ), (1- $(1-F(\tilde{a}_1))^{\lambda}$ )>

Obviously, the above operational results are still SVNTrLVs.

**Theorem3.4:** Let $\tilde{a}_1$ =<[ $s_{\theta(\tilde{a}_1)}$ , $s_{\rho(\tilde{a}_1)}$ , $s_{\mu(\tilde{a}_1)}$ , $s_{\nu(\tilde{a}_1)}$ ], ( $T(\tilde{a}_1)$ , $I(\tilde{a}_1)$ , $F(\tilde{a}_2)$ )> and $\tilde{a}_2$ =<[ $s_{\theta(\tilde{a}_2)}$ , $s_{\rho(\tilde{a}_2)}$ , $s_{\mu(\tilde{a}_2)}$ , $s_{\nu(\tilde{a}_2)}$ ],( $T(\tilde{a}_2)$ , $I(\tilde{a}_2)$ , $F(\tilde{a}_2)$ )> be any two single valued neutrosophic trapezoid linguistic variables, and $\lambda$ , $\lambda_1$, $\lambda_2$ ≥0, then the characteristics of single valued neutrosophic trapezoid linguistic variables are shown as follows:

1. $\tilde{a}_1 \oplus \tilde{a}_2 = \tilde{a}_2 \oplus \tilde{a}_1$
2. $\tilde{a}_1 \otimes \tilde{a}_2 = \tilde{a}_2 \otimes \tilde{a}_1$
3. $\lambda(\tilde{a}_1 \oplus \tilde{a}_2) = \lambda\tilde{a}_1 \oplus \lambda\tilde{a}_2$
4. $\lambda\tilde{a}_1 \oplus \lambda\tilde{a}_2 = (\lambda_1+\lambda_2)\,\tilde{a}_1$;
5. $\tilde{a}_1^{\lambda_1} \otimes \tilde{a}_1^{\lambda_2} = \tilde{a}_1^{\lambda_1+\lambda_2}$;
6. $\tilde{a}_1^{\lambda_1} \otimes \tilde{a}_2^{\lambda_1} = (\tilde{a}_1 \otimes \tilde{a}_2)^{\lambda_1}$

Theorem 3.4 can be easily proven according to **definition 3.3** (omitted).

To rank SVNTrLNs, we define the score function, accuracy function and certainty function of an SVNTrLN based on [7, 49], which are important indexes for ranking alternatives in decision-making problems.

**Definition 3.5.** $\tilde{a}$ =<[ $s_{\theta(\tilde{a})}$ , $s_{\rho(\tilde{a})}$ , $s_{\mu(\tilde{a})}$ , $s_{\nu(\tilde{a})}$ ], ( $T(\tilde{a})$ , $I(\tilde{a})$ , $F(\tilde{a})$ )> be a SVNTrLV. Then, the score function, accuracy function and certainty function of a SVNTrLN $\tilde{a}$ are defined, respectively, as follows:

$$E(\tilde{a}) = \frac{1}{3}(2+T(\tilde{a})-I(\tilde{a})-F(\tilde{a}))\times S_{\frac{(\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a}))}{4}}$$

$$= S_{\frac{1}{12}(2+T(\tilde{a})-I(\tilde{a})-F(\tilde{a}))\times(\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a}))} \qquad (1)$$

$$\text{H}(\tilde{a}) = (T(\tilde{a}) - F(\tilde{a})) \times S_{\frac{(\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a}))}{4}}$$

$$= S_{\frac{1}{4}(T(\tilde{a})-F(\tilde{a}))\times(\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a}))} \quad (2)$$

$$\text{C}(\tilde{a}) = (T(\tilde{a})) \times S_{\frac{(\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a}))}{4}}$$

$$= S_{\frac{1}{4}(T(\tilde{a}))\times(\theta(\tilde{a})+\rho(\tilde{a})+\mu(\tilde{a})+\nu(\tilde{a}))} \quad (3)$$

Based on definition 3.5, a ranking method between SVNTrLVs can be given as follows.

**Definition 3.6** Let $\tilde{a}_1$ and $\tilde{a}_2$ be two SVNTrLNs. Then, the ranking method can be defined as follows:

If E ($\tilde{a}_1$) > E ($\tilde{a}_2$), then $\tilde{a}_1 > \tilde{a}_2$

If E ($\tilde{a}_1$) = E ($\tilde{a}_2$) and H ($\tilde{a}_1$) > H ($\tilde{a}_2$), then $\tilde{a}_1 > \tilde{a}_2$,

If E($\tilde{a}_1$) = E($\tilde{a}_2$) and H($\tilde{a}_1$) = H($\tilde{a}_2$) and C($\tilde{a}_1$) > C($\tilde{a}_2$),then $\tilde{a}_1 > \tilde{a}_2$,

If E($\tilde{a}_1$) = E($\tilde{a}_2$) and H($\tilde{a}_1$) = H($\tilde{a}_2$) and C($\tilde{a}_1$) = C($\tilde{a}_2$),then $\tilde{a}_1 = \tilde{a}_2$,

## 4. SINGLE VALUED NEUTROSOPHIC TRAPEZOID LINGUISTIC AGGREGATION OPERATORS

Based on the operational laws in definition 3.3, we can propose the following weighted arithmetic aggregation operator and weighted geometric aggregation operator for SVNTrLNs, which are usually utilized in decision making.

*4.1 Single valued neutrosophic trapezoid linguistic weighted arithmetic Averaging operator***.**

**Definition 4.1.** Let $\tilde{a}_j$ =<[ $s_{\theta(a_j)}$, $s_{\rho(a_j)}$, $s_{\mu(a_j)}$, $s_{\nu(a_j)}$ ], ($T_{a_j}$, $I_{a_j}$, $F_{a_j}$)> (j=1,2,…,n) be a collection of SVNTrLNs. The single valued neutrosophic trapezoid linguistic weighted arithmetic **averaging** average SVNTrLWAA operator can be defined as follows and

SVNTrLWAA: $\Omega^n \longrightarrow \Omega$

SVNTrLWAA ($\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n$) = $\sum_{j=1}^{n} \omega_j \tilde{a}_j$ (4)

Where,$\omega_j = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weight vector of $\tilde{a}_j$ (j= 1,2,…,n), $\omega_j \in [0,1]$and $\sum_{j=1}^{n}\omega_j$ =1.

**Theorem 4.2 :** $\tilde{a}_j$ =<[ $s_{\theta(\tilde{a}_j)}, s_{\rho(\tilde{a}_j)}, s_{\mu(\tilde{a}_j)}, s_{\nu(\tilde{a}_j)}$ ], ($T_{\tilde{a}_j}, I_{\tilde{a}_j}, F_{\tilde{a}_j}$)> (j=1,2,…,n) be a collection of SVNTrLNs, Then by Equation (4) and the operational laws in Definition 3.3 , we have the following result

SVNTrLWAA ($\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n$)= <[ $s_{\sum_{j=1}^{n}\omega_j\theta(\tilde{a}_j)}$, $s_{\sum_{j=1}^{n}\omega_j\rho(\tilde{a}_j)}$, $s_{\sum_{j=1}^{n}\omega_j\mu(\tilde{a}_j)}$, $s_{\sum_{j=1}^{n}\omega_j\nu(\tilde{a}_j)}$ ], (1-

$\prod_{j=1}^{n}\left(1-T(\tilde{a}_j)\right)^{\omega_j}$, $\prod_{j=1}^{n}\left(I(\tilde{a}_j)\right)^{\omega_j}$, $\prod_{j=1}^{n}\left(F(\tilde{a}_j)\right)^{\omega_j}$ > (5)

Where, $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weight vector of $\tilde{a}_j$ (j= 1,2,…,n), $\omega_j \in [0,1]$ and $\sum_{j=1}^{n}\omega_j$ =1.

**Proof**

The proof of Eq.(5) can be done by means of mathematical induction

(1) When n=2, then

$\omega_1 \tilde{a}_1$ = <[ $s_{\omega_1\theta(\tilde{a}_1)}, s_{\omega_1\rho(\tilde{a}_1)}, s_{\omega_1\mu(\tilde{a}_1)}, s_{\omega_1\nu(\tilde{a}_1)}$ ], (1-$\left(1-T(\tilde{a}_1)\right)^{\omega_1}$, $\left(I(\tilde{a}_1)\right)^{\omega_1}$, $\left(F(\tilde{a}_1)\right)^{\omega_1}$ >

$\omega_2 \tilde{a}_2$ = <[ $s_{\omega_1\theta(\tilde{a}_2)}, s_{\omega_2\rho(\tilde{a}_2)}, s_{\omega_2\mu(\tilde{a}_1)}, s_{\omega_2\nu(\tilde{a}_2)}$ ], (1-$\left(1-T(\tilde{a}_2)\right)^{\omega_2}$, $\left(I(\tilde{a}_2)\right)^{\omega_2}$, $\left(F(\tilde{a}_2)\right)^{\omega 2}$ >

Thus,

SVNTrLWAA ($\tilde{a}_1, \tilde{a}_2$) = $\omega_1 \tilde{a}_1 \oplus \omega_2 \tilde{a}_2$

=< [ $s_{\sum_{j=1}^{2}\omega_j\theta(\tilde{a}_j)}$, $s_{\sum_{j=1}^{2}\omega_j\rho(\tilde{a}_j)}$, $s_{\sum_{j=1}^{2}\omega_j\mu(\tilde{a}_j)}$, $s_{\sum_{j=1}^{2}\omega_j\nu(\tilde{a}_j)}$ ], ( (1-$(1-T(\tilde{a}_1))^{\omega_1}$ +1-$(1-T(\tilde{a}_2))^{\omega_2}$ – (1-

$(1-T(\tilde{a}_1))^{\omega_1}$ ) (1-$(1-T(\tilde{a}_2))^{\omega_2}$ ), $(I(\tilde{a}_1))^{\omega_1}$ $(I(\tilde{a}_2))^{\omega_2}$, $(F(\tilde{a}_1))^{\omega_1}$ $(F(\tilde{a}_2))^{\omega_2}$ >

= <[ $s_{\sum_{j=1}^{2}\omega_j\theta(\tilde{a}_j)}$, $s_{\sum_{j=1}^{2}\omega_j\rho(\tilde{a}_j)}$, $s_{\sum_{j=1}^{2}\omega_j\mu(\tilde{a}_j)}$, $s_{\sum_{j=1}^{2}\omega_j\nu(\tilde{a}_j)}$ ], ((1-$(1-T(\tilde{a}_1))^{\omega_1}$ ) (1-$(1-T(\tilde{a}_2))^{\omega_2}$ ) ,

$\prod_{j=1}^{2}\left(F(\tilde{a}_j)\right)^{\omega_j}$, $\prod_{j=1}^{2}\left(F(\tilde{a}_j)\right)^{\omega_j}$ > (6)

(2) When n=k, by applying Eq.(5) , we get

SVNTrLWAA ($\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_k$)=<[ $s_{\sum_{j=1}^{k}\omega_j\theta(\tilde{a}_j)}$, $s_{\sum_{j=1}^{k}\omega_j\rho(\tilde{a}_j)}$, $s_{\sum_{j=1}^{k}\omega_j\mu(\tilde{a}_j)}$, $s_{\sum_{j=1}^{k}\omega_j\nu(\tilde{a}_j)}$ ], (1-

$\prod_{j=1}^{k}(1-T(\tilde{a}_j))^{\omega_j}$, $\prod_{j=1}^{k}(I(\tilde{a}_j))^{\omega_j}$, $\prod_{j=1}^{k}(F(\tilde{a}_j))^{\omega_j}$ )> (7)

(3) When n=k+1, by applying Eq.(6) and Eq.(7) , we can get

SVNTrLWAA ($\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_k, \tilde{a}_{k+1}$) =

<[ $s_{\sum_{j=1}^{k+1}\omega_j\theta(\tilde{a}_j)+\omega_{k+1}\theta(\tilde{a}_{k+1})}$, $s_{\sum_{j=1}^{k+1}\omega_j\rho(\tilde{a}_j)+\omega_{k+1}\rho(\tilde{a}_{k+1})}$, $s_{\sum_{j=1}^{k+1}\omega_j\mu(\tilde{a}_j)+\omega_{k+1}\mu(\tilde{a}_{k+1})}$, $s_{\sum_{j=1}^{k+1}\omega_j\nu(\tilde{a}_j)+\omega_{k+1}\nu(\tilde{a}_{k+1})}$ ],([1-

$\prod_{j=1}^{k}(1-T(\tilde{a}_j))^{\omega_j}$ +1- $(1-T(\tilde{a}_{k+1}))^{\omega_{k+1}}$ –(1- $\prod_{j=1}^{k}(1-T(\tilde{a}_j))^{\omega_j}$ ) (1- $\prod_{j=1}^{k}(1-T(\tilde{a}_{k+1}))^{\omega_{k+1}}$ ),

$\prod_{j=1}^{k+1}(I(\tilde{a}_j))^{\omega_j}$ , $\prod_{j=1}^{k+1}(F(\tilde{a}_j))^{\omega_j}$ >

=<[ $s_{\sum_{j=1}^{k+1}\omega_j\theta(\tilde{a}_j)}$, $s_{\sum_{j=1}^{k+1}\omega_j\rho(\tilde{a}_j)}$, $s_{\sum_{j=1}^{k+1}\omega_j\mu(\tilde{a}_j)}$, $s_{\sum_{j=1}^{k+1}\omega_j\nu(\tilde{a}_j)}$ ], (1- $\prod_{j=1}^{k+1}(1-T(\tilde{a}_j))^{\omega_j}$, $\prod_{j=1}^{k+1}(I(\tilde{a}_j))^{\omega_j}$,

$\prod_{j=1}^{k+1}(F(\tilde{a}_j))^{\omega_j}$ )>

Therefore, considering the above results, we have Eq.(5) for any. This completes the proof.

Especially when $\omega = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$ , then SVNTrLWAA operator reduces to a neutrosophic trapezoid linguistic arithmetic averaging operator for SVNTrLVs.

It is obvious that the SVNTrLWAA operator satisfies the following properties:

(1) **Idempotency** : Let $\tilde{a}_j$ (j=1,2,…,n) be a collection of SVNTrLVs. If $\tilde{a}_j$ (j=1,2,…,n) is equal, i.e $\tilde{a}_j = \tilde{a}$ for j=1,2,…,n, then

NTrFLWAA ($\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n$)= $\tilde{a}$ .

(2) **Boundedness:** Let $\tilde{a}_j$ (j=1,2,…,n) be a collection of SVNTrLVs and

$\tilde{a}_{\min} = \min(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)$ and $\tilde{a}_{\max} = \max(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)$ for j=1,2,…,n, $\tilde{a}_{\min}$ ≤ SVNTrLWAA($\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n$) ≤ $\tilde{a}_{\max}$ then be a collection of SVNTrLVs.

**(3) Monotoncity :** Let $\tilde{a}_j$ (j=1,2,…,n) be a collection of SVNTrLVs. If $\tilde{a}_j \le \tilde{a}_j^*$ for j= 1,2,…,n.Then SVNTrLWAA($\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n$) ≤ SVNTrLWAA($\tilde{a}_1^*, \tilde{a}_2^*, \ldots, \tilde{a}_n^*$).

**Proof.**

**(1)** Since $\tilde{a}_j = \tilde{a}$ for j=1,2,…n, we have

SVNTrLWAA($\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n$) = <[ $s_{\sum_{j=1}^{n}\omega_j\theta(\tilde{a}_j)}$, $s_{\sum_{j=1}^{n}\omega_j\rho(\tilde{a}_j)}$, $s_{\sum_{j=1}^{n}\omega_j\mu(\tilde{a}_j)}$, $s_{\sum_{j=1}^{n}\omega_j\nu(\tilde{a}_j)}$ ], (1- $\prod_{j=1}^{n}(1-T(\tilde{a}_j))^{\omega_j}$, $\prod_{j=1}^{n}(I(\tilde{a}_j))^{\omega_j}$, $\prod_{j=1}^{n}(F(\tilde{a}_j))^{\omega_j}$ >

= <[ $s_{\theta(\tilde{a})\sum_{j=1}^{n}\omega_j}$, $s_{\rho(\tilde{a})\sum_{j=1}^{n}\omega_j}$, $s_{\mu(\tilde{a})\sum_{j=1}^{n}\omega_j}$, $s_{\nu(\tilde{a})\sum_{j=1}^{n}\omega_j}$ ], (1- $(1-T(\tilde{a})^{\sum_{j=1}^{n}\omega_j}$ ), $(I(\tilde{a}))^{\sum_{j=1}^{n}\omega_j}$, $(F(\tilde{a}))^{\sum_{j=1}^{n}\omega_j}$ >

=<[ $s_{\theta(\tilde{a})}, s_{\rho(\tilde{a})}, s_{\mu(\tilde{a})}, s_{\nu(\tilde{a})}$ ],( $T_{\tilde{a}}$ , $I_{\tilde{a}}$ , $F_{\tilde{a}}$ )>

$= \tilde{a}$

**(2)** Since $\tilde{a}_{\min} = \min(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)$ and $\tilde{a}_{\max} = \max(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)$ for j=1,2,…,n, there is $\tilde{a}_{\min} \le \tilde{a}_j \le \tilde{a}_{\max}$. Thus, there exist is $\sum_{j=1}^{n}\omega_j\,\tilde{a}_{\min} \le \sum_{j=1}^{n}\omega_j\,\tilde{a}_j \le \sum_{j=1}^{n}\omega_j\,\tilde{a}_{\max}$. This is $\tilde{a}_{\min} \le \sum_{j=1}^{n}\omega_j\,\tilde{a}_j \le \tilde{a}_{max}$. i.e., $\tilde{a}_{\min} \le$ SVNTrLWAA ($\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n$) $\le \tilde{a}_{\max}$.

**(3)** Since $\tilde{a}_j \le \tilde{a}_j^*$ for j= 1,2,…,n. There is $\sum_{j=1}^{n}\omega_j\,\tilde{a}_j \le \sum_{j=1}^{n}\omega_j\,\tilde{a}_j^*$ Then INTRLWAA($\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n$) $\le$ SVNTrLWAA( $\tilde{a}_1^*$ , $\tilde{a}_2^*$ ,…, $\tilde{a}_n^*$ ).

Thus, we complete the proofs of these properties.

### *4.2 Single valued neutrosophic trapezoid linguistic weighted geometric averaging operator*

**Definition 4.3.** Let : $\tilde{a}_j$=<[ $s_{\theta(\tilde{a}_j)}, s_{\rho(\tilde{a}_j)}, s_{\mu(\tilde{a}_j)}, s_{\nu(\tilde{a}_j)}$ ], ( $T_{\tilde{a}_j}, I_{\tilde{a}_j}, F_{\tilde{a}_j}$ )> (j=1,2,…,n) be a collection of SVNTrLNs. The single valued neutrosophic trapezoid linguistic weighted geometric averaging SVNTrLWGA operator can be defined as follows:

SVNTrLWGA: $\Omega^n \longrightarrow \Omega$

$$\text{SVNTrLWGA}(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \prod_{j=1}^{n}\tilde{a}_j^{\,\omega_j} \tag{8}$$

Where, $\omega_j = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $\tilde{a}_j$ (j= 1,2,…,n), $\omega_j \in [0,1]$ and $\sum_{j=1}^{n}\omega_j = 1$.

**Theorem 4.4 : :** $\tilde{a}_j$ =<[ $s_{\theta(\tilde{a}_j)}$ , $s_{\rho(\tilde{a}_j)}$ , $s_{\mu(\tilde{a}_j)}$ , $s_{\nu(\tilde{a}_j)}$ ], ( $T_{\tilde{a}_j}$ , $I_{\tilde{a}_j}$ , $F_{\tilde{a}_j}$ )> (j=1,2,…,n) be a collection of SVNTrLS, Then by Equation (8) and the operational laws in Definition 3.3 , we have the following result

$$\text{SVNTrLWGA}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n) = <[s_{\prod_{j=1}^{n}\theta^{\omega_j}(\tilde{a}_j)}, s_{\prod_{j=1}^{n}\rho_i^{\omega_j}(\tilde{a}_j)}, s_{\prod_{j=1}^{n}\mu_i^{\omega_j}(\tilde{a}_j)}, s_{\prod_{j=1}^{n}\nu^{\omega_j}(\tilde{a}_j)}],$$
$$(\prod_{j=1}^{n}(T(\tilde{a}_j))^{\omega_j}, 1-\prod_{j=1}^{n}(1-I(\tilde{a}_j))^{\omega_j}, 1-\prod_{j=1}^{n}(1-F(\tilde{a}_j))^{\omega_j} > \quad (9)$$

Where, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $\tilde{a}_j$ (j= 1,2,…,n), $\omega_j \in [0,1]$and $\sum_{j=1}^{n}\omega_j$ =1.

By a similar proof manner of theorem 4.2, we can also give the proof of theorem 4.4 (omitted).

Especially when $\omega = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})^T$, then SVNTrLWGA operator reduces to a single valued neutrosophic trapezoid linguistic geometric averaging operator for SVNTrLVs.

It is obvious that the SVNTrLWGA operator satisfies the following properties:

(1) **Idempotency** : Let $\tilde{a}_j$ (j=1,2,…,n) be a collection of SVNTrLVs. If $\tilde{a}_j$ (j=1,2,…,n) is equal, i.e $\tilde{a}_j = \tilde{a}$ for j=1,2,…,n, then

SVNTrLWGA ( $\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n$ )= $\tilde{a}$ .

(2) **Boundedness:** Let $\tilde{a}_j$ (j=1,2,…,n) be a collection of SVNTrLVs and

$\tilde{a}_{\min} = \min(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)$ and $\tilde{a}_{\max} = \max(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n)$ for j=1,2,…,n, $\tilde{a}_{\min} \le$ SVNTrFLWGA ( $\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n$ ) $\le \tilde{a}_{\max}$ then be a collection of SVNTrLVs.

**(3) Monotonity :** Let $\tilde{a}_j$ (j=1,2,…,n) be a collection of SVNTrLVs. If $\tilde{a}_j \le \tilde{a}_j^*$ for j= 1,2,…,n. Then SVNTrLWGA ( $\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n$ ) $\le$ SVNTrLWGA ( $\tilde{a}_1^*$ , $\tilde{a}_2^*$ ,…, $\tilde{a}_n^*$ ).

Since the proof process of these properties is similar to the above proofs, we do not repeat it here.

## 5.DECISION –MAKING METHOD BY SVNTrLWAA AND SVNTrLWGA OPERATORS.

This section presents a method for multi attribute decision making problems based on the SVNTrLWAA and SVNTrLWGA operators and the score, accuracy, and certainty functions of SVNTrLVs under single valued neutrosophic trapezoid linguistic variable environment.

In a multiple attribute decision-making problem, assume that A={$A_1, A_2, A_3,\ldots, A_m$} is a setoff alternatives and C ={ $C_1, C_2,\ldots, C_n$} is a set of attributes. The weight vector of the attributes $C_j$ (j=1,2,…,n), entered by the decision maker, is $\omega=(\omega_1,\omega_2,\ldots,\omega_n)^T$ where $\omega_j \in$ [0,1]and $\sum_{j=1}^{n}\omega_j$ =1.In the decision process, the evaluation information of the alternatives $A_i$ (i=1,2,…,m) with respect to the attribute $C_j$ (j=1,2,…,n)is represented by the form of an SVNTrLS:

$$A_i=\{[s_{\theta_i(C_j)}, s_{\rho_i(C_j)}, s_{\mu_i(C_j)}, s_{\nu_i(C_j)}], (T_{A_i}(C_j), I_{A_i}(C_j), F_{A_i}(C_j)) \mid C_j \in \mathbf{C}\}$$

Where $[s_{\theta_i(C_j)}, s_{\rho_i(C_j)}, s_{\mu_i(C_j)}, s_{\nu_i(C_j)}] \in \hat{S}$, $T_{A_i}(C_j) \in$ [0.1], $I_{A_i}(C_j) \in$ [0.1], and $F_{A_i}(C_j) \in$ [0.1] with the condition $0 \le T_{A_i}(C_j) + I_{A_i}(C_j) + F_{A_i}(C_j) \le 3$ for j=1,2,..,n and i=1,2,…,m. For convenience, an SVNTrLV is a SVNTrLS is denoted by

$\tilde{d}_{ij}$= <[$s_{\theta_{ij}}, s_{\rho_{ij}}, s_{\mu_{ij}}, s_{\nu_{ij}}$], ($T_{ij}, I_{ij}, F_{ij}$)> (i=1=1,2,..m) j=1,2,…,n) thus, one can establish a single valued neutrosophic trapezoid linguistic decision matrix D = $(\tilde{d}_{ij})_{m\times n}$.

Using the SVNTrLWAA or SVNTrLWGA operator, we now formulate an algorithm to solve multiple attribute decision making problem with single valued neutrosophic linguistic information.

**Step1 :** Calculate the individual overall value of the SVNTrLV $\tilde{d}_i$ for $A_i$ (i=1,2,…,m) by the following aggregation formula:

$$\tilde{d}_i = \langle [s_{\theta_i}, s_{\rho_i}, s_{\mu_i}, s_{\nu_i}], (T_i, I_i, F_i) \rangle$$

$$= \text{SVNTrLWAA}(\tilde{d}_{i1}, \tilde{d}_{i2}, \ldots, \tilde{d}_{in})$$

$$= \langle [s_{\sum_{j=1}^{n}\omega_j\theta_{ij}}, s_{\sum_{j=1}^{n}\omega_j\rho_{ij}}, s_{\sum_{j=1}^{n}\omega_j\mu_{ij}}, s_{\sum_{j=1}^{n}\omega_j\nu_{ij}}], (1-\prod_{j=1}^{n}(1-T_{ij})^{\omega_j}, \prod_{j=1}^{n}(I_{ij})^{\omega_j}, \prod_{j=1}^{n}(F_{ij})^{\omega_j}) \rangle \quad (10)$$

$$\tilde{d}_i = \langle [s_{\theta_i}, s_{\rho_i}, s_{\mu_i}, s_{\nu_i}], (T_i, I_i, F_i) \rangle$$

$$= \text{SVNTrLWGA}(\tilde{d}_{i1}, \tilde{d}_{i2}, \ldots, \tilde{d}_{in}) = \langle [s_{\prod_{j=1}^{n}\theta_{ij}^{\omega_j}}, s_{\prod_{j=1}^{n}\rho_{ij}^{\omega_j}}, s_{\prod_{j=1}^{n}\mu_{ij}^{\omega_j}}, s_{\prod_{j=1}^{n}\nu_{ij}^{\omega_j}}], (\prod_{j=1}^{n}(T_{ij})^{\omega_j}, 1-\prod_{j=1}^{n}(1-I_{ij})^{\omega_j}, 1-\prod_{j=1}^{n}(1-F_{ij})^{\omega_j} \rangle \quad (11)$$

**Step 2 :**Calculate the score function E($\tilde{d}_i$) (i=1,2,…,m) (accuracy function H($\tilde{d}_i$) and certainty function C($\tilde{d}_i$) by applying Eq,(1) (Eqs.(2) and (3)).

**Step 3 :**Rank the alternatives according to the values of E($\tilde{d}_i$) (H($\tilde{d}_i$) and C($\tilde{d}_i$)) ((i=1,2,…,m) by the ranking method in Definition 3.5, and then select the best one(s).

**Step 4 :** End

## 6.ILLUSTARTIVE EXAMPLE

An illustrative example about investment alternatives problem adapted from [18] is used to demonstrate the applications of the proposed decision –making method under single valued neutrosophic trapezoid linguistic environment. There is an investment company, which wants to invest a sum of money in the best option. To invest the money, there is a panel with four possible alternatives: (1) $A_1$ is car company; (2) $A_2$ is food company; (3) $A_3$ is a computer company; (4) $A_4$ is an arms company. The investement company must take a decision according to the three attributes: (1) $C_1$ is the risk; (2) $C_2$ is the growth; (3) $C_3$ is a the environmental impact. The weight vector of the attributes is ω= $(0.35, 0.25,0.4)^T$.The expert evaluates the four possible alternatives of $A_i$ (i=1,2,3,4) with respect to the three attributes of $C_j$ (j=1,2,3), where the evaluation information is expressed by the form of SVNTrLV values under the linguistic term set S={$s_0$=extremely poor, $s_1$=very poor, $s_2$= poor, $s_3$= medium, $s_4$= good, $s_5$= very good, $s_6$= extremely good}.

The evaluation information of an alternative $A_i$ (i=1,2, 3,4) with respect to an attribute $C_j$ (j=1, 2, 3) can be given by the expert. For example, the SVNTrL value of an alternative $A_1$ with respect to an attribute $C_1$ is given as <$[s_{1.4}, s_{2.7}, s_3, s_{5.3}]$ (0.4 ,0.2, 0.3)> by the expert, which indicates that the mark of the alternative $A_1$ with respect to the attribute $C_1$ is about the trapezoid linguistic value $[s_{1.4}, s_{2.7}, s_3, s_{5.3}]$ with the satisfaction degree 0.4 indeterminacy degree 0.2, and dissatisfaction degree 0.3, similarly, the four possible alternatives with respect to the three attributes can be evaluated by the expert, thus we can obtain the following single valued neutrosophic trapezoid linguistic decision matrix:

D=$(d_{ij})_{m\times n}$

$$\begin{bmatrix} <([s_{1.8},s_{3.4},s_{4.5},s_{5.5}],(0.7,0.3,0.4) & <([s_{1.3},s_{2.3},s_{4.4},s_{5.4}],(0.4,0.2,0.4) & <([s_{0.8},s_{2.2},s_{3.8},s_{5.1}],(0.3,0.4,0.4) \\ <([s_{1.4},s_{2.8},s_{3.8},s_{5.1}],(0.4,0.3,0.4) & <([s_{1.5},s_{2.5},s_{4.5},s_{5.5}],(0.4,0.5,0.4) & <([s_{1.1},s_{2.1},s_{1.4},s_{5.4}],(0.6,0.6,0.5) \\ <([s_{2.1},s_{3.2},s_{4.5},s_{5.7}],(0.6,0.2,0.4) & <([s_{1.8},s_{2.8},s_{4.4},s_{5.5}],(0.6,0.5,0.4) & <([s_{1.4},s_{2.7},s_{3},s_{5.3}],(0.6,0.3,0.5) \\ <([s_{1.5},s_{3.1},s_{4.7},s_{5.9}],(0.8,0.2,0.5) & <([s_{1.7},s_{2.9},s_{4},s_{5.4}],(0.7,0.3,0.4) & <([s_{1.2},s_{1.8},s_{4.3},s_{5.3}],(0.6,0.7,0.4) \end{bmatrix}$$

The proposed decision –making method can handle this decision –making problem according to the following calculation steps:

**Step1:** By applying Eq.(10), we can obtain the individual overall value of the SVNTrLV $\tilde{d}_i$ for $A_i$ (i=1,2,.3,4).

$\tilde{d}_1$=<[ $s_{1.275}$, $s_{2.645}$, $s_{4.195}$, $s_{5.315}$ ],(0.4933, 0.1397, 0.400)>

$\tilde{d}_2$=<[ $s_{1.305}$, $s_{2.445}$, $s_{3.015}$, $s_{5.320}$ ],(0.4898, 0.2612, 0.4373)>

$\tilde{d}_3$=<[ $s_{1.745}$, $s_{2.900}$, $s_{3.875}$, $s_{5.490}$ ],(0.600, 0.2460, 0.4373)>

$\tilde{d}_4$=<[ $s_{1.430}$, $s_{2.530}$, $s_{4.365}$, $s_{5.535}$ ],(0.7079, 0.4379, 0.4325)>

**Step 2**: By applying Eq.(1) , we can obtain the score value of E($\tilde{d}_1$) (i=1,2,3,4)

E($\tilde{d}_1$)= $s_{2.1931}$, E($\tilde{d}_2$) = $s_{1.8040}$, E($\tilde{d}_3$)= $s_{2.2378}$ , E($\tilde{d}_4$)= $s_{2.1224}$

**Step 3 :** since E($\tilde{d}_3$) > E($\tilde{d}_4$) > E($\tilde{d}_1$) > E($\tilde{d}_2$), the ranking order of four alternatives . Therefore, we can see that the alternative $A_3$ is the best choice among all the alternative.

On the other hand, we can also utilize the SVNTrLWGA operator as the following computational steps:

**Step 1**:By applying Eq.(11) , we can obtain the individual overall value of the SVNTrLV $\tilde{d}_i$ for $A_i$ (i=1,2,.3,4)

$\tilde{d}_1$=<[ $s_{1.200}$, $s_{2.591}$, $s_{4.182}$, $s_{5.312}$ ],(0.4337, 0.3195, 0.4000)>

$\tilde{d}_2$=<[ $s_{1.293}$, $s_{2.426}$, $s_{2.659}$, $s_{5.317}$ ],(0.4704, 0.4855, 0.4422)>

$\tilde{d}_3$=<[ $s_{1.718}$, $s_{2.892}$, $s_{3.805}$, $s_{5.487}$ ],(0.6, 0.3527, 0.4422)>

$\tilde{d}_4$=<[ $s_{1.416}$, $s_{2.453}$, $s_{4.356}$, $s_{5.528}$ ],(0.690, 0.477, 0.437)>

**Step2**: By applying Eq.(1), we can obtain the score value of E( $\tilde{d}_i$ ) (i=1,2,3,4)

E( $\tilde{d}_1$ )= $s_{1.8978}$ , E( $\tilde{d}_2$ ) = $s_{1.5035}$ , E( $\tilde{d}_3$ )= $s_{2.1146}$ , E( $\tilde{d}_4$ )= $s_{2.0354}$

**Step 3** : since E( $\tilde{d}_3$ ) > E( $\tilde{d}_4$ ) > E( $\tilde{d}_1$ ) > E( $\tilde{d}_2$ ), the ranking order of four alternatives . Therefore, we can see that the alternative $A_3$ is the best choice among all the alternative.

Obviously, we can see that the above two kinds of ranking orders of the alternatives are the same and the most desirable choice is the alternative $A_3$ .

## 7-CONCLUSION

In this paper, we have proposed some single valued neutrosophic trapezoid linguistic operators such as single valued neutrosophic trapezoid linguistic weighted arithmetic averaging SVNTrLWAA and single valued neutrosophic trapezoid linguistic weighted geometric averaging SVNTrLWGA operator. We have studied some desirable properties of the proposed operators, such as commutativity, idempotency and monotonicity, and applied the SVNTrLWAA and SVNTrLWGA operator to decision making with single valued neutrosophic trapezoid linguistic information. Finally, an illustrative example has been given to show the developed operators.

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